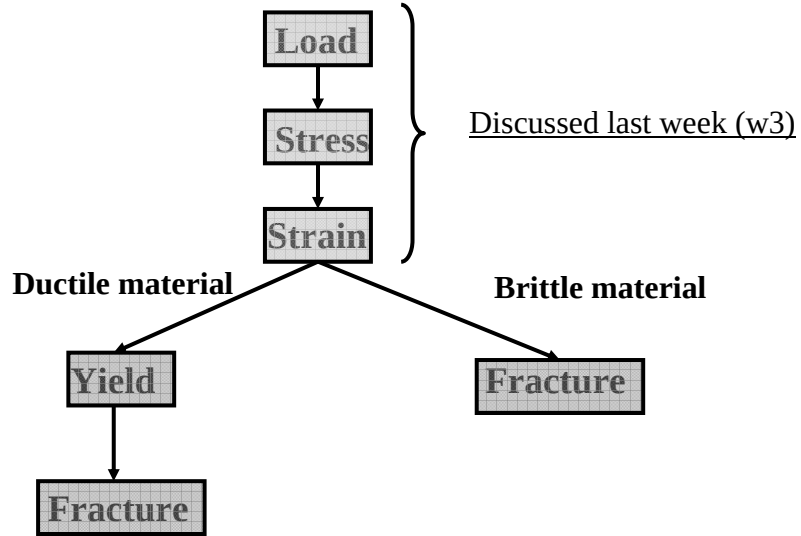


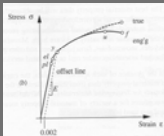
Path to static failure of machine components



Dr. P. Buyung Kosasih, Spring 2008

**Name some of ductile and brittle materials
Commonly used in engineering applications**

DUCTILE



- Steel
- Plastics
- Rubbers

BRITTLE (S_v close to S_u)



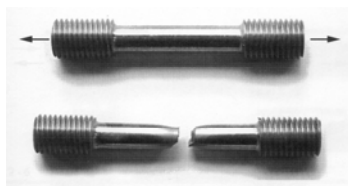
- **Cast iron**
- **Hardened tool steel**
- **Concrete**
- **Woods**
- **Ceramics**

Lecture 4 Failure under static load

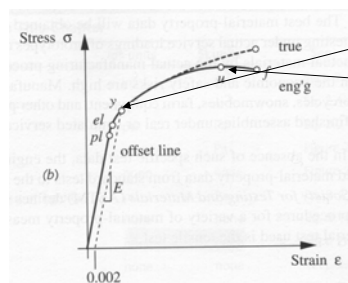
- Failure of ductile material
(Ch. 5 p. 239 – 253)
- Failure of brittle material
(Ch. 5 p.254 - 261)

3

Most common test to determine material strength is Tensile test.



What do we know from tensile test ?



- Yield tensile stress (S_y)
- Ultimate tensile stress (S_u)

4

Mechanical properties of some material are listed in Appendix C (Norton).

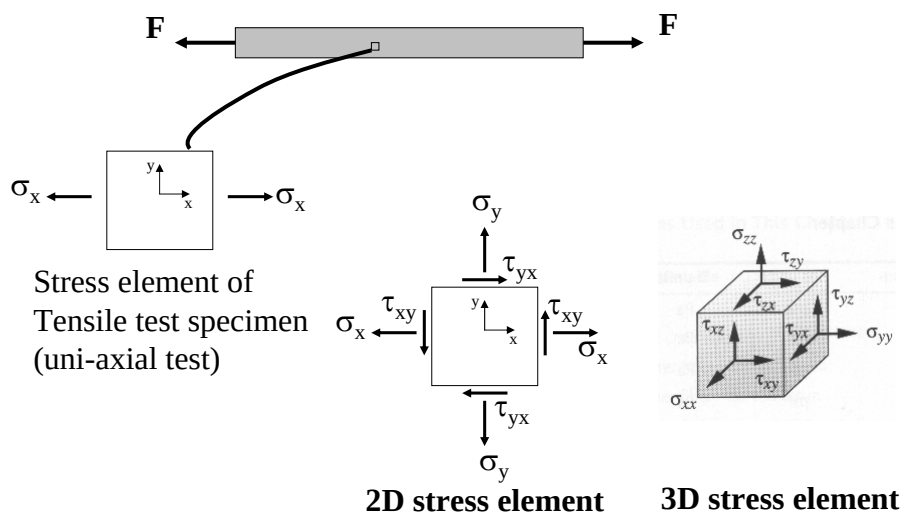
Table C-4 Mechanical Properties for Some Wrought- and Cast-Copper Alloys
Data from INCO.^{*} Approximate Values. Consult Material Manufacturers for More Accurate Information

Copper Alloy	Condition	Tensile Yield Strength (0.2% offset)		Ultimate Tensile Strength		Elongation over 2 in %	Brinell or Rockwell Hardness
		kpsi	MPa	kpsi	MPa		
CA110—Pure Copper	strip annealed	10	69	32	221	45	40HRF
	spring temper	50	345	55	379	4	60HRB
CA170—Beryllium Copper	strip annealed plus age	145	1 000	165	1 138	7	35HRC
	hard plus age	170	1 172	190	1 310	3	40HRC
CA220—Commercial Bronze	strip annealed	10	69	37	255	45	53HRF
	spring temper	62	427	72	496	3	78HRB
CA230—Red Brass	strip annealed	15	103	40	276	50	50HB
	hard temper	60	414	75	517	7	135HB
CA260—Cartridge Brass	strip annealed	11	76	44	303	66	54HRF
	spring temper	65	448	94	648	3	91HRB
CA270—Yellow Brass	strip annealed	14	97	46	317	65	58HRF
	spring temper	62	427	91	627	30	90HRB
CA510—Phosphor Bronze	annealed	19	131	47	324	64	73HRF
	spring temper	80	552	100	689	4	95HRB
CA614—Aluminum Bronze	soft	45	310	82	565	40	84HRB
	hard	60	414	89	614	32	87HRB
CA655—High Silicon Bronze	annealed	21	145	56	386	63	76HRF
	spring temper	62	427	110	758	4	97HRB
CA675—Manganese Bronze	soft	30	207	65	448	33	65HRB
	half-hard	60	414	84	579	19	90HRB
Leaded-Tin Bronze	as cast	19	131	34	234	18	60HB
	as cast	20	138	50	345	40	85HB
Nickel-Tin Bronze	as cast	20	138	50	345	40	85HB
	cast and heat treated	55	379	85	586	10	180HB

^{*} Properties of Some Metals and Alloys, International Nickel Co., Inc., New York.

5

How the yield tensile stress related to failure of part ?



6

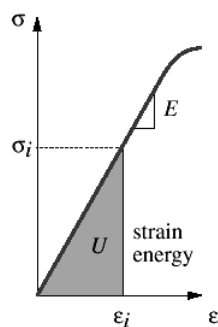
The answer is :

Using

- For ductile materials
 - a1) The von Mises-Hencky theory (distortion energy theory)
 - a2) The maximum Shear-Stress theory
 - a3) Maximum Normal-Stress theory
- For brittle materials
 - b1) Maximum Normal-Stress theory
 - b2) The Coulomb-Mohr theory
 - b3) The Modified-Mohr theory

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a1) The von Mises-Hencky or Distortion-Energy Theory



The **total strain energy stored** per unit volume in the material

$$U = \left(\frac{1}{2}\right) \sigma \varepsilon \text{ (J/m}^3\text{)}$$

Extending to 3D stress state

$$U = \frac{1}{2} (\sigma_1 \varepsilon_1 + \sigma_2 \varepsilon_2 + \sigma_3 \varepsilon_3)$$

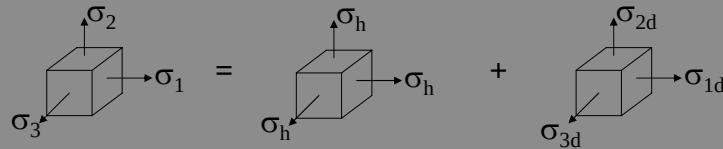
In terms of principle normal stresses:

$$U = \frac{1}{2E} [\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\nu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_1\sigma_3)]$$

8

Component of strain energy = hydrostatic + distortion components

$$U = U_h + U_d$$



It is shown that the hydrostatic stress is (page 243)

$$\sigma_h = (\sigma_1 + \sigma_2 + \sigma_3) / 3$$

9

Where U_h is due to hydrostatic loading i.e. equal principal stresses (volume change without changing shape $\rightarrow$ no shear)

$$U_h = \frac{1}{2E} [\sigma_h^2 + \sigma_h^2 + \sigma_h^2 - 2\nu(\sigma_h\sigma_h + \sigma_h\sigma_h + \sigma_h\sigma_h)]$$

$$U_h = \frac{1-2\nu}{6E} [\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + 2(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_1\sigma_3)]$$

U_d is due to (angular) distortion (shape change $\rightarrow$ shear)

$$U_d = U - U_h$$

$$U_d = \frac{1+\nu}{3E} [\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \sigma_1\sigma_2 - \sigma_2\sigma_3 - \sigma_1\sigma_3]$$

10

From tensile test, the uni-axial stress state at yield gives $\sigma_1 = S_y$,

$\sigma_2 = \sigma_3 = 0$. So,

$$U_{d@yield} = \frac{1+\nu}{3E} S_y^2$$

Comparing this with general stress state, so to prevent failure

$$U_{d@yield} > U_d$$

or

$$S_y > \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \sigma_1\sigma_2 - \sigma_2\sigma_3 - \sigma_1\sigma_3}$$

von Mises stress
or
effective stress
or
equivalent stress
 σ'

11

The von Mises (effective) stress, σ' , can also be expressed in terms of applied stresses.

$$\sigma' > \sqrt{\frac{(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)}{2}}$$

For 2D case:

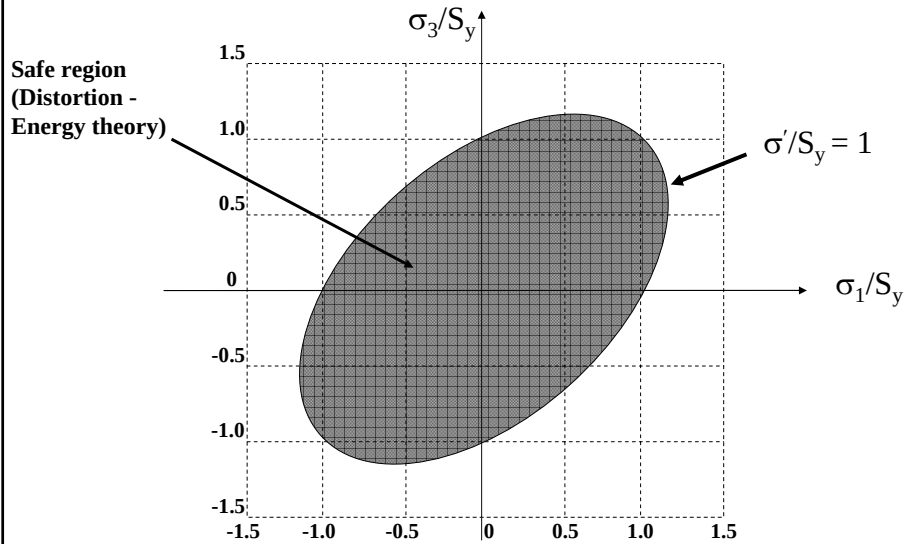
$$\sigma' = \sqrt{\sigma_1^2 - \sigma_1\sigma_3 + \sigma_3^2}$$

$$\sigma' = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + 3\tau_{xy}^2}$$

Factor of safety : $N = S_y / \sigma'$

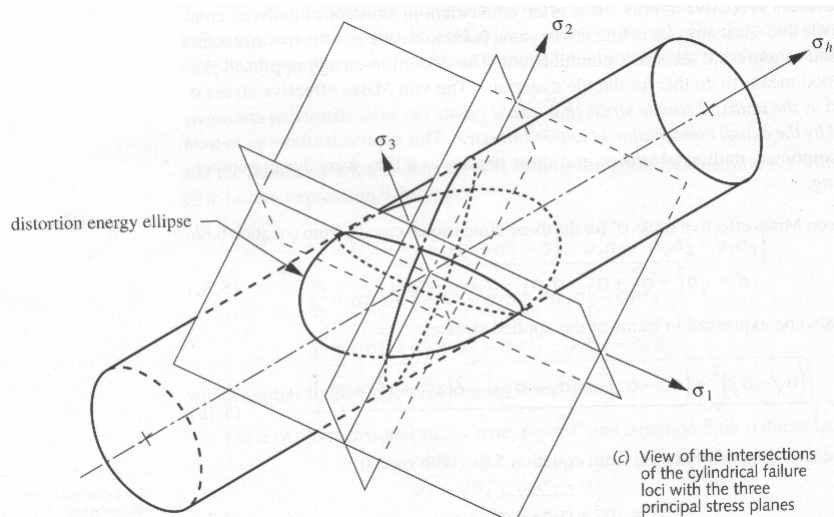
12

2D visualization of the distortion energy failure theory.



13

3D visualization of the distortion energy failure theory.



14

a2) The maximum shear stress theory

The theory states that *failure occurs when the maximum shear stress in a part exceeds the shear stress in a tensile specimen at yield (one-half of the tensile yield strength).*

$$S_{ys} = 0.5 S_y$$

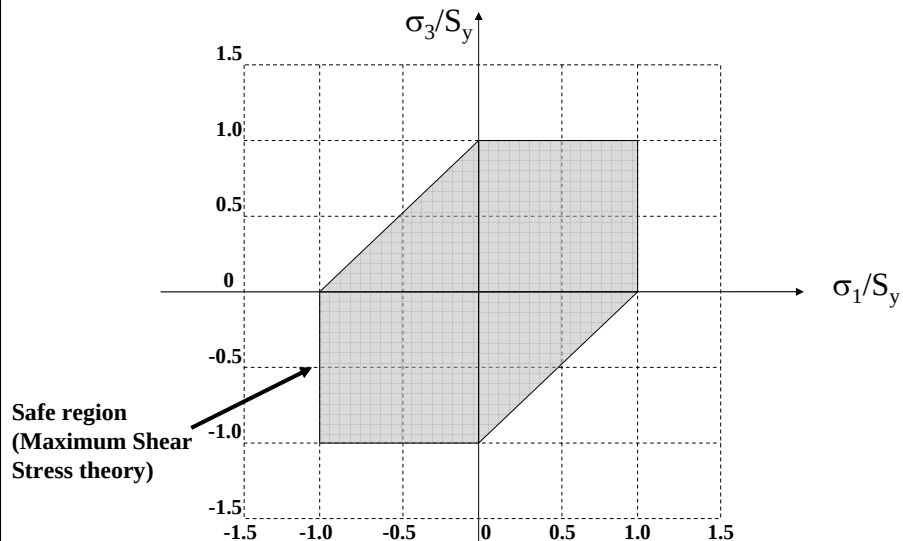
i.e. part fails when

$$\tau_{\max} > S_{ys}$$

Factor of safety : $N = S_{ys} / \tau_{\max}$

15

2D visualization of the maximum shear stress failure theory.



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a3) Maximum normal stress theory

The theory states that
failure will occur when the normal stress in the specimen reaches some limit on normal strength such as tensile yield strength or ultimate tensile strength.

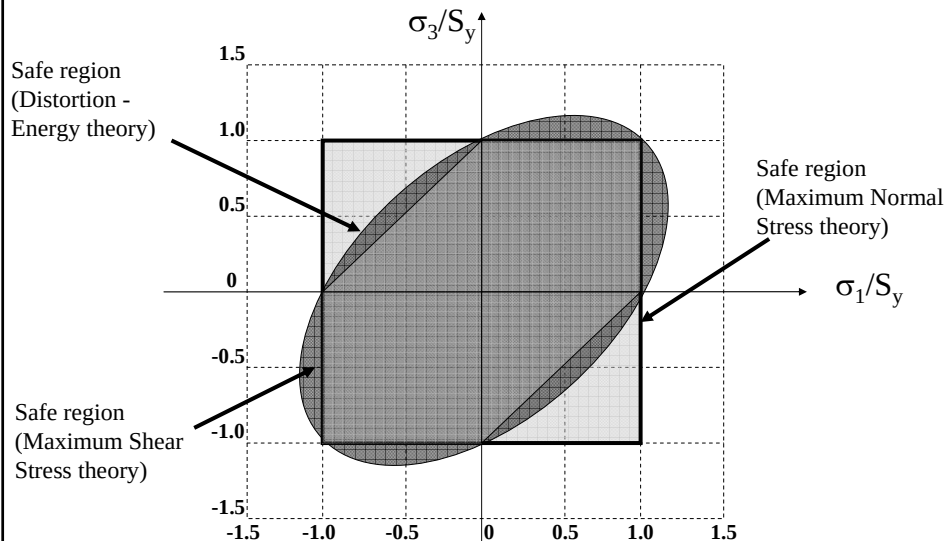
i.e. part fails when

$$\{\sigma_1, \sigma_2, \sigma_3\} > S_y$$

Factor of safety : $N = S_y / \max\{\sigma_1, \sigma_2, \sigma_3\}$

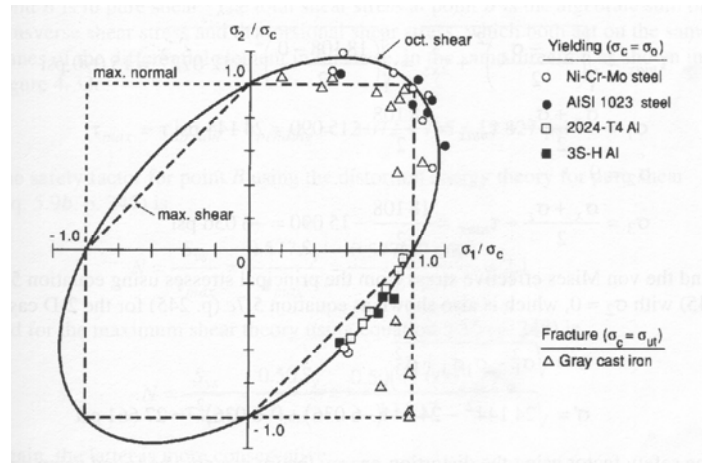
17

2D visualization of the three (ductile) failure theories.



18

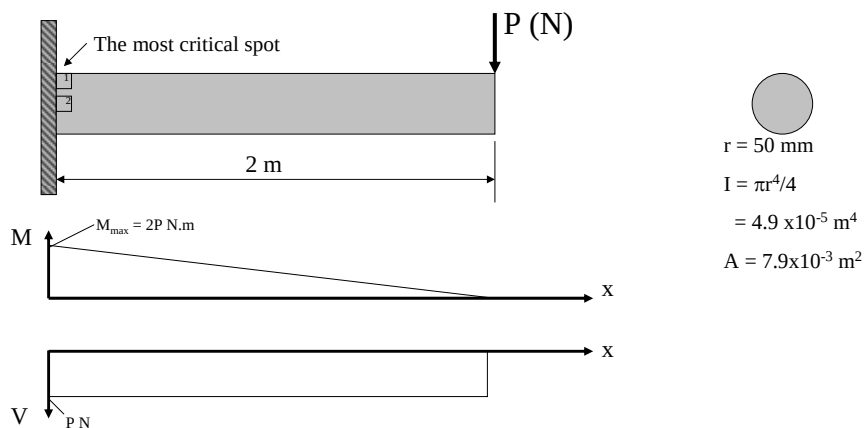
Which theory should we choose ?



19

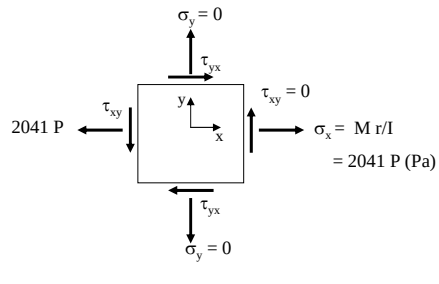
Application:

For the loaded cantilever beam made of a material with tensile yield strength of 30 MPa. What is the maximum P that the beam can support without permanent yield ?



20

Element 1



$$\sigma_1 = \frac{2041P + 0}{2} + \sqrt{\left(\frac{2041P - 0}{2}\right)^2} = 2041P$$

$$\sigma_2 = \frac{2041P + 0}{2} - \sqrt{\left(\frac{2041P - 0}{2}\right)^2} = 0$$

$$\tau_{\max} = \frac{\sigma_1 - \sigma_2}{2} = 1020P$$

21

Based on the distortion energy theory $\sigma' = \sqrt{\sigma_1^2 - \sigma_1\sigma_3 + \sigma_3^2}$

The von Mises stress at point 1 is $\sigma' = 2041P$

For

$$\sigma' = 2041P$$

$$S_y > \sigma'$$

$$30 \times 10^6 > 2041P$$

$$P < 14.7 \text{ kN}$$

22

Based on the maximum shear stress theory

$$S_{ys} = 0.5 S_y = 15 \text{ Mpa}$$

For

$$\tau_{\max} = 1020 P < S_{ys} = 15 \times 10^6$$

$$P < 14.7 \text{ kN}$$

Based on the maximum normal stress theory

$$P < 14.7 \text{ kN}$$

(verify yourself)

23

Element 2

$$\sigma_1 = \frac{0+0}{2} + \sqrt{\left(\frac{0-0}{2}\right)^2 + (168.8P)^2} = 168.8 P$$
$$\sigma_2 = \frac{0+0}{2} - \sqrt{\left(\frac{0-0}{2}\right)^2 + (168.8P)^2} = -168.8 P$$
$$\tau_{\max} = \frac{\sigma_1 - \sigma_2}{2} = 168.8 P$$

24

Based on the distortion energy theory

The von Mises stress at point 2 is $\sigma' = 292.4 P$

For

$$\sigma' = 292.4 P$$

$$S_y > \sigma'$$

$$30 \times 10^6 > 292.4 P$$

$$P < 308 \text{ kN}$$

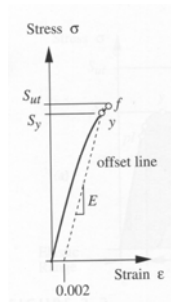
25

So the answer to the question “What is the maximum P that the beam can support without permanent yielding ?”

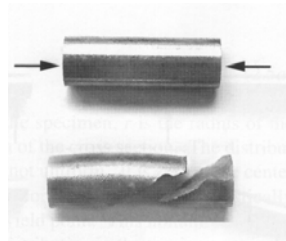
$$P < 14.7 \text{ kN}$$

26

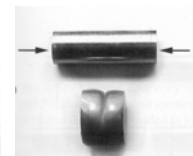
Failure mechanisms of brittle materials



On tension fracture is due to normal stress alone



On compression fracture is due to combination of normal stress and shear stress.



Ductile materials do not fracture on compression

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Characteristics of brittle materials

- Failure mechanisms.
In tension : due to normal stress alone
In compression : due to combination of normal and shear stress
- Their yield strength (S_y) and ultimate strength (S_u) are almost identical. So failure of brittle materials is normally associated with fracture rather than yield. And strength refers to S_u .
- S_{ut} may be equal or not to S_{uc} . When $S_{uc} > S_{ut}$, the materials are said to be **uneven materials**, or else they are known as **even materials**.
- Their shear strength (S_{us}) can be greater than their tensile strength (S_{ut}) unlike ductile materials where $S_{us} \cong 0.5 S_{ut}$

28

Failure theories for brittle materials

- Even materials:

b1) Maximum Normal-Stress Theory

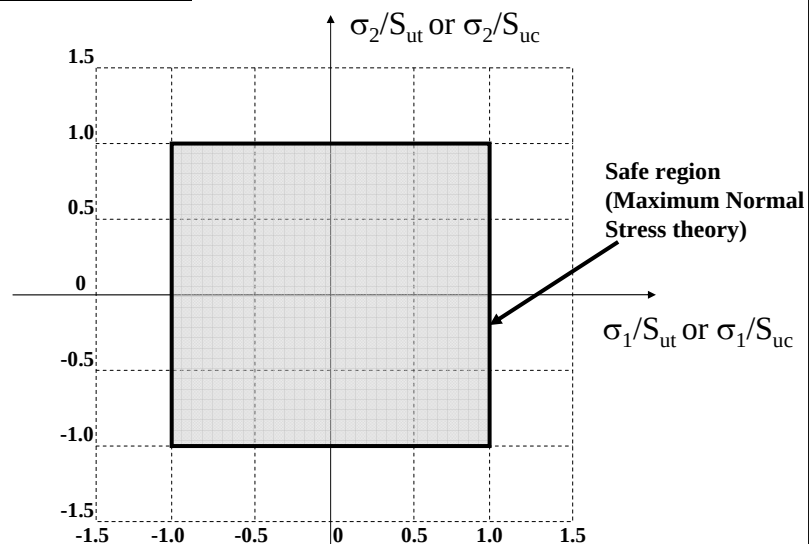
- Uneven materials:

b2) The Coulomb-Mohr Theory

b3) The Modified-Mohr Theory

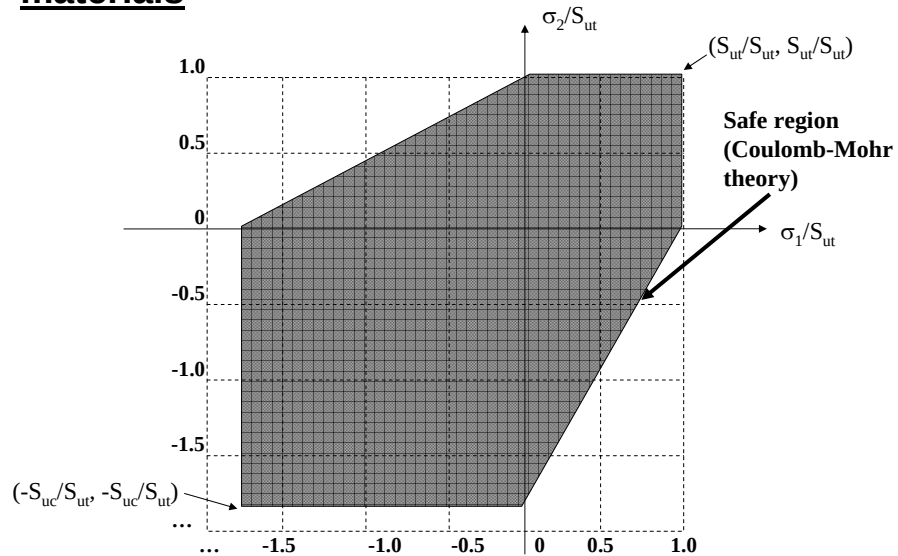
29

b1) Maximum Normal-Stress Theory for even materials



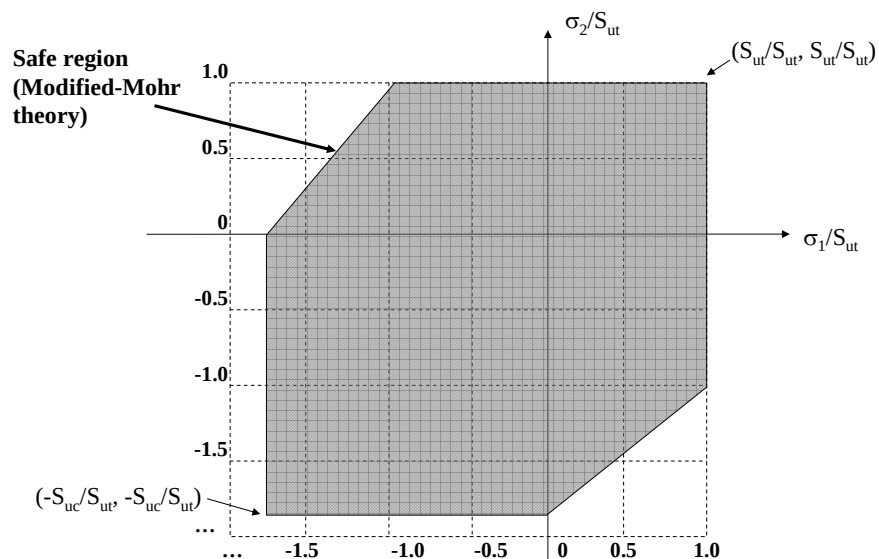
30

b2) The Coulomb-Mohr theory for uneven materials



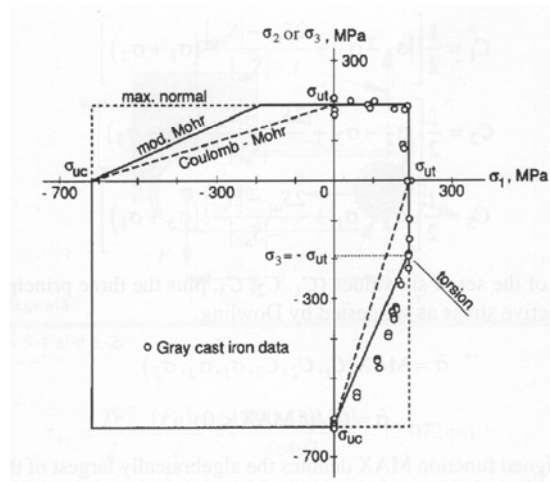
31

b3) Modified-Mohr theory for uneven materials



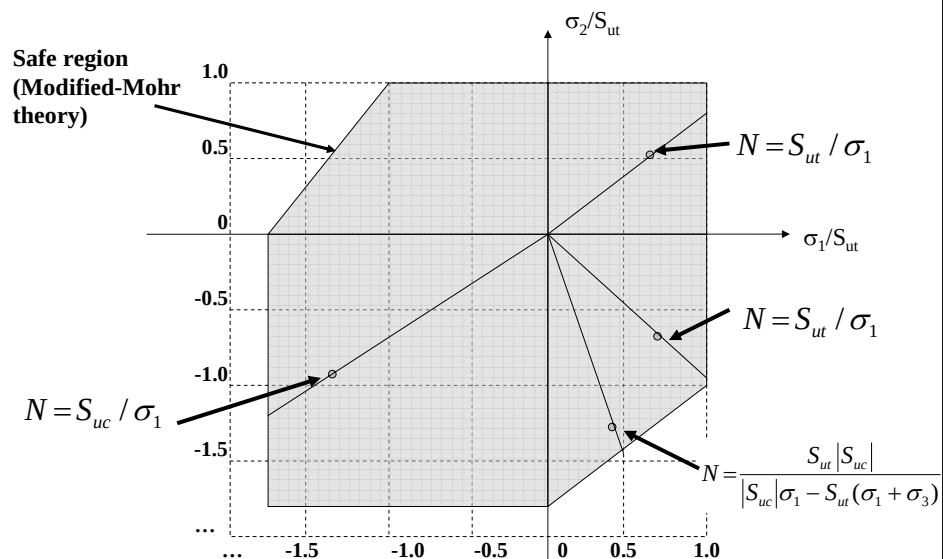
32

Which theory should we use for uneven materials?



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Calculation of factor of safety



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Modified-Mohr effective stress by Dowling (for general 3D case and usually programmed)

$$N = \frac{S_{ut}}{\tilde{\sigma}}$$

$$\tilde{\sigma} = \max(C_1, C_2, C_3, \sigma_1, \sigma_2, \sigma_3)$$

$$C_1 = \frac{1}{2} \left[|\sigma_1 - \sigma_2| + \frac{2S_{ut} - |S_{uc}|}{-|S_{uc}|} (\sigma_1 + \sigma_2) \right]$$

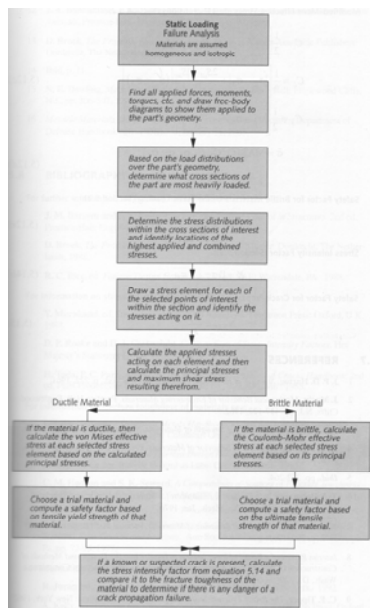
$$C_2 = \frac{1}{2} \left[|\sigma_2 - \sigma_3| + \frac{2S_{ut} - |S_{uc}|}{-|S_{uc}|} (\sigma_2 + \sigma_3) \right]$$

$$C_3 = \frac{1}{2} \left[|\sigma_3 - \sigma_1| + \frac{2S_{ut} - |S_{uc}|}{-|S_{uc}|} (\sigma_3 + \sigma_1) \right]$$

$$\tilde{\sigma} = 0 \text{ if } \max < 0$$

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Summary



Lecture 1
Static loading
analysis

Lecture 3
Stress analysis of
statically loaded
parts

Lecture 4
Failure analysis of
stressed parts

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Type of loadings

Statically loaded parts

Dynamically loaded parts

Lecture 5 : Fatigue failure