

Ways of Writing Numbers

Scientific Notation, Significant Figures, and Decimal Places

INTRODUCTION

First, a short review of our *decimal system* (also called *base 10*). This system is so efficient that we can use it to write any number we like, no matter how big or how small. But, when numbers get either TOO big or small, then the amount of digits becomes unwieldy. That's where *scientific notation*, *significant figures* and *decimal places* all come in. You probably remember from school that the place value of a number is important because it determines how big the number is. For example, here are some of the names we give to whole number place values:

Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones/Units
----------	-------------------	---------------	-----------	----------	------	------------

Reading from the right (units), each column to the left has been obtained by multiplying by 10 (each column represents a number that is 10 times bigger than the one on its right). For example, 10 is 10×1 , 100 is 10×10 , 1000 is 10×100 and so on. Going in the other direction, each column to the right has been obtained by dividing the one on its left by 10.

The decimal point which goes straight after the units column tells us that we no longer have a whole number, and that the unit has been divided by 10. That's why 0.1 is the same as $1 \div 10$, or $\frac{1}{10}$. 1 has been divided by 10 in the same way as $100 \div 10 = 10$, or $1000 \div 10 = 100$. The number is moved across a column to make it smaller. So, we get new headings for our place values AFTER (to the right of) the decimal point.

Thousands	Hundreds	Tens	Ones/Units	.	Tenths	Hundredths	Thousandths
-----------	----------	------	------------	---	--------	------------	-------------

One tenth can be written as $\frac{1}{10}$ or 0.1, a hundredth can be written as $\frac{1}{100}$ or 0.01, and a thousandth can be written as $\frac{1}{1000}$ or 0.001.

DECIMAL PLACES

Decimal places are the positions of numbers that appear to the right of the decimal point. They can give us a way of stating how accurately we want to measure something. For example, when you fill up the car at the petrol pump, you know that the price of petrol is given to three places of decimals (in cents), yet we round that to two places when we pay – in fact we even round to the nearest 5 cents. So the calculation that the machine does is accurate right up until the very last step. For example, my car might take 32.4 litres when the price is 139.9 cents per litre. If we calculate this (and of course you can use your calculator!) we get 4532.76. Thankfully, this is cents, so we need to divide by 100 and get \$45.3276. Now, rounding to two decimal places (the nearest cent), we look at the third decimal place – that is the 7 – and ignore everything that comes after it. If the third decimal place is 5, 6, 7, 8, or 9, then we “round up” and we



would get \$45.33*. This is because the 7 is more than halfway up to the next number (this is similar to 327 being closer to 330 than it is to 320). You might like to put a little line to the right of the number you're going to round to: e.g. 45.32|76 to help you.

Let's say we have an answer to some other calculation and it looks like this: 5894.874128. That is too many digits for the answer to make sense in our calculation so let's try to round it to four decimal places. So we look at 5894.8741|28. This time the 2 to the right of the line tells us to keep the fourth number just as it is, so we get 5894.8741. So our number represents 5 thousands, 8 hundreds, 9 tens, 4 ones, 8 tenths, 7 hundredths, 4 thousandths, and 1 ten thousandth. We rounded off the 2 hundred thousandths and 8 millionths! Of course this all depends on how precise we want an answer to be.

Here are some for you to try for yourself: You can check your answers to these using the solutions at the end of the resource.

EXERCISES

1. Round the following numbers to two decimal places:
a. 14.803 b. 167.1509 c. 38.598

2. How much would you pay for 1.8 kg apples if they cost \$4.59 per kg?

3. A sheet of glass measures 650 mm by 455 mm. Calculate the area in square metres, correct to 3 decimal places. (Hint: change the mm measurements to metres first by dividing by 1000, then multiply.)

4. Calculate the tax on \$15825.14 at 35 cents in the dollar. (Hint: change the 35 cents to dollars first and multiply.)

SIGNIFICANT FIGURES

Significant figures are different from decimal places, although both involve the use of "rounding". Significant figures apply to whole numbers as well as decimals. Significant figures are the digits that are regarded as "important" when stating a number or measurement.

All non-zero numbers are "significant". Zeros, 0s, are significant when they are between two non-zero digits, for example in 504 or 3001 have 3 and 4 significant digits respectively, or at the end of numbers that have a decimal point, for example 3.50 or 2.010 also have 3 and 4 significant digits respectively. Zeros or not significant if they are at the end of a whole number or at the beginning of a decimal number, in this case they are seen as a place holder and not counted as significant. An example of this might be: the crowd at a game of football is accurately counted as 38521. Do you think it is more likely that a commentator would say: "the crowd today is thirty-eight thousand, five hundred

* Of course if paying cash we would actually pay \$43.35.



and twenty-one”, or would they say: “the crowd is nearly forty thousand”? The forty thousand is a lot easier to say, and for the listener to understand. 40000 has only 1 significant figure (the 4) and the zeros tell us its place value – that it is in the ten thousands column. The accurate number (38521) has 5 significant figures (ALL OF THEM!) and when we read or hear it we know that everyone in the crowd has been counted.

Another example: Round 0.005218 to 2 significant figures. As with decimal places, we can put a line to the right of the number we need to round at, so 0.0052|18 (the zeros are not significant, and as the 5 and 2 are the first 2 significant figures we are only interested in them). The 1 tells us to leave the 2 alone, so $0.005218 \approx 0.0052$ correct to 2 significant figures.

Here are some for you to try. You can check your answers from the back of the handout.

EXERCISES

5. Round each of the following to 3 significant figures:
 - a. 58942
 - b. 632.1
 - c. 0.04793
 - d. 409100

6. The distance from the Earth to the Moon is 384400 km. Express this distance correct to 2 significant figures.

7. The diameter of a golf ball is 4.267 cm. Express this correct to 3 significant figures.

SCIENTIFIC NOTATION

To explain scientific notation we must return to the origin of our decimal system. This time in our table we will express the place value using its power of 10, this is because interpreting powers of 10 is vital when reading scientific notation:

Thousands	Hundreds	Tens	Ones/Units	.	Tenths	Hundredths	Thousandths
10^3	10^2	10^1	10^0		10^{-1}	10^{-2}	10^{-3}

Recalling that 10^2 is $10 \times 10 = 100$, we see that this one does line up nicely. The $10^3 = 10 \times 10 \times 10 = 1000$, so that is also correct, and we can continue increasing the power by 1 every time we want to move to the left (this is just like multiplying by 10). Likewise when moving to the right we are decreasing the power by 1 (just like dividing by 10). We can continue doing this past the decimal point (since we are still dividing by 10 as we move). If you want more information about this, and negative powers, please refer to *Powers and Logarithms – 1. Powers, Indices, Exponents*.

We can use these powers of ten to write large or small numbers in a more efficient manner. This efficient manner is called *scientific notation* or *standard form*.

For example, we may wish to write the distance of the Earth to the Moon, 384400 km, more efficiently. We see that the 3 is really 3 hundred thousand, and is in the column of 10^5 . We put a decimal point after the 3 and call our number 3.844×10^5 . To check this result, you can multiply 3.844 by $10 \times 10 \times 10 \times 10 \times 10$ (so you would move the decimal point to the right 5 times). In general, you might like to use the reverse method where you place a decimal point after the first non-zero digit, then work out how many times you would multiply by 10 (by moving the decimal point) until you get back to the original number. This has the benefit that moving the decimal point to the right (which is our usual positive direction) results in a positive power.



Second Example: The diameter of the Sun is 1390000 km, write this in scientific notation.

So, we put a decimal point after the first digit 1.39 (we don't need the zeros on the end as they are just place holders and are not "significant"). We then see that we would need to move the decimal point 6 spots to the right to return to our original notation, so 1390000 km in scientific notation is 1.39×10^6 km. You could also check that the first digit of the original notation has a place value of millions, and one million is 10^6 .

Decimal numbers (numbers that are between 0 and 1) will always have negative powers of 10 as their multiplier. This is because we would need to move the decimal point to the left, but we are counting how far we move it right.

Example: The thickness of a human hair can be as small as 0.000017 metres. To write this in scientific notation, again we can put the decimal point after the first non-zero digit, which is a 1, to get 1.7. We could use a table to look at where the 1 lines up with our powers of 10, but we would need a large table to do this (you might use the patten of the powers to see it would be 10^{-5}). Instead of doing this we note that to get from 1.7 to 0.000017 we move the decimal point 5 places to the left, i.e. in the negative direction. So, 0.000017 m is 1.7×10^{-5} m.

In summary, numbers smaller than 1 are expressed with negative powers of 10, but numbers equal to or larger than 10 are expressed with positive powers of 10. In the case of a number being between 1 and 10, we would write this with a power of 0. For example, $9.42 = 9.42 \times 10^0$.

Scientific notation also helps with working out significant figures, because when using scientific notation all digits are significant!

This also helps clear up a slight ambiguity in significant figures. Suppose you are counting crowd numbers and you find that there are exactly 39000 people in the stadium. In this case, the zeros are significant, because they are not just being used as place holders, but we have no way of specifying this in normal notation. In scientific notation, however, we could write the crowd figure as 3.9000×10^4 , which makes it clear those trailing zeros are significant.

Here are some for you to try. You can check your answers with the solutions at the end of this resource.

EXERCISES

8. Write these numbers using scientific notation (i.e. in standard form):
- a. 38950000 b. 0.00421 c. 128.53 d. 0.093
9. The distance from the Earth to the Sun has been calculated as 149597870.7 km. Write this distance in scientific notation.
10. A large flea is approximately 0.0033 m long. Write this measurement in scientific notation.
11. Are the following numbers very large or very small?
- a. 6.73×10^{24} b. 1.89×10^{-16}

If you need help with any of the maths covered in this resource (or any other maths topic), you can make an appointment with Learning Development through reception: phone (02) 4221 3977, or Level 2 (top floor), Building 11, or through your campus.



SOLUTIONS TO EXERCISES

1. Round the following numbers to two decimal places:
- $14.803 \approx 14.80$
 - $167.1509 \approx 16.15$
 - $38.598 \approx 38.60$ (we must round the 9 up, so it would become 10, which means we have to carry the 1 and add it to the 5)

2. How much would you pay for 1.8 kg apples if they cost \$4.59 per kg?
 $1.8 \times 4.59 = 8.262$ so we round this to 2 decimal places to obtain **\$8.26**.

3. A sheet of glass measures 650 mm by 455 mm. Calculate the area in square metres, correct to 3 decimal places. (Hint: change the mm measurements to metres first by dividing by 1000, then multiply.)
650 mm gets changed to 0.65 m, and 455 mm gets changed to 0.455 m. Therefore the area is

$$\begin{aligned} A &= 0.65 \text{ m} \times 0.455 \text{ m} \\ &= 0.29575 \text{ m}^2 \\ &\approx 0.296 \text{ m}^2 \end{aligned}$$

4. Calculate the tax on \$15825.14 at 35 cents in the dollar. (Hint: change the 35 cents to dollars first and multiply.)

35 cents is \$0.35, so the tax on \$15825.14 is:

$$\begin{aligned} \text{Tax} &= \$15825.14 \times 0.35 \\ &= \$5538.799 \\ &\approx \$5538.80 \end{aligned}$$

5. Round each of the following to 3 significant figures:

- $58942 \approx 58900$
- $632.1 \approx 632$
- $0.04793 \approx 0.0479$
- $409100 \approx 409000$

6. The distance from the Earth to the Moon is 384400 km. Express this distance correct to 2 significant figures.
 $384400 \text{ km} \approx 380000 \text{ km}$

7. The diameter of a golf ball is 4.267 cm. Express this correct to 3 significant figures.
 $4.267 \text{ cm} \approx 4.27 \text{ cm}$

8. Write these numbers using scientific notation (i.e. in standard form):

- $38950000 = 3.895 \times 10^7$
- $0.00421 = 4.21 \times 10^{-3}$
- $128.53 = 1.2853 \times 10^2$
- $0.093 = 9.3 \times 10^{-2}$

9. The distance from the Earth to the Sun has been calculated as 149597870.7 km. Write this distance in scientific notation.

$$149597870.7 \text{ km} = 1.495978707 \times 10^8 \text{ km}$$

We might also like to round this number if we don't need so much accuracy. To three significant figures this is $1.50 \times 10^8 \text{ km}$.



10. A large flea is approximately 0.0033 m long. Write this measurement in scientific notation.

$$0.0033 \text{ m} = 3.3 \times 10^{-3} \text{ m}$$

11. Are the following numbers very large or very small?

- a. 6.73×10^{24} is **very large**, we are multiplying by a one followed by 24 zeros! Equivalently we need to move the decimal point 24 spots to the right.
- b. 1.89×10^{-16} is **very small**, we are moving the decimal point 16 spots to the left.

