Centre for Statistical and Survey Methodology

The University of Wollongong

(Keynote lecture presented at International Conference on Quality Management of Official Statistics, Korea 6-7 September 2007)

Working Paper

11-08

Design and Analysis of Repeated Surveys

David Steel and Craig McLaren

Centre for Statistical & Survey Methodology, University of Wollongong, Wollongong NSW 2522. Phone +61 2 4221 5435, Fax +61 2 4221 4845. Email: anica@uow.edu.au

Design and Analysis of Repeated Surveys

David Steel[1] and Craig McLaren[2]
[1] Centre for Statistical and Survey Methodology, Australia
University of Wollongong, e-mail : dsteel @uow.edu.au
[2] Craig McLaren, Office for National Statistics, U.K.
e-mail : craig.mclaren @ons.gov.uk

Abstract— **This lecture will review the major issues associated with the design and analysis of repeated surveys. The interaction between the design of a repeated survey and the methods used for estimation and analysis will be examined. The choice of rotation pattern will be considered in terms of the impact on the estimation of levels and changes. Composite and other forms of estimators will be reviewed and the interaction between design and estimation explored. Estimation of seasonally adjusted and trend estimates from repeated surveys will also be considered.**

I. Overview of Issues in Repeated Surveys

Sampling over time enables analysis of social and economic processes through estimation and analysis of changes in variables of interest. In addition to the usual design issues we need to consider the frequency of sampling, the spread and pattern of inclusion of units over time, the use of overlapping or non-overlapping samples over time and the precise pattern of overlap. Examples of types of surveys using sampling in time include: repeated, panel and longitudinal surveys; rotating panel surveys; split panel surveys and rolling samples.

Factors affecting the design of a sample over time include the key estimates to be produced, the type and level of analyses to be carried out, cost, data quality and reporting load. The interaction between sampling over time and features of the design, such as stratification and cluster sampling also needs to be decided. Duncan and Kalton (1987), Kalton and Citro (1993) and Steel (2004) give a general review of issues in the design and analysis of repeated surveys. Kasprzyk *et al*. (1989) cover may of the important issues associated with panel surveys. Smith (1978), Binder and Hidiroglou (1988) and Fuller (1990) review estimation issues for repeated surveys.

Time series may be produced from repeated surveys. The analysis of these time series may involve seasonal adjustment and trend estimation.

High quality surveys are based on probability sampling methods that provide estimates of characteristics of a population and permit analysis of relationships between variables. A probability sampling methods ensures that all population members have a known, non-zero probability of selection. Common methods are simple random sampling, probability proportional to size selection, stratification, cluster and multi-stage sampling. Standard errors are estimated and used to make inferences by constructing confidence intervals. Analytical outputs such as regression coefficients can also be produced enabling relationships between variables to be analyzed.

Samples of people are often obtained by selecting a sample of dwellings and including the households and people in the selected dwellings. Sampling is also used for surveys of other entities such as hospitals, schools and businesses. Sampling of physical units, such as areas of land, can also be used.

The frequency of sampling depends on the purpose of the survey. A repeated survey enables estimation of changes for the population as well as cross-sectional estimate. Monitoring and detecting important changes will usually be a key reason for sampling in time. Common frequencies for repeated survey are monthly, quarterly and annual. More frequent sampling may be adopted *e.g.* opinion polls leading up to an election and monitoring TV ratings.

Some examples of repeated surveys are monthly labour force surveys in Australia, US, Canada, Japan and Retail Trade Survey in Australia. Quarterly surveys include the Labour Force Survey in UK and Ireland and many business surveys.

Factors in deciding the frequency of sampling are how quickly changes are likely to occur, how quickly decisions are needed and the budget available. Sampling should not take place so often that the sample is registering unimportant short-term movements of no practical interest.

In a stratified, multi-stage design the sample should include each stratum and Primary Sampling Unit (PSU) in each period. Cost considerations may lead to each PSU being included in only one month or time period.

The reference period is a fundamental part of the definition of the variables of interest. A long reference period may increase the number of episodes or incidents included in the survey but the impact of telescoping has to be considered. Other recall errors must be taken into account, which will depend on the specific variables. For some variables a twelve-month reference period might be feasible, whereas for other variables a one-day reference period might be appropriate. Some variables are defined at the time of interview, for example an opinion.

The population frame must be updated to incorporate changes in the population as quickly as possible. The sample should be updated to give new units a chance of selection and to remove defunct units that may affect sampling errors. Systems to update the sampling frame and sample need to be developed. In household surveys this can be done through a master sampling frame which is updated regularly to add new housing and reflect other changes to the population. A rotation pattern can be implemented by dividing the frame into rotation groups. Use of a master sampling frame means that we can control overlap between different surveys using the same frame. For surveys of businesses the list or register has to be maintained and the sample updated. Rotation and overlap between the samples of different surveys can be controlled using permanent random number sampling.

In a panel survey an initial sample is selected and interviewed on several occasions. It can provide estimates of change for variables for which information is collected at each occasion and can provide estimates for different variables over time. Cost savings arise because the first time a unit is included there are higher costs.

A distinction can be made between a repeated survey and a longitudinal survey. Binder(1998) provides a review of longitudinal surveys. In a longitudinal survey an initial sample is selected and at each occasion, or wave, an attempt is made to include all the members of the initial sample. Longitudinal survey permits analysis of changes at a micro level. Examples of longitudinal surveys include: British Household Panel Survey (UK); Survey of Family, Income and Employment (New Zealand); National Longitudinal Surveys (US); Households, Income and Labour Dynamics in Australia Survey (Australia), Survey of Income and Program Participation (US). A longitudinal survey is a form of panel survey, which are designed for analysis of changes at the unit level.

In a repeated survey there is not necessarily any overlap of the sample for different occasions. A rotating panel surveys also uses a sample that is followed over time, but the focus is on estimates at aggregate levels. When the emphasis is on estimates for the population an independent sample may be used on each occasion, which is often the case when the interval between the surveys is quite large. An option is to use the same sample at each occasion, with additions so that the sample estimates refer to the current population. For monthly or quarterly surveys the sample is often designed with considerable overlap between successive surveys. The sample overlap will reduce the sampling variance of estimates of change and reduce costs.

Many important surveys are conducted repeatedly to give estimates of the level or mean for several time periods. When one of the objectives is to estimate changes over time a number of issues arise. Estimates of change between time periods may be as important as, or more important than, estimates of the levels or means. There may be interest in change in the level of a variable between two adjacent time periods and changes between periods *s* time periods apart. The time periods are usually months or quarters, but may be days or years

The question arises of whether we should we use the same sample at each time period, or independent samples, or partially overlapping samples? Cost and the standard errors of estimates of movements are usually minimized by having complete overlap of the samples.

Respondent load, attrition, conditioning and declining response rate usually lead to some degree of replacement or rotation of the sample from one period to the next.

The main interest may be in the time series, which leads to issues of seasonal adjustment and trend estimation.

A longitudinal survey can be used to provide estimates of changes at aggregate levels but these estimates refer to the population at the time of the initial sample selection unless the sample has been updated to make it representative of the current population. The main purpose of a longitudinal survey is to enable estimates of changes at the unit level.

In a rotating panel survey the panel aspect is often implemented at the dwelling level, which implies that people and households are not followed when they leave a selected dwelling. People and households moving into a selected dwelling are included in the survey. This approach is suitable when the main objective is to provide unbiased aggregate estimates.

In a rotating panel survey the focus is on aggregate estimates of change. However, any overlapping sample can also be used to analyze change at the micro-level. A table can be produced from the matched sample showing the change of a variable between two time periods. An example is when a table of change in status is produced, which is referred to as a Gross Flows table.

It is possible to create longitudinal data from rotating panel surveys. The length of the total time period and the time interval between observations are determined by the rotation pattern used. The resulting sample of individuals for which a longitudinal data are available will be biased away from people who move permanently or are temporarily absent. An alternative to a rotating panel survey is a split panel survey, which involves a panel survey supplemented on each occasion by an independent sample. This approach permits longitudinal analysis from the panel survey for more periods than would be possible in a rotating panel design, but also cross sectional estimates obtained from the entire sample. In general the three dimension of space, time and variables need to be considered (Kish, 1998).

A survey may be conducted continuously but the sample size in any time period may not be sufficient to provide reliable estimates for that period, at least for sub national estimates. By cumulating sample over several time periods reliable estimates may be produced and in this approach sample overlap is detrimental. The sample design can be developed so that it is a rolling sample with non-overlapping samples that over time cover many areas and eventually all areas. This approach can be useful in producing sub-national and small area estimates. A related approach is rolling estimates For example in the UK Labour Force Survey a non-overlapping sample is interviewed in each week of the quarter. Each month estimates based on an average of the latest 13 weeks are produced (Caplan *et al.*, 1999).

In section II some relevant theory is reviewed. Correlation modelling is considered in section III and rotation patterns and their impact are considered in section IV. Section V considers composite estimation and time series methods are mentioned ion section VI.

II. Some Basic Theory

Repeated surveys can provide estimates for each time periods, y_t. A major value of repeated surveys is in their ability to provide estimates of change. The simplest analysis of change is the estimate of one period change, $y_t - y_{t-1}$. In a monthly survey this corresponds to one-month change. For a survey conducted annually this corresponds to annual change. In general the change *s* time periods apart can be estimated, using

$$y_t - y_{t-s} = \Delta^{(s)} y_t \,.$$

The focus is often on *s*=1, but for a survey repeated on a monthly basis changes for *s*=2, 3, 12 are also commonly examined. Having sample overlap at lag *s* will usually lead to a positive correlation between the estimates. Since

$$Var(\Delta^{(s)} y_t) = Var(y_t) + Var(y_{t-s}) - 2\sqrt{Var(y_t)}\sqrt{Var(y_{t-s})}Corr(y_t, y_{t-s}) \qquad (1)$$

having sample overlap reduces the variance of $\Delta^{(s)} y_t$ compared with having no sample overlap. Holt and Skinner (1989) consider the components of change in a repeated survey.

A positive correlation between the estimates will reduce the variance and can often be achieved through sample overlap. If comparisons are made with time periods for which there are no sample units in common then the variance of the estimate of change will be the sum of the variances, which will often be approximately twice the variance of the estimate of the level for a particular time period. These considerations result in designing the sampling so that there is overlap between the samples for time periods between which the movements are of major interest. So, if there is strong interest in monthly movement then there should be high sample overlap between successive months. If there is also interest in changes 12 months apart then consideration should be given to designs that induce sample overlap at this lag. However, for many variables the correlation at the unit level 12 months apart may not be high enough for there to be appreciable gains from doing so.

If the samples are independent between the two time periods, then $corr(y_t, y_{t-s}) = 0$. In general there will be overlap between the samples and we would expect the degree of overlap to be a factor influencing correlation. For the very simple situation of a stable population, that is no births and death, and simple random sampling with negligible sampling fractions $corr(y_t, y_{t-s}) = \sqrt{k_t k_{t-s}} R_{t,t-s}$, where $R_{t,t-s}$ is the individual level correlation and k_t is the proportion of the sample at time *t* that is common between periods *t* and *t-s*. Many rotation patterns are set up so that $k_t = k_{t-s} = k(s)$, so that $corr(y_t, y_{t-s}) = k(s) R_{t,t-s}$. If the patterns of change do not vary over time, then $R_{t,t-s} = R(s)$. Usually we would expect the unit level correlation to be positive. More complex models for the correlation of the sampling errors are considered in section III. These results suggest that the higher the sample overlap the higher the correlation between the estimates and this leads to rotation designs with high sample overlap between periods for which the change is of interest. So for analysis of one period changes the high overlap between adjacent periods is desirable.

Averaging of estimates can be used to produce more stable estimates when the original estimates have high sampling variances, for example for small sub-groups or domains in the population. A particular case is estimates for small geographic areas. Averaging over time changes the length of the time period to which the estimate refers, which will hide any variation within the period over which the average is calculated.

If the interest is in averages then positive correlations increase the variance. For example the average of three consecutive months would have variance

$$Var\left(\frac{y_{t-2} + y_{t-1} + y_t}{3}\right) = \frac{1}{9}\begin{bmatrix} Var(y_{t-2}) + Var(y_{t-1}) + Var(y_t) \\ + 2Cov(y_{t-2,} y_{t-1}) + 2Cov(y_{t-1,} y_t) \\ + 2Cov(y_{t-2,} y_t) \end{bmatrix}.$$

If the series of sampling errors is weakly stationary so that and the sample overlap depends only on the gap between two periods, so that $corr(y_t, y_{t-s}) = r(s)$ then

$$Var\left(\frac{y_{t-2} + y_{t-1} + y_t}{3}\right) = \frac{Var(y_t)}{9}\left[3 + 4r(1) + 2r(2)\right]$$

For averages of estimates positive correlation between the estimates increases sampling variance, so it is better to average uncorrelated estimates, which can be obtained from independent or non-overlapping samples.

If both averages and differences of estimates are of interest then the relative importance of each type of estimate has to be considered and the impact of different options assessed on both types.

In general for an estimator that is a linear combination of the estimates, $\mathbf{l}^T \mathbf{y}_T$ where $\mathbf{y}_T = (y_1, \ldots y_T)^T$ is the vector containing the values of the time series up to time *T*, then $Var(\mathbf{l}^T \mathbf{y}_T) = \mathbf{l}^T Var(\mathbf{y}_T)\mathbf{l}$. For example the change in the most recent non-overlapping 3 month averages corresponds to $\mathbf{l} = \left(0,0,.....,0,-\frac{1}{3},-\frac{1}{3},-\frac{1}{3},\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)^T$. In an evaluation of options for a UK Monthly Labour Force Survey, Steel (1996) considers estimates of the current month's level, changes between months for s=1,3, 12, three month averages and one and three month changes in three month averages, 12 month averages and 1 and 12 month changes in 12 month averages. To determine the variance of $\mathbf{l}^T \mathbf{y}_T$ we need to know, estimate or model $Var(\mathbf{y}_T)$.

The best overlap pattern depends on what you want to estimate. If we want to estimate the average over two or more time periods we do not want any positive covariance and independent samples are best. This principle applies generally: the best design depends on what you want to estimate, that is the target(s) in inference. It is important to decide if you are interested in estimating differences or averages or both. If independent samples are used the correlations are zero. Averaging independent estimates over several periods can reduce standard errors but it is important to realize that the estimate refers to an average not individual time periods.

If the population correlation is high even a small amount of rotation can greatly increase the variance of the movement estimator. This is because nearly all the variance is coming from the unmatched parts of the sample. Usually the population correlations decrease as the gap between the time periods in question increases, although seasonality can lead to some appreciable correlations for example12 months apart. In some cases we assume that the population. correlation depends only on gap between the periods. We can then try to estimate and perhaps model using say an autoregressive process, AR (1) or Markov process.

III. Correlation Models

Using sampling to obtain time series means that each observed value y_t has a component attributed to the sampling error. The use of rotation patterns means that the sampling error will have a complex structure and it will be correlated over time. We need estimation of realistic correlation models to accurately reflect what is happening to the sampled time series. They can also help us determine the impact of the sample design on the estimates. Once we have a correlation model we can also use this approach to evaluate the choice of different rotation patterns.

In developing the correlation models we need to include the effect of the rotation pattern. A simple model is $corr(y_t, y_{t-s}) = R(s) = k(s)r(s)$. Here k(s) is the sample overlap factor and r(s) is the individual level correlation at lag s. For a household survey the overlap factor is what the overlap would be for selected dwellings if there were no new dwellings added. We can modify *k*(*s*) to allow for other factors such as non-response at each stage of the survey. As an example for *in-for-8, k* (1) = 7/8 *k*(2) = 6/8….*k*(7) = 1/8, *k*(8) = 0. For *s* > 8 gives *k*(s) = 0.

To allow for the fact that rotation often occurs within PSUs more complex models such as R(s)=d(s)+k(s)(r(s)-d(s)) may be developed. Here d(s) is the correlation between estimates for the same rotation group when a rotation has occurred. Scott *et al.* (1977) consider this model. Bell (1998) give correlation models for estimate of the proportion of employed persons and proportion of unemployed persons for the Australian Labour Force Survey. Lee (1990) provides estimates correlations for the Canadian Labour Force Survey. Pfeffermann *et al*. (1998) develop a method for estimating correlations over time using panels estimates.

Steel and McLaren (2000a) developed a correlation models for the Australian Retail Trade survey. Analysis of correlations showed differences between different states and size strata were not important. The main differences were correlations across industries and so separate correlation models were estimated for each industry.

Correlation models are used to try and remove the survey error component. If not removed we may end up with spurious correlations as part of our time series estimates. Typically this can affect the trend and seasonal components of a time series. Once we can generate our estimates we can then use them in a model framework to account for these complexities, see Peffermman (1991) and Pfeffermann *et al*. (1998).

IV. Rotating Panel Surveys- Impact on Sampling Variances

Having sample overlap results in sampling variances on estimates of change being reduced because the variation due to including different units is reduced. The reduction in variances depends on the correlation of the variable at the unit level over time and the degree of sample overlap. These considerations would lead to maximizing the sample overlap at each time period at almost 100 percent, with the only change in sample arising from the updating the sample to represent units moving in and out of the population. Such a design would lead to selected units being included in the survey indefinitely. In practice a limit needs to be placed on how many times a unit is surveyed to spread the reporting load and maintain response rates and data quality. In deciding the degree of sample overlap these considerations need to be balanced.

An overlapping sample design can be implemented using a rotation pattern or design. A rotation pattern can be designed to manage the sample over time. Rotation patterns can be developed that have the same proportion of the sample in common between any two time periods the same time apart and the same proportion of sample rotated out and into the sample at each period.

The rotation design should ensure that the cross-sectional estimates are unbiased, while reducing costs and sampling variances on important estimates of change. A rotation pattern should ensure that at each time the sample is balanced on the number of times a unit has been included in the survey, because of the potential times in survey effect (Bailer, 1975).

A rotation sampling design can be implemented using rotation groups and panels. The sample will consist of several rotation groups. A panel is the set of selected units that enter and leave the sample at the same time. When a panel leaves the sample it is replaced from the same rotation group. For example in the Australian Labour Force Survey the PSUs are allocated to eight rotation groups. In a particular month the dwellings in one of the rotation groups are rotated out of the survey and replaced by an equivalent sample of dwellings in the same PSU.

There are many different rotation patterns in use and many that can be considered. Consider a monthly survey. The simplest rotation pattern is when a unit is included for *a* months. A more general class is *a*-*b*-*a*(*m*): a unit is included initially for a months, leaves the sample for *b* month and returns for a further *a* months. The pattern is repeated until the unit is included for a total of *m* months (Rao anf Graham, 1964).. *S*etting *b*=0 to give an *in-for-m* rotation pattern. More generally a pattern of the form *a1-b1-a2-b2-…am*(*c*) can be considered, where the number of months included and excluded from the survey varies. Park et el. (2001) discuss general designs of rotation patterns.

The US Current Population Survey uses a 4-8-4(8) rotation pattern, the Australian Labour Force Survey uses in-for-8 rotation pattern, which can also be denoted 8(8) and the Canadian Labour Force Survey uses an in-for-6 rotation pattern.

By using sampling in time an analysis of changes can be carried out. Consider a variable of interest which is estimated for time period t by y_t. The simplest analysis focusses on the estimates of one period change, $y_t - y_{t-s} = \Delta^{(s)} y_t$. In a monthly survey this is one-month change. For a survey conducted annually this is annual change. For a monthly survey looking at 3 monthly and 12 monthly changes can be useful.

A positive covariance reduces the variance, which can be achieved through sample overlap. These considerations result in designing the sampling so that there is overlap between the samples for time periods between which the movements are of major interest. If there is strong interest in monthly movement then there should be high sample overlap between successive months. If there is also interest in changes 12 months apart then consideration should be given to designs that induce sample overlap at this lag. For many variables the correlation 12 months apart may not be high enough for there to be appreciable gains from doing so.

Rotation patterns vary in the number of times a unit is included in the survey and the time interval between inclusions. For example the Australian Labour Force Survey includes selected dwellings for eight consecutive months One eighth of households are removed each month and replaced with nearby dwellings. After 8 months we have a completely new sample.

In the Australian Retail Survey Some businesses are always in sample and some businesses are sampled. Every three months the sampling frame is updated and on twelfth of businesses rotated out of the sample in each quarter. This ensures that no sampled business remains in survey for more than 36 months. We can define this as an *in-for*-36(*q*) rotation pattern. We need rotation patterns to achieve a balance between refreshing the sample and enabling comparisons over time. There is a trade off between the type of rotation pattern you use and the estimates you are interested in

Consider monthly rotation patterns. A general class of rotation patterns is to s elect dwellings for *a* consecutive months, remove for *b* months and then include again for a further *a* months for a total of *m* months, which is Denoted by *a-b-a(m)..* Now consider some common monthly rotation patterns and the sample overlap that they produce.

An *in-for-m* has overlap of 1-*s/m* between samples *s* months apart for *s*=1,…*m*-1 and no overlap for months *m* or more apart. Unless *m* exceeds 12 there will be no overlap for months a year apart. This type of scheme is used by the Australian Bureau of Statistics and Statistics Canada for their Labour Force surveys.

An 1-2-1*(m)* rotation pattern leads to no sample overlap between months one or two months apart and overlap of 1-*s*/(3*m*) for *s*=3,6,…,3*m* The overlap between months a year apart is 1-4/*m* provided *m* is 5 or more*. We* can have 1-*b*-1(*m*) variants for different choices of *b* (see Steel and McLaren, 2002).

The 4-8-4(8) rotation pattern leads to an overlap of 1-*s*/4 for months *s* months apart, *s*=1,2,3. For *s*=12 the overlap factor is 4/8 and the sample overlap is 4/8 – |*s*-12|/8 for *s*=9,…,15 and there is no overlap for *s*=4,…,8 This is used by the Bureau of Labour Statistics (United States) for the Current Population Survey.

The variance of these estimates will be determined by the overlap pattern and the correlations between estimates. The standard errors of monthly change depend on the correlation between the two estimates, which depends on the overlap in the sample between the two months and the correlation of the variable being estimated in the population. The higher the sample overlap the lower the standard error on movement estimates and the cost of the survey is reduced.

The in-for-8 rotation pattern is good for estimation of short term changes. The 4-8-4(8) rotation pattern is good for short term changes and better for quarterly averages. The 1-2-1(8) rotation pattern is poor for changes, s=1, 2 but good for changes *s*=3, 6 and very good for 3 average monthly and changes in them, such as overlapping averages and non-overlapping average.

Results from Steel (1996) show that for estimating monthly change the higher the monthly overlap the better, although the further gains diminish as the overlap increases. Because of the higher monthly correlation the gain from having monthly overlap is higher for employment estimates than estimates of unemployment.

For estimating quarterly averages and the changes in them the 1-2-1(m) patterns are better than the in-for-m patterns, because they result in independent monthly samples within the quarter and the overlap is concentrated at the 3 month lag. The 4-8-4(8) design is worse than the *in-for*-6 design for monthly change, but better for annual changes in monthly estimates

To decide on a rotation pattern we must decide on the relative importance of different estimates. Consider in-for-6, 1-2-1(5) and 4-8-4. Steel (1996) found for UK data for the monthly change in unemployment the 1-2-1(5) and 4-8-4 schemes have variance 2.30 and 1.13 times larger than the in-for-6 scheme For the 3 month change in the quarterly average unemployment the in-for-6 and 4-8-4 schemes have variance 1.98 and 2.3 times larger than the 1-2-1(5) scheme.

In summary we see that the rotation pattern determines how a unit is included in the sample over time. There are many choices of rotation patterns each with different properties in terms of overlap and time out of the survey. Thus we need a balance in the choice of rotation pattern. Usually the samples have a degree of overlap from period to period, which reduces costs and the standard errors associated with estimates of change between consecutive time periods. The rotation patterns currently used have been designed for levels and/or movements of original or seasonally adjusted estimates.

The rotation pattern chosen affects the correlation structure of the sampling errors over time, which can affect the properties of seasonally adjusted and trend estimates. Different rotation patterns produce different correlation structures, (McLaren and Steel, 2000b). Using linear approximations to the seasonally adjusted and trend estimates produced by X11 and X11 ARIMA McLaren and Steel (2000) considered the impact of different rotation patterns on the sampling variance of seasonally adjusted and trend estimates obtained by applying Henderson moving averages to the seasonally adjusted estimates. They found that the popular rotation designs that focus on obtaining high sample overlap between adjacent time periods are good for the estimated of changes between the adjacent periods for the original and seasonally adjusted estimates, but that for the level and change in trend estimates rotation pattern with little or no monthly overlap such as 2-2-2(8) and 1-2-1(8) give considerable gains. Rotation patterns such as 1-2-1(80 are also good for estimates of three month average of the seasonally adjusted estimates and changes in them. Steel and McLaren (2000b) reached similar conclusions for the mean squared error of the revision of trend estimates. McLaren and Steel (2001) considered the impact of different rotation patterns when estimates for the rotation groups are available. Steel and McLaren (2002) provides an overview of these results. The results suggest that there is a trade off when designing for movement estimates for seasonally adjusted and trend estimates. The recommended rotation pattern depends on users needs which can be wide and varied. These results suggest using 1-2-1(*m*) rotation pattern for monthly repeated surveys unless the one-month change in seasonally adjusted estimates is the key statistic to be analysed.

V. Composite Estimation

In overlapping samples the sampling errors are usually correlated over time. The sampling variance of the survey estimates can be reduced by exploiting the correlations over time between the estimates from each rotation group through various forms of least squares and composite estimation methods. These methods enable us to exploit the data from previous time periods through the correlation effectively increasing the sample on which the estimate is based.

In composite estimation the sample for the previous time periods is used along with the sample for the current period. In its simplest form the estimate for the current period is obtained by updating the estimate of the previous period using an estimate of the change that has occurred in which the matched and non-matched samples are given different weights. More generally the estimates for each rotation group can be determined and combined in an efficient manner taking into account the correlation structure of these estimates over time. However, issues of time in survey bias need to be considered (Bailer 1975).

Composite estimation methods have mainly been applied in monthly labour force surveys. Regression composite estimators have been developed to combine the benefits of composite estimation and regression estimation, which is a technique for exploiting extra information on auxiliary variables (Singh, Kennedy, and Wu, 2001). Wolter (1979) develops composite estimation for the 2 level schemes as used in the US Retail Trade Survey in which selected businesses report every three months giving data for the current and the previous months.

In assessing the potential gains from using composite estimation attention is usually focused on the estimate of level for the most recent period and the movement between the two most recent time periods. The gains arising from composite estimation depend on the degree of sample overlap and the individual level correlation. The gains for estimates of levels are greatest when the degree of overlap is moderate and the correlation high. For estimates of movement high sample overlap is still preferred.

Estimators can be developed that use data for all the T time periods and the correlation structure induced by the rotation pattern. We focus on rotation designs in which the sample in any period of G panels. When a panel is rotated out of the survey it is replaced by another panel. The set of panels related in this way is referred to as a rotation group. In a particular period elementary estimates can be calculated from each panel in the survey. Gurney and Daley (1965), Smith (1978) and Wolter (1979). Yansaneh and Fuller (1998) considered rotation pattens that are balanced in terms of the number of times units have been included in the surveys.

Suppose the sample is divided into rotation groups and we can calculate an estimate from each rotation group at time *t*, y_{tg} ,which is unbiased for Y_t, $E(y_{tg}) = Y_t$ .Stacking the G rotation group estimates gives the data vector $\mathbf{y} = (y_{11},..., y_{T1},..., y_{1G,},...y_{TG})^T$. Then $E(\mathbf{y}) = \mathbf{XY}_T$ where $\mathbf{Y}_T = (Y_1,...,Y_T)^T$ is the vector containing the series of population values and $\mathbf{X} = \mathbf{1}_G \otimes \mathbf{I}_T$. The rotation pattern leads to $Var(\mathbf{y}) = \mathbf{V}$ which also depends on the population correlations. The Best Linear Unbiased Estimator (BLUE) of $\mathbf{Y}_t$ is then $\left(\mathbf{X}^T\mathbf{V}^{-1}\mathbf{X}\right)^{-1}\mathbf{X}^T\mathbf{V}^{-1}\mathbf{y}$, which has variance $\left(\mathbf{X}^T\mathbf{V}^{-1}\mathbf{X}\right)^{-1}$. We need to determine or estimate $\mathbf{V}$ to calculate the BLUE.

The BLUE uses data for the entire series and involves storing elementary estimates for the entire length of the series and inversion of a TGxTG matrix. So for a monthly series of 40 years involving 8 rotation groups the matrix is 3640x3640. In practice methods that approximate the full BLUE can be used. For example, restricting the calculation to the most recent 36 months (Yansaneh and Fuller, 1998) or 7 months (Bell, 2001). The revision strategy also has to be considered, as when data for additional periods are added the estimated for previous periods change.

Yansaneh and Fuller (1998) noted that computation of the BLUE becomes complicated as the number of periods increases. The usual approach is to use as method of composite estimation in which the estimate for a particular period is obtained by updating the estimates for the preceding periods by taking into account the different correlations for the overlapping and non-overlapping parts of the sample. They developed a recursive regression estimator to produce an estimate equivalent to the BLUE based on estimates for *m* periods which avoids the complexity of the direct BLUE approach. Anther way to reduce the computational burden associated with the BLUE approach is to restrict the calculation the calculation for the last *m* periods. They compared the sampling variances of the CPS composite estimation with the restricted BLUE with m=12 and 16 and the recursive estimation for estimates of level and s month change, for s=1,…,12 for employment and unemployment, A selection of rotation patterns were considered, including 4-8-4(8), in-for-6 and in-for-8 rotation patterns. They commented that m=36 gave virtually the same efficiency as the recursive restricted estimator. The results suggest that for 4-8-4(8) rotation pattern useful gains can be made using BLUEs and that m=16 is almost as efficient as the recursive regression estimator.

Bell (1999) also considered restricted window BLUEs with m=7 and 1-2-1(8),2-2-2-(8) and 4-8-4(8) rotation patterns for the Australian Labour Force Survey. He found that m=7 gave nearly all the available gains but that smaller windows gave noticeable higher standard errors. The evaluation considered a range of estimators for employment and unemployment: level, one month movement, quarterly average, non-overlapping movement in quarterly average, seasonally adjusted, and one month movement in seasonally adjusted, trend estimate at the end of the series, one month movement in the trend estimates at the end of the series, revision of the movement in trend. This method has been introduced for the Australian Labour Force Survey (ABS 2007).

An approach that avoids the complexity of storing all the elementary estimates and inversion of a large matrix is to use composite estimation. The basic approach is to consider two estimates for the current month; the first is the usual estimate calculated form the data collected in the month, y_t and the second is obtained by taking the estimate from the previous month, y_{t-1}^K and adding an estimate of change based on the panels that have not been rotated, $y_{t-1}^K + (y_t^M - y_{t-1}^M)$, where y_t^M is the estimate for time t calculated from the matching sample. A weighed average of the two estimates is then calculated,

$$y_t^K = (1-K)y_t + K\left(\hat{y}_{t-1} + (y_t^M - y_{t-1}^M)\right)$$

This is the composite estimator initially used in the US CPS with *K*=0.5. An additional term is also added to further reduce variance and ameliorate the impact of times in survey effect, which is the difference between the estimates for the current month based on the new panel, y_t^{UM} , and the panels matching to the previous month, y_t^M giving the AK estimator (Gurney and Daley, 1965,) with *K*=0.4 and *A*=0.2,

$$y_t^{AK} = (1-K)y_t + K\left(\hat{y}_{t-1} + (y_t^M - y_{t-1}^M)\right) + A\left(y_t^{UM} - y_t^M\right) \qquad)$$

The composite estimator can give gains in efficiency because it exploits the correlation between estimates from the same panel over time and can give additional weight to the estimate for the current month that uses the common sample. The recursive nature of the estimator means that it is using the elementary estimates for the entire length of the series and therefore using some information from samples that have been rotated out of the sample. While the implicit weights are not the same as would be obtained from the BLUE, the efficiency can be close. The recursive nature means that the estimation for any month only needs access to the data for two consecutive months and the previous months estimate and troublesome matrix inversion is avoided.

The values of A and K need to be decided. For any particular variable they can be chosen to minimize the variance of y_t^K. Relevant variance formulas are given in (Cantwell, 1990). So in general they depend on the variable and time period. Compromise values are chosen based on analysis. Higher values are better for estimating employment levels because of the higher correlation over time..

A further refinement is called composite weighting. This approach uses values of A and K chosen separately for the estimation of employment and unemployment to produce marginal totals. The standard GREG weights are then adjusted so that the estimated agree with the margins obtained from the AK estimator (Lent *et al*. 1999).

The generalized composite estimator (see Cantwell, 1990 for example) is

$$y_t^{GCE} = \sum_{g=1}^{G} a_g y_{tg} - \omega \sum_{g=1}^{G} b_g y_{t-1,g} + \omega y_{t-1}^{GCE} \qquad (97)$$

The coefficients are constrained do that $\mathbf{1}^{\mathbf{T}}\mathbf{a} = \mathbf{1}^{\mathbf{T}}\mathbf{b} = 1$. Park, Kim and Lee (2001) present the general theory for choosing **a** and **b** for fixed ω when estimating 4 types of quantities; the current level, the change between levels, aggregates of several time periods and changes in aggregates. Kim, Park and Kim (2005) consider generalized composite estimator for three level data.

The Canadian LFS also uses a version of composite estimation that can be implemented in a standard GREG estimation system by clever specification of the auxiliary variables. The methods is referred to as modified regression estimation (Singh, Kennedy and Wu, 2001). Implementation is described by Gambino, Kennedy and Singh (2001). See also Fuller and Rao (2001) and Bell (2001). Cantwell and Caldwell (1998) consider composite estimation in a survey or retail and wholesale trade.

VI. Time Series Methods and Analysis

Producing estimates at regular intervals enables trends to be assessed. Methods of estimating trends are available using model-based or filter-based methods. For surveys producing monthly or quarterly estimates seasonal adjustment can remove the impact of regular factors operating at different times of the year. Producing seasonally adjusted estimates can assist in assessing the underlying direction or trends in the series. Analysis of time series aids in forecasting (prediction), identifying relationships between series, identifying turning points and assessing the underlying direction. Seasonal adjustment is a widely applied procedure used by government statistical agencies and has three main purposes: aid in short term forecasting, aid in relating time series to other series or extreme events and allow comparability in the time series from month to month.

The trend component is the underlying movement in a time series and reflects the underlying general level of the time series. The trend can be due to influences such as population growth, price inflation and general economic development. We assume that the trend contains the long term business cycle.

Seasonal adjustment involves the estimation and removal of systematic calendar related effects from the original estimates to give the seasonally adjusted estimates. Trend is derived by smoothing the seasonally adjusted estimates although there is no unique definition of trend. Filter based and model based approaches to seasonal adjustment are available, such as X11 (Shisken *et al.*, 1967), X12ARIMA (Findley *et al*.. 1998) and TRAMO-SEATS. Kenny and Durbin (1992) review issues associated with trend estimation. The Australian Bureau of Statistics publishes trend estimates obtained by applying Henderson moving averages to the seasonally adjusted series (ABS, 1993)

The rotation pattern chosen affects the correlation structure of the sampling errors over time, which affects the properties of seasonally adjusted and trend estimates. When undertaking time series analysis we need to take into account as much as we know to help improve the estimation process. Variances on seasonally adjusted and trend estimates are easily obtained from model based approaches.

Model based seasonal adjustment approaches not used extensively in government statistical agencies. Most other agencies use a filter based approach (X11 variant) and so we need methods of calculating variances that accommodate this type of approach. Methods for producing variance estimates for seasonally adjusted and trend estimates have been considered by Wolter and Monsour (1981) and Pfeffermann (1994) and reviewed by Scott *et al.* (2005).

Time series methods can help combine data across time and space to produce small area estimates from rotating panel surveys. See Pfeffermann and Burk (199), Pfeffermann and Bleur (1993) and Pfeffermann *et al*. (1998).

Methods of estimation assuming a time series structure for the population mean or total have also been developed (Binder and Dick 1989). Considering a time series approach for the population mean allows better estimates of the population means or the components of the series. Blight and Scott (1973) suggested a time series approach based on an AR(1) process and derived a recursive form of estimation for the unknown true values. They found the optimal overlap to minimise the variance of the one month difference in between two estimates. Scott and Smith (1974) and Scott *et al*. (1977) developed time series approach for both non-overlapping and overlapping samples where the sampling errors were assumed to be independent. Scott *et al.* (1977) extended this approach for more complex designs involving non-overlapping and overlapping samples. Jones (1980) unified the time series into a general form for the minimum mean squared estimator (MMSE), which involves large matrix calculations. Binder and Hidoroglou (1988) simplified this approach by using state space models and the Kalman Filter to allow efficient computation, which avoids the large matrix inversion and involves recursive estimation. Pfeffermann (1991) extended this approach and Tiller r(1992) describes it application to the estimation of state level estimates in the US. Feder (2001) provides a good review of the approach and further refinements for accounting for correlated sampling errors are given in Pfeffermann and Tiller (2006).

IX. References

Australian Bureau of Statistics (1993) *A Guide to Interpreting Time Series - Monitoring “Trends, An Overview*” catalogue no 1348.0 , Canberra, Australia: Australian Bureau of Statistics.

Australian Bureau of Statistics (2007). *Forthcoming changes to Labour Force Statistics* catalogue no 6292 , Canberra, Australia: Australian Bureau of Statistics.

Bailar, B. A. (1975). The effects of rotation group bias on estimates from panel surveys. *Journal of the American Statistical Association*, 70, 23-30

Bell, P.A. (1999). The impact of sample rotation patterns and composite estimation on survey outcomes. Australian Bureau of Statistics. Working Paper. Catalogue number 1351.0, no99/1 May 1999

Bell, Philip (2001). Comparison of alternative Labour Force Survey estimators. *Survey Methodology*, 27, 53-63.

Binder, D. A. and Dick, J. P. (1989). Modelling and estimation for repeated surveys. *Survey Methodology*, 15, 29-45

Binder, D. A. and Hidiroglou, M. A. (1988). Sampling in time *Handbook of Statistics Volume 6: Sampling*, 187-211 *Krishnaiah, P. R. (ed.) and Rao, C. R. (ed.) Elsevier/North-Holland [Elsevier Science Publishing Co., New York; North-Holland Publishing Co., Amsterdam] (New York; Amsterdam)*

Binder, David A. (1998). Longitudinal surveys: Why are these surveys different from all other surveys? *Survey Methodology*, 24, 101-108.

Blight, B. J. N. and Scott, A. J. (1973). A stochastic model for repeated surveys. *Journal of the Royal Statistical Society, Series B: Methodological*, 35, 61-66

Breu, P. and Ernst, L, (1983). Alternative estimators to the current composite estimators, *Proceedings of the survey research methods section, American Statistical Association,* 397-402.

Cantwell, P. J and Caldwell, (1998). Examining the revisions in monthly retail and wholesale trade surveys under a rotation panel design. *Journal of Official Statistics*, 14, 47-54.

Cantwell, P. J. (1990). Variance formulae for composite estimators in rotation designs. *Survey Methodology*, 16, 153-163

Caplan, D., Haworth, M. and Steel, D. G. (1999). UK labour market statistics: combining continuous survey data into monthly reports. *Bulletin of the International Statistical Institute,* 52nd Session, contributed papers, Book 2 p 25-26.

Duncan, G. J. , and Kalton, G. (1987). Issues of design and analysis of surveys across time. *International Statistical Review*, 55, 97-117.

Feder, M. (2001). Time series analysis of repeated surveys: The state-space approach. *Statistica Neerlandica*, 55, 182-199.

Findley, D.F., Monsell, B.C., Bell, W.R., Otto, M.C. and Chen B. (1998). New Capabilities and Methods of the X-12-ARIMA Seasonal-Adjustment Program. *Journal of Business and Economic Statistics*, Vol. 16, No. 2., 127-177.

Fuller, W. A. (1990). Analysis of repeated surveys *Survey Methodology*, 16, 167-180

Gambino, J. , Kennedy, B. , and Singh, M. P. (2001). Regression composite estimation for the Canadian Labour Force Survey: Evaluation and implementation Survey Methodology, 27 (1), 65-74

Gurney, M. and Daley. J.F. (1965). A mul,tivairet approach to estimation in priodiv canple surveys. *Proceedings of the survey research methods section, American Statistical Association,* 247-257.

Holt, D. and Skinner, C, J, (1983). Components of change in repeaters surveys. International Statistical Review, 57. 1-18.

Jones, R. G. (1980). Best linear unbiased estimators for repeated surveys. *Journal of the Royal Statistical Society, Series B: Methodological*, 42, 221-226

Kalton, G., and Citro, C. F. (1993). Panel surveys: Adding the fourth dimension. *Survey Methodology*, 19, 205-215.

Kasprzyk, D., Duncan, G., Kalton, G., and Singh, M. P. (eds) (1989),. *Panel Surveys*. John Wiley & Sons, New York.

Kenny, P.B., and Durbin, J. (1982). Local Trend Estimation and Seasonal Adjustment of Economic and Social Time Series. *Journal of the Royal Statistical Society*, Series A, 145, 1-41.

Kim, K. W., Park. Y. S. and Kim, N. Y. (2005,) l-Step Generalized Composite Estimator under 3-Way Balanced Rotation Design.. *Journal of the Korean Statistical Society*, 34, 219 - 233.

Kish, L. (1998). Space/time variations and rolling samples. *Journal of Official Statistics*, 14, 31-46.

Lee, H. (1990). Estimation of panel correlations the Canadian Labour Force. Survey Methodology, 16, 283-292.

Lent, J., Miller, S. and Cantwell, P. (1996). Effect of composite weights on some estimates for the Current Population Survey. *Proceeding of the survey reseach methods section, American Statistical Association,* 130-139.

McLaren, C. H. and Steel, D. G. (2000). The impact of different rotation patterns on the sampling variance of seasonally adjusted and trend estimates. *Survey Methodology*, 26, 163-172

McLaren, C. H. and Steel, D. G. (2001). Rotation patterns and trend estimation for repeated surveys using rotation group estimates. *Statistica Neerlandica*, 55, 221-238

Park, Y. S., Kim, K. W. and Choi, J. W. (2001). One-level rotation design balanced on time in monthly sample and in rotation group. *Journal of the American Statistical Association*, 96, 1483-1496

Pfeffermann, D. (1991). Estimation and seasonal adjustment of population means using data from repeated surveys. Journal of Business and Economic Statistics, **9**, 163-175.

Pfeffermann, D. (1994). A general method for estimating the variances of X-11 seasonally adjusted estimators. *Journal of Time Series Analysis*, 15, 85-116

Pfeffermann, D. and Bleuer, S. R. (1993). Robust joint modelling of labour force series of small areas. *Survey Methodology*, 19, 149-163

Pfeffermann, D. and Burck, L. (1990). Robust small area estimation combining time series and cross-sectional data. *Survey Methodology*, 16, 217-237

Pfeffermann, D. and Tiller, R. (2006). Small-area estimation with state–space models subject to benchmark constraints. *Journal of the American Statistical Association*, 101, 1387-1397

Pfeffermann, D., Feder, M. and Signorelli, D. (1998). Estimation of autocorrelations of survey errors with application to trend estimation in small areas. *Journal of Business & Economic Statistics*, 16, 339-348

Rao, J.N.K. and Graham, J. E (1964). Rotation designs for sampling on repeated occasions. *Journal of the American Statistical Association*, 69, 492-501.

Scott, A. J. and Smith, T. M. F. (1974). Analysis of repeated surveys using time series methods. *Journal of the American Statistical Association*, 69, 674-678

Scott, A. J., Smith, T. M. F. and Jones, R. G. (1977). The application of time series methods to the analysis of repeated surveys. *International Statistical Review*, 45, 13-28

Scott, S., Sverchkov, M.l and Pfeffermann, D. (2005). Variance measures for X-11 seasonal adjustment: A summing up of empirical work. *ASA Proceedings of the Joint Statistical Meetings*, 3534-3545. *American Statistical Association (Alexandria, VA)*

Shiskin, J., Young, A. H. and Musgrave, J.C. (1967). The X11 Variant of the Census Method II Seasonal Adjustment Program. Technical Paper 15, Bureau of the Census, U.S. Department of Commerce, Washington, D.C.

Singh, A.C. , Kennedy, B. , and Wu, S. (2001). Regression composite estimation for the Canadian Labour Force Survey with a rotating panel design. *Survey Methodology*, **27**, 33-44.

Smith, T. M. F. (1978). Principles and problems in the analysis of repeated surveys. *Survey Sampling and Measurement*, 201-216. *Namboodiri, N. Krishnan (ed.) Academic Press (New York; London)*

Steel D. G. and McLaren C. H. (2000a). Designing for Trend Estimation in Repeated Business Surveys. *Proceedings of the Second International Conference on Establishment Surveys*, Invited Papers, pp799-808. American Statistical Association, Alexandria, VA.

Steel D.G. and McLaren C.H., (2000b) “The effect of different rotation patterns on the revision of trend estimates”, Journal of Official Statistics, Vol 16, No. 1, pp 61-76.

Steel, D. (1997). Producing monthly estimates of unemployment and employment according to the international labour office definition (Disc: p33-46). *Journal of the Royal Statistical Society*, Series A, 160 , 5-33.

Steel, D. G (2004). Sampling in Time. in *Encyclopaedia of Social Measurement*, pp823-828 Academic Press.

Steel, D. G. and McLaren, C. H. (2000). The effect of different rotation patterns on the revisions of trend estimates. *Journal of Official Statistics*, 16, 61-76

Steel, D.G. (1996). Options for Producing Monthly Estimates of Unemployment According to the ILO Definition. London: Central Statistical Office.

Steel, D.G. and McLaren, C.H. (2002). In Search of a Good Rotation Pattern, in *Advances is Statistics, Combinatorics and Related Areas*. Chandra Gulati, Yan-Xia Lin, Satya Mishra and John Rayner (eds), pp309-319. Singapore: World Scientific.

Tiller, R.B. (1992). Time Series Modeling of Sample Survey Data from the U.S. Current Population Survey. *Journal of Official Statistics,* Vol. 8. No. 2, 149-166.

Wolter, K.M. and Monsour, N.J. (1981). On the Problem of Variance Estimation for a Deseasonalised Series. In Current Topics in Survey Sampling, (eds. D. Krewski, R. Platek and J.N.K. Rao), New York, Academic Press, 367-403.

Yansaneh, I. S. and Fuller, W. A. (1998). Optimal recursive estimation for repeated surveys. *Survey Methodology*, 24, 31-40.