

IMIA Operator Algebra Seminar
University of Wollongong

Title: The Klein-Podleś bottle as a non-trivial bundle over the quantum real projective space $\mathbb{R}\mathbb{P}_q(2)$

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Time and Dates: 1:30pm, Thursday August 18, 2011

Location: Room 1.G03

Abstract: We tensor the C^* -algebra of the equatorial Podleś quantum sphere with the algebra of continuous functions on the unit circle, act on the tensor product with the diagonal antipodal $\mathbb{Z}/2$ -action, and consider the invariant subalgebra. This gives a $U(1)$ - C^* -algebra A with the quantum real projective space C^* -algebra $C(\mathbb{R}\mathbb{P}_q(2))$ as its $U(1)$ -invariant part. Using the identity representation of $U(1)$, we associate with it a finitely generated projective module over $C(\mathbb{R}\mathbb{P}_q(2))$. Combining methods of topology and operator algebras, we prove that this module is not free. This implies that A cannot be a crossed product of $C(\mathbb{R}\mathbb{P}_q(2))$ and the integers. To compute the K-theory of A , we present it as a pullback of two copies of the tensor product of the equatorial sphere C^* -algebra with the algebra of continuous functions on the interval $[0, 1]$. We put these two copies together by the identity and antipodal automorphisms of the quantum sphere applied at 0 and 1, respectively. Hence A can be viewed as defining a non-trivial bundle over a circle with the fibre being the equatorial quantum sphere. Since replacing the quantum sphere with the unit circle and the antipodal action with the complex conjugation would yield the Klein bottle, we call A the C^* -algebra of the Klein-Podleś bottle.

This is joint work with P.F. Baum.