

ENGG252 : Engineering Fluid Mechanics

Week 6

Fluid Kinematics

Cengel and Cimbala – Chapter 4 (4-1 to 4-5)

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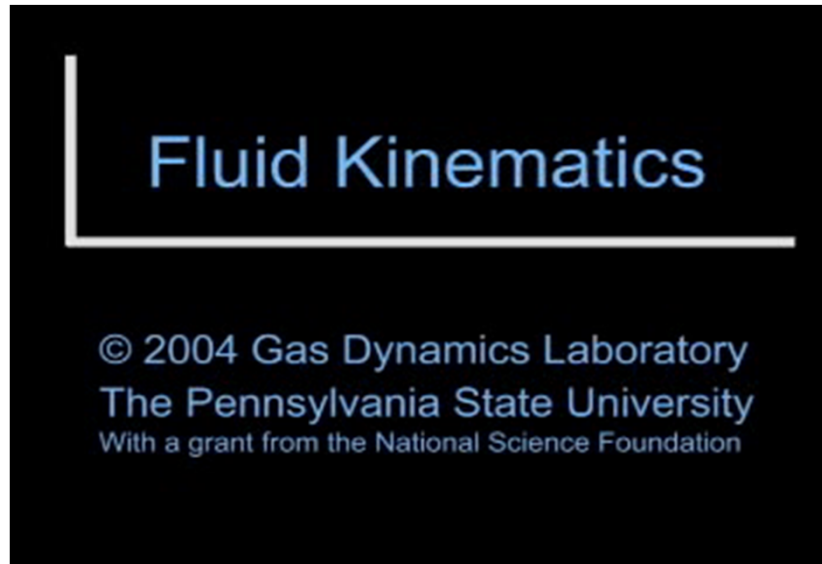
Recap from Week 4

In last weeks' lecture we covered

- Hydrostatic forces on curved surfaces
 - Analyse in the x and y direction using plane surfaces
 - Include the weight of the fluid in the element
- Buoyancy and stability
 - Floating / neutrally buoyant / sinking bodies
- Rigid-body motion
 - Accelerated bodies
 - Rotational bodies

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ENGG252 : Fluid Kinematics



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Agenda

Fluid Kinematics deals with the motion of fluids without considering the forces and moments which create the motion.

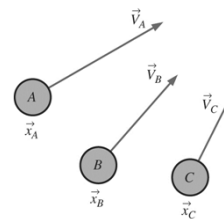
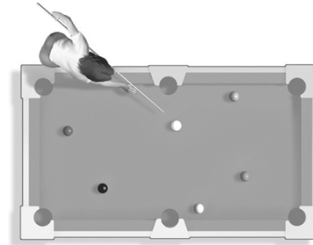
This week

- Lagrangian and Eularian descriptions
- Flow visualisation
 - Streamlines, pathlines, timelines etc
- Plots of fluid flow data
- Fundamental kinematic properties of fluid motion and deformation
 - Linear and shear strain rate
 - Vorticity and rotationality

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Lagrangian Description

- Track the position and velocity of individual particles in a fluid flow.
- Based on Newton's laws of motion.
- Difficult to use for practical flow analysis.
 - Fluids are composed of *billions* of molecules.
 - Interaction between molecules hard to describe/model.
- However, can be useful for specialized applications
 - Sprays, particles, bubble dynamics, rarefied gases.
 - Coupled Eulerian-Lagrangian methods.



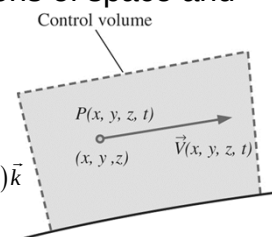
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Eulerian Description

- Eulerian description of fluid flow: a **flow domain** or **control volume** is defined by which fluid flows in and out.
- No need to keep track of position and velocity of a mass of particles
- We define **field variables** which are functions of space and time.
 - Pressure field, $P = P(x, y, z, t)$
 - Velocity field, $\vec{V} = \vec{V}(x, y, z, t)$

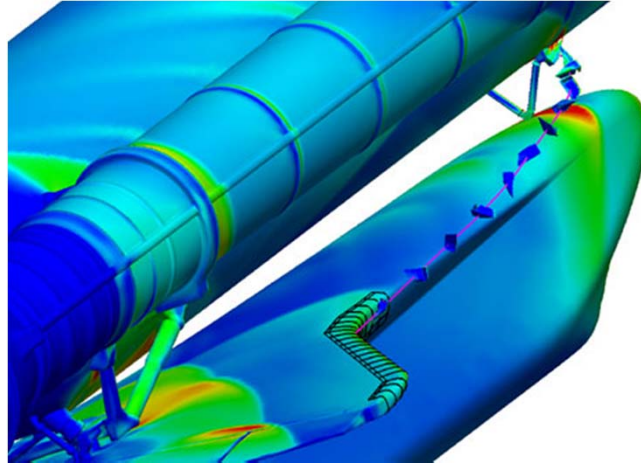
$$\vec{V} = u(x, y, z, t)\vec{i} + v(x, y, z, t)\vec{j} + w(x, y, z, t)\vec{k}$$
 - Acceleration field, $\vec{a} = \vec{a}(x, y, z, t)$

$$\vec{a} = a_x(x, y, z, t)\vec{i} + a_y(x, y, z, t)\vec{j} + a_z(x, y, z, t)\vec{k}$$
 - These (and other) field variables define the **flow field**.



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Example: Coupled Eulerian – Lagrangian Method



Forensic analysis of Columbia accident: simulation of shuttle debris trajectory using Eulerian CFD for flow field and Lagrangian method for the debris.

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Example 1

A steady, incompressible two dimensional velocity field

x and y in metres, $\mathbf{V}$ is in m/s

$$\vec{V} = (u, v) = (0.5 + 0.8x)\vec{i} + (1.5 - 0.8y)\vec{j}$$

(a) Determine if there are any stagnation zones

(b) sketch velocity vectors between $x = -2$ to 2 m and $y = 0$ to 5 m

Solution

(a) A stagnation point is a point in the flow field where the velocity is identically zero

For a stagnation point to exist $\mathbf{V}$ must equal zero

$$u = 0.5 + 0.8x = 0 \Rightarrow x = -0.625 \text{ m}$$

$$v = 1.5 - 0.8y = 0 \Rightarrow y = 1.875 \text{ m}$$

Location of stagnation point

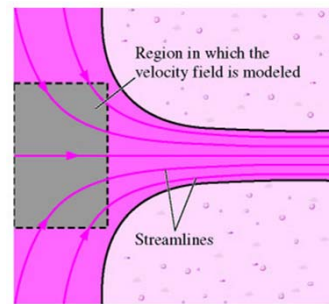
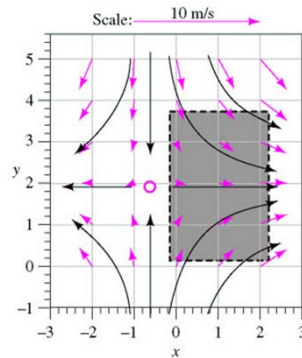
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Example 1 (Cont.)

$$\vec{V} = (u, v) = (0.5 + 0.8x)\vec{i} + (1.5 - 0.8y)\vec{j}$$

(b) to sketch velocity vectors, sub x and y coordinates into the velocity field equation within the specified range

(e.g. at x=2m, y=3m) u = 2.1 m/s and v = -0.9 m/s $\Rightarrow$ magnitude is 2.28 m/s



Bell mouth inlet of a hydro-electric dam

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Acceleration Field

- Consider a fluid particle and Newton's second law,

$$\vec{F}_{particle} = m_{particle} \vec{a}_{particle} \quad \text{EQN 4-5}$$

- The acceleration of the particle is the time derivative of the particle's velocity.

$$\vec{a}_{particle} = \frac{d\vec{V}_{particle}}{dt} \quad \text{EQN 4-6}$$

- However, particle velocity at a point is the same as the fluid velocity, $\vec{V}_{particle} = \vec{V}(x_{particle}(t), y_{particle}(t), z_{particle}(t), t)$

- To take the time derivative, the chain rule must be used.

$$\vec{a}_{particle} = \frac{\partial \vec{V}}{\partial t} \frac{dt}{dt} + \frac{\partial \vec{V}}{\partial x} \frac{dx_{particle}}{dt} + \frac{\partial \vec{V}}{\partial y} \frac{dy_{particle}}{dt} + \frac{\partial \vec{V}}{\partial z} \frac{dz_{particle}}{dt} \quad \text{EQN 4-7}$$

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Acceleration Field

- Since $\frac{dx_{particle}}{dt} = u, \frac{dy_{particle}}{dt} = v, \frac{dz_{particle}}{dt} = w$

$$\vec{a}_{particle} = \frac{d\vec{V}}{dt} = \frac{\partial \vec{V}}{\partial t} + u \frac{\partial \vec{V}}{\partial x} + v \frac{\partial \vec{V}}{\partial y} + w \frac{\partial \vec{V}}{\partial z} \quad \text{EQN 4-8}$$

- In vector form, the acceleration can be written as

$$\vec{a}(x, y, z, t) = \frac{d\vec{V}}{dt} = \frac{\partial \vec{V}}{\partial t} + (\vec{V} \cdot \nabla) \vec{V} \quad \text{EQN 4-9}$$

- **First term** is called the **local acceleration** and is nonzero only for unsteady flows.
- **Second term** is called the **advective acceleration** and accounts for the effect of the fluid particle moving to a new location in the flow, where the velocity is different.

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Acceleration Field

In Cartesian coordinates the components of the acceleration vector are

$$a_x = \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z}$$

$$a_y = \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z}$$

$$a_z = \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z}$$

EQN 4-11

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Example 2

A nozzle is 10cm long, inlet diameter is 1cm, outlet diameter is 0.5cm. The volumetric flow rate through the nozzle is $\dot{V} = 5 \times 10^{-5} \text{ m}^3/\text{s}$ and the flow is steady.

Estimate the magnitude of the acceleration of the fluid down the centreline of the nozzle

Solution

u is along centreline, $v = w = 0$ (radial)

using

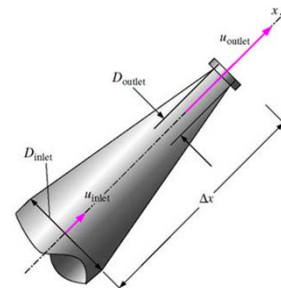
$$a_x = \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z}$$

$$a_x = u \frac{\partial u}{\partial x} \equiv u_{\text{avg}} \frac{\Delta u}{\Delta x}$$

$$a_x = \left(\frac{u_{\text{outlet}} + u_{\text{inlet}}}{2} \right) \left(\frac{u_{\text{outlet}} - u_{\text{inlet}}}{\Delta x} \right) = \frac{u_{\text{outlet}}^2 - u_{\text{inlet}}^2}{2\Delta x} = 30.4 \text{ m/s}^2$$

$$u_{\text{inlet}} = \frac{\dot{V}}{A_{\text{inlet}}} = \frac{4\dot{V}}{\pi D_{\text{inlet}}^2} = \frac{4 \times (5 \times 10^{-5})}{\pi \times 0.01^2} = 0.64 \text{ m/s}$$

$$\text{similarly } u_{\text{outlet}} = 2.55 \text{ m/s}$$



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Material Derivative

- The total derivative operator d/dt is called the **material derivative** and is often given special notation, D/Dt , emphasising following a particle as it moves through the flow field

$$\frac{D\vec{V}}{Dt} = \frac{d\vec{V}}{dt} = \frac{\partial \vec{V}}{\partial t} + (\vec{V} \cdot \vec{\nabla})\vec{V} \quad \text{EQN 4-12}$$

- Apply the material derivative to the velocity field, Eqn 4-9 becomes **material acceleration**

$$\vec{a}(x, y, z, t) = \frac{D\vec{V}}{Dt} = \frac{d\vec{V}}{dt} = \frac{\partial \vec{V}}{\partial t} + (\vec{V} \cdot \vec{\nabla})\vec{V} \quad \text{EQN 4-13}$$

- Advective acceleration is nonlinear: source of many phenomenon and primary challenge in solving fluid flow problems.
- Provides "transformation" between Lagrangian and Eulerian frames.
- Other names for the material derivative include: **total**, **particle**, **Lagrangian**, **Eulerian**, and **substantial** derivative.

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Example 3

$$\vec{V} = (u, v) = (0.5 + 0.8x)\vec{i} + (1.5 - 0.8y)\vec{j}$$

Consider the velocity field from Example 1 (page 7). Calculate the material acceleration at the point $(x=2\text{m}, y=3\text{m})$. Also sketch the material acceleration vectors on the same size array as Example 1.

Solution

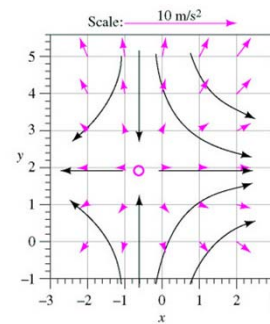
Use equation for material acceleration in Cartesian coordinates

$$a_x = \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z}$$

and

$$a_y = \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z}$$

At $(x=2\text{m}, y=3\text{m})$ $a_x = 1.68 \text{ m/s}^2$, $a_y = 0.72 \text{ m/s}^2$



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Break

Flow Visualization

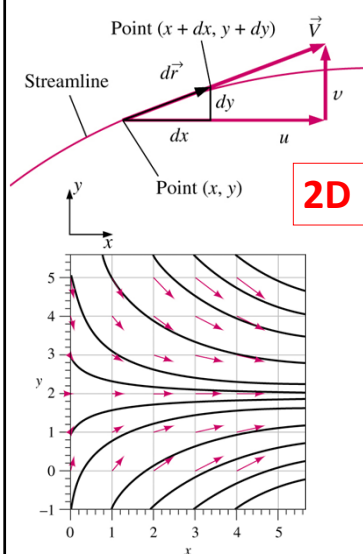
- Flow visualization is the visual examination of flow-field features.
- Important for both physical experiments and numerical (CFD) solutions.
- Numerous methods
 - Streamlines and streamtubes
 - Pathlines
 - Streaklines
 - Timelines
 - Refractive techniques
 - Surface flow techniques

Much can be learned from flow visualisation – the visual examination of flow fields

The first thing an engineer will do after running a simulation is view the flow visualisation, taking in the whole picture, not just the numbers generated

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Streamlines



- A **Streamline** is a curve that is everywhere tangent to the *instantaneous* local velocity vector.

- Consider an arc length

$$d\vec{r} = dx\vec{i} + dy\vec{j} + dz\vec{k}$$

- $d\vec{r}$ must be parallel to the local velocity vector $\vec{V} = u\vec{i} + v\vec{j} + w\vec{k}$

- Geometric arguments results in the equation for a streamline

$$\text{scalars} \quad \frac{dr}{V} = \frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w} \quad \text{EQN 4-15}$$

- Integrating, streamline in the xy-plane is

$$\left(\frac{dy}{dx} \right)_{\text{along a streamline}} = \frac{v}{u} \quad \text{EQN 4-16}$$

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Example 4

$$\vec{V} = (u, v) = (0.5 + 0.8x)\vec{i} + (1.5 - 0.8y)\vec{j}$$

Using the same velocity field as in Example 1, plot several streamlines in the right half ($x > 0$). Compare to velocity vectors

Assumptions

Steady, incompressible flow, 2D flow – no z component

Solution

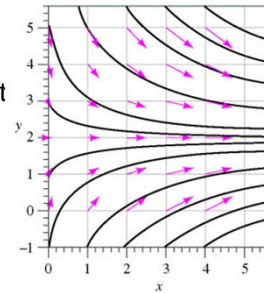
Use eqn 4-16 $\left(\frac{dy}{dx}\right)_{\text{along a streamline}} = \frac{v}{u}$

$$\frac{dy}{dx} = \frac{1.5 - 0.8y}{0.5 + 0.8x}$$

Solve this by integration $\frac{dy}{1.5 - 0.8y} = \frac{dx}{0.5 + 0.8x} \Rightarrow \int \frac{dy}{1.5 - 0.8y} = \int \frac{dx}{0.5 + 0.8x}$

After some algebra $y = \frac{C}{0.8(0.5 + 0.8x)} + 1.875$

By varying C, multiple streamlines can be evaluated

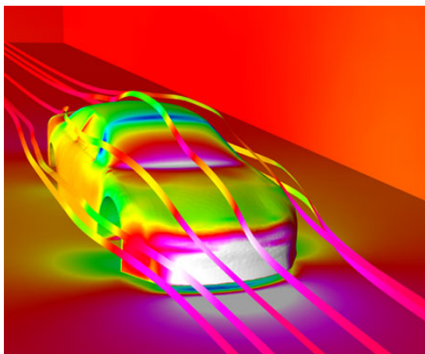


Notice that the velocity vectors point tangent to the streamline everywhere

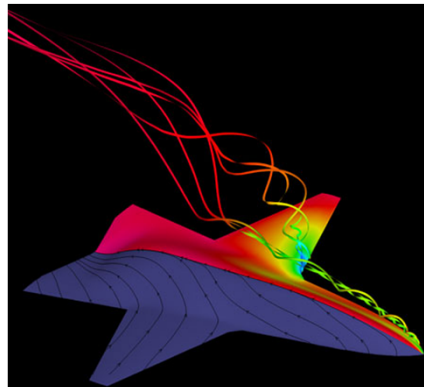
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Streamlines

NASCAR surface pressure contours and streamlines



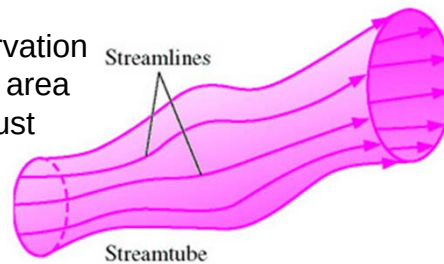
Airplane surface pressure contours, volume streamlines, and surface streamlines



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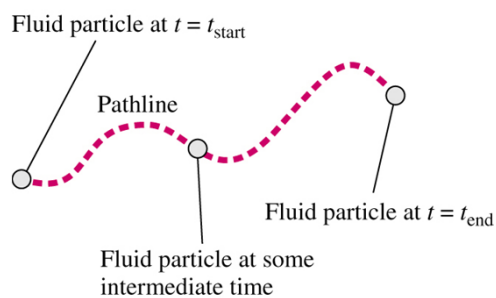
Streamtubes

- A streamtube consists of a bundle of streamlines
- Fluid within a streamtube must stay there, not crossing the boundary
- Streamtubes and streamlines are instantaneous quantities. They are defined at a particular instant by the velocity field at that instant
- In unstable flow, the conservation of mass must be obeyed. If area decreases, then velocity must increase and vice-versa



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Pathlines



- A **Pathline** is the actual path travelled by an individual fluid particle over some time period.

- Same as the fluid particle's material position vector

$$(x_{particle}(t), y_{particle}(t), z_{particle}(t))$$

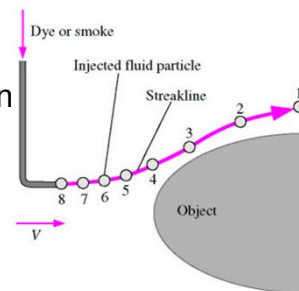
- Particle location at time t :

$$\vec{x} = \vec{x}_{start} + \int_{t_{start}}^t \vec{V} dt \quad \text{EQN 4-17}$$

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Streaklines

- A **Streakline** is the locus of fluid particles that have passed sequentially through a prescribed point in the flow.
- Easy to generate in experiments: dye in a water flow, or smoke in an airflow.
- The figure shows particles being injected at regular intervals
- Change in distance is due to acceleration
- Join the dots – this is the streakline



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Confusion

- Streaklines often confused with streamlines or pathlines.
- The three flow patterns are identical for steady flow BUT can be very different for unstable flow
- A streamline represents an instantaneous flow pattern at a given instant
- A streakline and a pathline are flow patterns that have some age (time history)
- Streakline: instantaneous snapshot of a time-integrated flow pattern
- Pathline: time-exposed flow path of an individual particle

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Example 5 Comparison of Flow Patterns

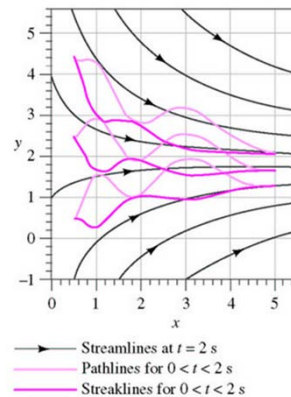
Unsteady, incompressible, 2D velocity field

Velocity field has a periodic term added

$$\vec{V} = (u, v) = (0.5 + 0.8x)\vec{i} + (1.5 + 2.5\sin(\omega t) - 0.8y)\vec{j}$$

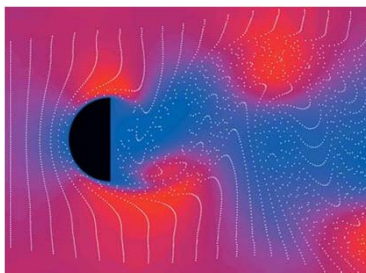
Angular frequency, $\omega = 2\pi$ rad/s

Compare instantaneous streamline at $t=2$ s with pathlines and streaklines during the time $t=0$ s to $t=2$ s



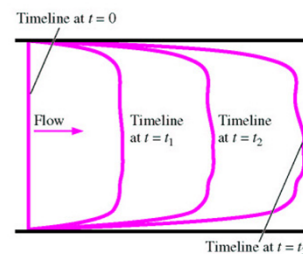
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Timelines



- A **Timeline** is the locus of fluid particles that have passed sequentially through a prescribed point in the flow
- Timelines can be generated using a hydrogen bubble wire

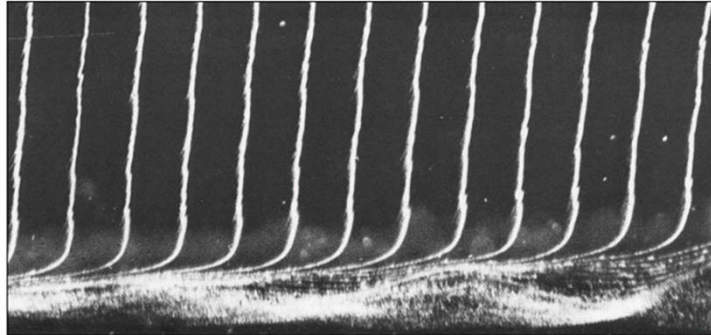
- Timelines are useful to examine the uniformity of a flow (or lack of)
- Away from the walls, the timeline distorts as a result of the fluid velocity



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Timelines

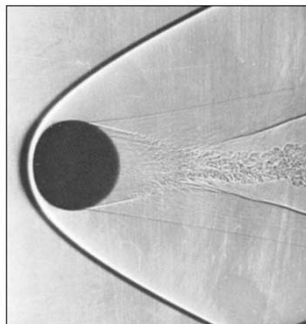
- The hydrogen bubbles are formed by passing an electric current through a cathode wire. Electrolysis results, forming tiny hydrogen bubbles with negligible buoyancy.



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Refractive Flow Visualisation

- Shadowgraphs and Schlieren photography
- Relies on the refractive properties of light waves



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Plots of Flow Data

- It is usually necessary to plot flow data to allow someone to visualise what is happening in time and or space
- Already explained timelines, now there are three others to consider
 - Profile plots
 - Vector plots
 - Contour plots

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Profile Plot

- A **Profile plot** indicates how the value of a scalar property varies along some desired direction in the flow field
- Easiest to understand
- Standard xy-plots that you are already familiar with
- They are plots of scalar variables (e.g. pressure, temperature, density)
- Most common is the velocity profile

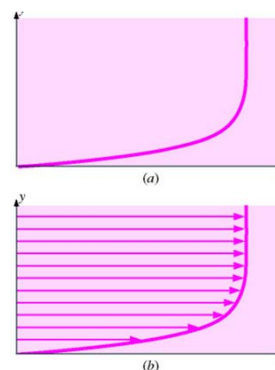
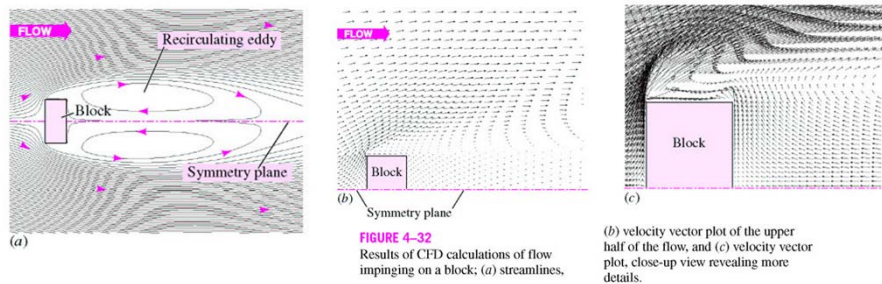


FIGURE 4-31
 Profile plots of the horizontal component of velocity as a function of vertical distance; flow in the boundary layer growing along a horizontal flat plate: (a) standard profile plot and (b) profile plot with arrows.

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Vector Plot

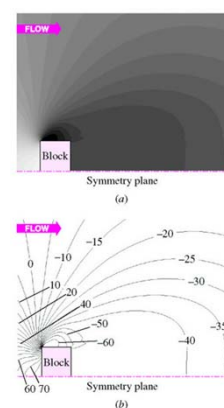
- A **Vector plot** is an array of arrows indicating the magnitude and direction of a vector property at an instant in time
- Streamlines show direction but not magnitude
- Already seen velocity and acceleration vector plots
- Can be generated analytically, experimentally or numerically



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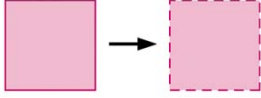
Contour Plot

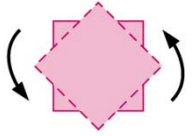
- A **Contour plot** shows curves of constant values of a scalar property for magnitude of a vector property at an instant in time
- Like contour maps of mountains
- Same principle applies to fluid mechanics, for scalar quantities such as pressure, temperature, velocity magnitude
- Two types, filled contour plot and contour line plot
- Dark regions represent low pressure
- Lines indicate gauge pressure



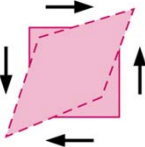
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Kinematic Description

(a) 

(b) 

(c) 

(d) 

- In fluid mechanics, an element may undergo four fundamental types of motion.
 - a) Translation
 - b) Rotation
 - c) Linear strain
 - d) Shear strain
- Because fluids are in constant motion, motion and deformation is best described in terms of rates
 - i. velocity: rate of translation
 - ii. angular velocity: rate of rotation
 - iii. linear strain rate: rate of linear strain
 - iv. shear strain rate: rate of shear strain

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Break

Rate of Translation

- To be useful, these rates must be expressed in terms of velocity and derivatives of velocity
- The **rate of translation vector** is described as the velocity vector. In Cartesian coordinates:

$$\vec{V} = u\vec{i} + v\vec{j} + w\vec{k} \quad \text{EQN 4-19}$$

(which we have seen before)

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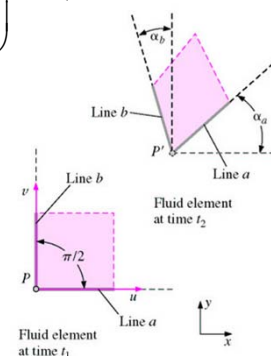
Rate of Rotation

- **Rate of rotation** at a point is defined as the average rotation rate of two initially perpendicular lines that intersect at that point. The rate of rotation vector in Cartesian coordinates:

$$\vec{\omega} = \frac{1}{2} \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right) \vec{i} + \frac{1}{2} \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right) \vec{j} + \frac{1}{2} \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) \vec{k} \quad \text{EQN 4-21}$$

- If a particle rotates and deforms, follow the path of the two lines a and b
- **a** has rotated by angle α_a and **b** by α_b
- Average rotation angle is $(\alpha_a + \alpha_b)/2$
- Rate of rotation about point P

$$\omega = \frac{d}{dt} \left(\frac{\alpha_a + \alpha_b}{2} \right) = \frac{1}{2} \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) \quad \text{EQN 4-20}$$



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Linear Strain Rate

- **Linear Strain Rate** is defined as the rate of increase in length per unit length.

- In Cartesian coordinates

$$\epsilon_{xx} = \frac{\partial u}{\partial x}, \epsilon_{yy} = \frac{\partial v}{\partial y}, \epsilon_{zz} = \frac{\partial w}{\partial z} \quad \text{EQN 4-23}$$

- If you stretch an object in one direction it generally shrinks in another.
This is also true in fluids
- For incompressible flow – net volume remains constant, hence stretches in one direction and shrinks in another
- For compressible flow – the volume may increase or decrease as density changes

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Linear Strain Rate

- The rate of increase of volume per unit volume of a fluid is called **volumetric strain rate**
In Cartesian coordinates

$$\frac{1}{V} \frac{DV}{Dt} = \epsilon_{xx} + \epsilon_{yy} + \epsilon_{zz} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \quad \text{EQN 4-24}$$

- Since the volume of a fluid element is constant for an incompressible flow, the volumetric strain rate must be zero

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Shear Strain Rate

- **Shear Strain Rate** at a point is defined as *half of the rate of decrease of the angle between two initially perpendicular lines that intersect at a point.*
- Shear strain rate can be expressed in Cartesian coordinates as:

$$\epsilon_{xy} = \frac{1}{2} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right), \epsilon_{zx} = \frac{1}{2} \left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \right), \epsilon_{yz} = \frac{1}{2} \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right)$$

EQN 4-26

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Shear Strain Rate

We can combine linear strain rate and shear strain rate into one symmetric second-order tensor called the **strain-rate tensor**.

$$\epsilon_{ij} = \begin{pmatrix} \epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz} \\ \epsilon_{yx} & \epsilon_{yy} & \epsilon_{yz} \\ \epsilon_{zx} & \epsilon_{zy} & \epsilon_{zz} \end{pmatrix} = \begin{pmatrix} \frac{\partial u}{\partial x} & \frac{1}{2} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) & \frac{1}{2} \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right) \\ \frac{1}{2} \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right) & \frac{\partial v}{\partial y} & \frac{1}{2} \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right) \\ \frac{1}{2} \left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \right) & \frac{1}{2} \left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \right) & \frac{\partial w}{\partial z} \end{pmatrix}$$

EQN 4-27

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Example 6 – Bringing it all together

- The 2D, steady, incompressible velocity field from Ex. 1

$$\vec{V} = (u, v) = (0.5 + 0.8x)\vec{i} + (1.5 - 0.8y)\vec{j}$$

- There is a stagnation point at (-0.625, 1.875)
- Units in m, s and m/s
- Determine
 - Rate of translation
 - Rate of rotation
 - Linear strain rate
 - Shear strain rate
 - Volumetric strain rate
 - Verify the flow is incompressible

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Example 6 – Bringing it all together

- The rate of translation is simply the velocity vector

$$u = 0.5 + 0.8x \quad v = 1.5 - 0.8y \quad w = 0$$

- Rate of rotation, use Eqn 4-21. $w = 0$ and u and v do not vary with z , so

$$\vec{\omega} = \frac{1}{2} \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) \vec{k} = \frac{1}{2} (0 - 0) \vec{k} = 0$$

- Therefore there is no net rotation

- Linear strain rate, use Eqn 4-22

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} = 0.8s^{-1} \quad \varepsilon_{yy} = \frac{\partial v}{\partial y} = -0.8s^{-1} \quad \varepsilon_{zz} = \frac{\partial w}{\partial z} = 0$$

- This means the fluid particles stretch in the x direction and shrink in the y direction

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Example 6 – Bringing it all together

- Shear strain rate, use Eqn 4-26. 2D, therefore non zero strain rates can only occur in the xy plane

$$\varepsilon_{xy} = \frac{1}{2} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) = \frac{1}{2} (0 + 0) = 0$$

- Particle deforms but stays rectangular
- Volumetric strain, use Eqn 4-24

$$\frac{1}{V} \frac{DV}{Dt} = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = (0.8 - 0.8 + 0) s^{-1} = 0$$

- Since the volumetric strain rate is zero everywhere, we can verify that the flow is incompressible

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Vorticity and rotationality

- The **vorticity vector** is defined as the curl of the velocity vector $\vec{\zeta} = \vec{\nabla} \times \vec{V} = \text{curl}(\vec{V})$
- Vorticity is equal to twice the angular velocity of a fluid particle $\vec{\zeta} = 2\vec{\omega}$

Cartesian coordinates

$$\vec{\zeta} = \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right) \vec{i} + \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right) \vec{j} + \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) \vec{k} \quad \text{EQN 4-30}$$

Cylindrical coordinates

$$\vec{\zeta} = \left(\frac{1}{r} \frac{\partial u_z}{\partial \theta} - \frac{\partial u_\theta}{\partial z} \right) \vec{k}_r + \left(\frac{\partial u_r}{\partial z} - \frac{\partial u_z}{\partial r} \right) \vec{k}_\theta + \frac{1}{r} \left(\frac{\partial(r u_\theta)}{\partial r} - \frac{\partial u_r}{\partial \theta} \right) \vec{k}_z \quad \text{EQN 4-32}$$

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Vorticity and rotationality

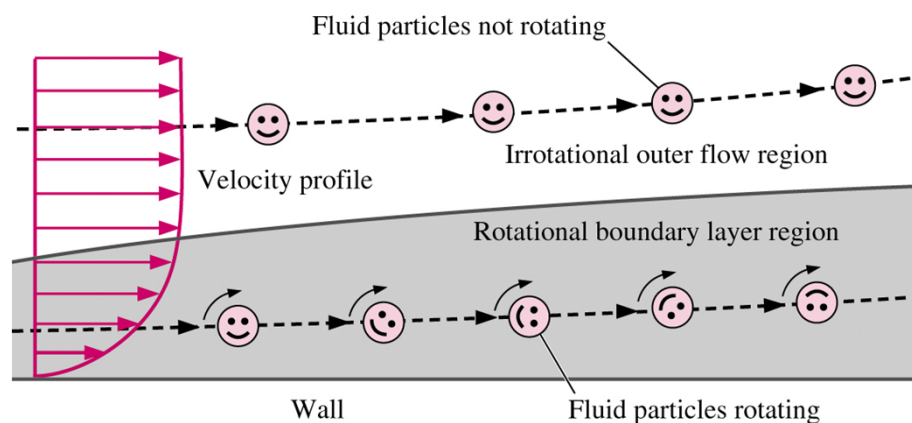
- Two dimensional flow in cylindrical coordinates has no z component so Eqn 4-32 simplifies and is given by

$$\vec{\zeta} = \left(\frac{\partial(ru_\theta)}{\partial r} - \frac{\partial u_r}{\partial \theta} \right) \vec{k}_z \quad \text{EQN 4-33}$$

- In regions where $\zeta = 0$, the flow is called **irrotational**
- Elsewhere, the flow is called **rotational**

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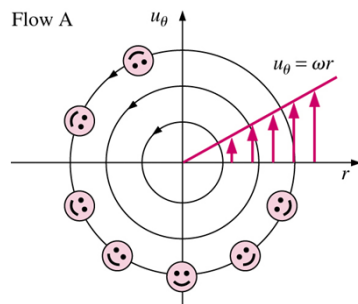
Vorticity and rotationality



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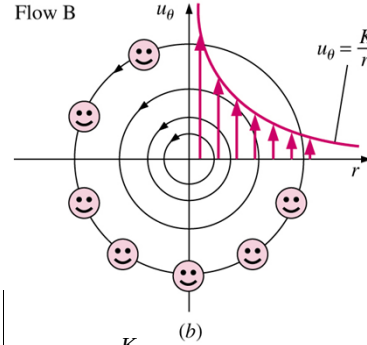
Vorticity and rotationality

Special case: consider two flows with circular streamlines



$$u_r = 0, u_\theta = \omega r$$

$$\vec{\zeta} = \frac{1}{r} \left(\frac{\partial(ru_\theta)}{\partial r} - \frac{\partial u_r}{\partial \theta} \right) \vec{k}_z = \frac{1}{r} \left(\frac{\partial(\omega r^2)}{\partial r} - 0 \right) \vec{e}_z = 2\omega \vec{k}_z$$



$$u_r = 0, u_\theta = \frac{K}{r}$$

$$\vec{\zeta} = \frac{1}{r} \left(\frac{\partial(ru_\theta)}{\partial r} - \frac{\partial u_r}{\partial \theta} \right) \vec{k}_z = \frac{1}{r} \left(\frac{\partial(K)}{\partial r} - 0 \right) \vec{e}_z = 0 \vec{k}_z$$

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Comparison of Two Circular Flows



(a)

Rotational flow



(b)

Irrotational flow

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Tuts and what is coming up?

4-18, 4-19, 4-37, 4-38, 4-73, 4-74, 4-79, 4-83

REMEMBER

THERE IS A TUTORIAL QUIZ THIS WEEK

Next week lecture is revision lecture, we
will review topic covered from weeks 1-6
to prepare for the mid-session

[Please have a look through the material up to the week 6 lecture.](#)

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