Divisibility

INTRODUCTION

If a number can be exactly divided by a second number, with no remainder, then we say that the first number is *divisible* by the second number. For example, 6 can be divided by 3 (with zero remainder) so we say that 6 *is divisible by* 3. (We can also say that 3 is a *factor* of 6 and 6 is a *multiple* of 3, but that is not what this handout is about!) Note that 0 divided by anything (except 0) is 0 with no remainder, so 0 is divisible by all numbers (except 0).

It can be difficult to divide numbers that are large or perhaps unfamiliar to you. However, there are procedures that enable us to quickly work out whether a number is divisible by other numbers. (The methods don't give the answer for the division problem, they are simply a guide to see if the number CAN be divided by the other number.) This is useful for simplifying fractions, or in a division problem to check if your answer should have a remainder or not.

You need to know that the word *digit* refers to symbols for numbers: 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. For example, the number 453872 contains the digits 2, 3, 4, 5, 7 and 8. We can add the value of each of those digits together or do other maths operations on them. So, if we add all the digits in 453872, we get $4 + 5 + 3 + 8 + 7 + 2$, and that gives 29. If we add the digits in 1253226, we get $1 + 2 + 5 + 3 + 2 + 2 + 6$, which is 21.

DIVISIBILITY TESTS SUMMARY

The following table gives a summary of all the divisibility tests. If for a given number the answer to a question is "**yes**" then it is **divisible** by the corresponding number in the table, but if it is "**no**" then it is **not divisible**. The tests will be explained further in the following section and examples given.

Dividing By	Test
2	Is the number even? i.e. does it end in 0, 2, 4, 6, or 8
3	Is the sum of the digits divisible by 3?
4	Do the last two digits form a number that is divisible by 4?
5	Does the number end in 0 or 5?
6	Is the number even and divisible by 3?
7	Remove the last digit and subtract twice its value from the remaining digits, is the result divisible by 7?
8	Do the last three digits form a number that is divisible by 8?
9	Is the sum of the digits divisible by 9?
10	Does the number end in 0?
11	Add the digits in the odd positions, then add the digits in the even positions, is the difference divisible by 11?
12	Is the number divisible by both 3 and 4?

2

If a number is *even* (that is, if tis last digit is 0, 2, 4, 6, or 8) then, and only then, is it divisible by 2.

For a number that is as large as 3694806, it is now easy to see that we can divide it exactly by 2 (because it ends in a 6). This could also be stated as: if the last digit is divisible by 2, then it is divisible by 2.

3

To determine whether a number is divisible by 3, we add up all the digits in the number. If that total is divisible by 3, then and only then is the original number divisible by 3.

For our example above, 3694806, we would add up the digits, $3 + 6 + 9 + 4 + 8 + 0 + 6 = 36$, and remember that 36 is divisible by 3, hence 3694806 is also divisible by 3. (To calculate what the value is when 3694806 is divided by 3, we would need to do the division procedure, but at least we know there should not be any remainder!)

4

Similar to the 2 case, to determine if a number is divisible by 4, we only need to check if the last two digits are divisible by 4. That is: a number is divisible by 4 if its last two digits, read as a two digit number, is divisible by 4.

For our example above, 3694806, we take the last two digits 06, which is the same as 6, and see this is not divisible by 4, so 3694806 is not divisible by 4 (if we tried the division we would get a remainder). In contrast, the for the number 3257816 we immediately see the its last two digits (16) are divisible by 4 and hence 3257816 is divisible by 4. A fast way to divide by 4 is to divide the original number by 2, then divide by 2 again. In our example you would get 1628908 after the first division, then 814454 after the second.

5

If the number ends in 0, or 5 then and only then is it divisible by 5.

This is also similar to the 2 test and also could be phrased as: if the last digit is divisible by 5, then the number is divisible by 5. So our example of 3694806 is not divisible by 5 (since it ends in a 6), while 489124665 is divisible by 5 (since it ends in a 5). Of course we would still need to do the division in this second case to obtain an answer, but at least we know there will be no remainder.

6

A number is divisible by 6 if and only if it is divisible by both 2 and 3. That is, if it is even and the sum of the digits can be divided by 3.

We have already seen that our example of 3694806 is divisible by both 2 and 3, so this tells us it is divisible by 6. Let's do the division in this case (by our rule we should not end up with a remainder):

$$\begin{array}{r} 61\ 5\ 801 \\ \hline 6)\ 369^{3}4^{4}806 \end{array}$$

7

Unfortunately there is no straightforward rule for testing divisibility by 7. It is often easiest to just try the division and see if you end up with 0 remainder. However, there is one rule that may be useful:

Subtract twice the last digit from the remaining digits. If the result is divisible by 7, then (and only then) the original number was divisible by 7.

After doing the subtraction you may end up with a number that is negative or zero. Zero is divisible by 7 (since every non-zero number divides zero) and to deal with negative numbers we can just ignore the sign. You may have to repeat this rule several times.

Let's start with a simple example: Is 896 divisible by 7?

We take the last digit (6), double it (12) and subtract it from the remaining digits (89). This gives:

$$89 - 2 \times 6 = 89 - 12 = 77$$

We know that 77 is divisible by 7 (since $11 \times 7 = 77$), so this means that 896 is also divisible by 7.

Now, let's try our previous example of 3694806:

We remove the 6 and subtract twice it from the remaining digits:

$$369480 - 2 \times 6 = 369468$$

We still can't tell if this number is divisible by 7 or not so we repeat the procedure:

$$36946 - 2 \times 8 = 36930$$

Let's go again:

$$3693 - 2 \times 0 = 3693$$

This is taking a long time:

$$369 - 2 \times 3 = 363$$

Still not quite sure:

$$36 - 2 \times 3 = 30$$

Finally we see that 30 is not divisible by 7, so neither is 3694806 (or for that matter any of our other numbers we found along the way).

With the amount of calculations it may have been easier to just do the division, let's try that:

$$\begin{array}{r} 5\ 2\ 7\ 8\ 2\ 9 \\ \hline 7)\ 36^{1}9^{5}4^{5}8^{2}0^{6}6^{3} \end{array}$$

We see here we are left with 3 remainder, so it is not divisible by 7 as we found out earlier.

8

A number is divisible by 8 if and only if the number formed by its last three digits is divisible by 8.

This test still involves dividing a three digit number by 8, but it can be useful if the number is ten digits long. A rule to help in dividing by 8 is that you divide the number by 2, then 2 again, and finally divide by 2 a third time (that is you halve the number, then halve that number, then halve THAT number).

For example our number 3694806 is not divisible by 8 as 806 is not divisible by 8 (you would find it has 6 remainder if you do the division, or that after halving it you get 403 which can't be halved again without going into decimals).

9

To determine whether a number is divisible by 9, we add up all the digits in the number. If that total is divisible by 9, then and only then is the original number divisible by 9.

Notice that this is just like our rule for divisibility by 3. For our example above, 3694806, we have already added up the digits to obtain 36, which is divisible by 9, hence 3694806 is also divisible by 9.

10

A number is divisible by 10 if and only if it ends in a 0.

You probably know this one already. We see that 3694806 is not divisible by 10, while 3694810 is divisible by 10.

11

Add every second digit of the number (i.e. all the ones in the odd positions), then add the digits you skipped (i.e. all the digits in the even positions). If the difference in the two numbers is divisible by 11 then and only then was the original number divisible by 11.

This is another strange one. Again you have to remember that zero is divisible by all numbers, and that if it is negative we can ignore the negative sign. Another method to get the number is to alternate adding and subtracting the digits.

For example: Is 3694806 divisible by 11?

We sum all the odd positioned digits: $3 + 9 + 8 + 6 = 26$

We sum all the even positioned digits: $6 + 4 + 0 = 10$

We find their difference: $26 - 10 = 16$

Since 16 is not divisible by 11 we can determine that 3694806 is not divisible by 11.

Another example: Is 53834 divisible by 11?

This time let's do it in one calculation by alternating adding and subtraction:

$$5 - 3 + 8 - 3 + 4 = 11$$

Since this is divisible by 11 this means that 53834 is divisible by 11.

12

A number is divisible by 12 if and only if it is divisible by both 4 and 3. That is, if its last two digits are divisible by 4 and the sum of the digits can be divided by 3.

We have already seen that 3694806 is not divisible by 4, so it is also not divisible by 12. Similarly 3257816 is not divisible by 12 since it is not divisible by 3 (the sum of its digits is 32). However, 3257820 is divisible by 12 since it is both divisible by 3 (its digits sum to 27) and divisible by 4 (its last two digits make the number 20).

Here are some exercises for you to try. You can check your answers with the solutions at the end of the resource.

EXERCISES

1. Complete the following divisibility table for 4620.

4620 is divisible by:	2	5	8	9	11	12
Yes/No	Yes					

2. Complete the following divisibility table for 180473.

180473 is divisible by:	3	4	7	9	12
Yes/No					

3. Complete the following divisibility table for 13320.

13320 is divisible by:	2	5	8	9	11	12
Yes/No						

4. Complete the following divisibility table for 386250.

386250 is divisible by:	3	4	5	7	9	11	12
Yes/No							

If you need help with any of the maths covered in this resource (or any other maths topics), you can make an appointment with Learning Development through reception: phone (02) 4221 3977, or Level 2 (top floor), Building 11, or through your campus.

SOLUTIONS TO EXERCISES

1. Complete the following divisibility table for 4620.

4620 is divisible by:	2	5	8	9	11	12
Yes/No	Yes	Yes	No	No	Yes	Yes

2. Complete the following divisibility table for 180473.

180473 is divisible by:	3	4	7	9	11
Yes/No	No	No	No	No	No

3. Complete the following divisibility table for 13320.

13320 is divisible by:	2	5	8	9	11	12
Yes/No	Yes	Yes	Yes	Yes	No	Yes

4. Complete the following divisibility table for 386250.

386250 is divisible by:	3	4	5	7	9	11	12
Yes/No	Yes	No	Yes	No	No	No	No