Title: "*On robust testing in the Cox model*"

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Abstract

In biostatistics ``robust'' inference in the Cox model typically refers to the use of the Lin and Wei sandwich variance estimator (*LW)* in conjunction with the partial likelihood estimator (*PLE*). It usually means that under certain misspecifications of the Cox model, e.g. covariate omission or slight deviations from the proportional hazard assumption, the test size is approximately preserved. In modern robustness theory a testing procedure has to be stable in a neighbourhood of the assumed model both under the null and alternative hypothesis to be declared locally robust. This property is required to draw safe inference in data with outliers such as abnormal long-term survivors, a particularly harmful type of contamination in the Cox model. As expected, the *LW* procedure is not robust under that paradigm as it relies solely on the *PLE.* It is clearly outperformed by its robust counterpart based on the adaptive robust estimator (*ARE)* proposed by Bednarski and colleagues over the years.; see Bednarski (1993), Minder and Bednarski (1996), and Bednarski (1999, 2007). However, the Wald test based on the *ARE* has limited robustness properties. It can only maintain its level in a neighbourhood of the null hypothesis that shrinks at the usual rate of $O(n^{-1/2})$. In a fixed contamination rate neighbourhood, the Wald test usually breaks down although the underlying estimator does not. To make matter worse, the bigger the sample size, the worse the distortion to the level in general. This unusual finding is caused by a small asymptotic bias to the *ARE* that may not be critical for the estimation problem but definitely is for the testing problem. A closer look at the two approaches also shows that the asymptotic variance matrix for the *ARE* is literally a weighted version of the *LW* formula. This work brings new insight to the difficult problem of robust testing in the Cox model that is far more complex than anticipated from our knowledge of robust inferential procedures in fully parametric models.

References

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