Percentages

Understanding and Using Percentages

If you haven't done maths for a while, it might be best for you to start with *Fractions – 4. Fractions, Decimals, and Percentages.*

WHAT ARE THEY?

Percentages occur EVERYWHERE! You just need to take a walk around a suburban shopping centre to find them – shops (discounts, mark-ups), supermarkets (food packaging – nutrition), banks (interest rates), estate agents (commissions, stamp duties), Medicare (rebates), and so on. We even find percentages mentioned in sport (goals scored out of shots taken, for example), weather (humidity) and of course the marks you obtain in the subjects you are studying. In other words, it is important to understand and to be able to use percentages.

So, what is a percent(age)? *per cent* means *out of* 100. If you look closely (and use your imagination!) you can see that the percent sign (%) consists of a 1 and two 0s. That should remind you that it has something to do with 100!

Here is a little exercise. It will tell you that you know quite a lot about percentages already! You can check your answers with the solutions at the end.

EXERCISES

1. Match a percent to all these common phrases. For many of them, there is not an exact answer, but for some there are (you will recognize them!).

Phrase	%	Phrase	%
Just about all		About a third	
Hardly any		The lot	
A large proportion		Half	
Most of		None	
Twice as much		A quarter	

Again, % means *out of* 100, or *as a proportion of* 100, *over* 100, or *divided by* 100. Each of these definitions gives us a way of viewing what percentages represent. From the table, you should realise that you already have a good knowledge of these proportions. 80%, for example, is more than half, so it is a fairly large percentage. If you gained 80% in an assessment task, you should be very happy with the result because you would not have very much wrong! Imagine 80% of the population voting for a politician – that politician would be rated as very popular! But, in this case, it does not mean that exactly 100 people voted and that 80 of those people voted for that politician; it means that out of every 100 people who voted, 80 voted for the politician. There could have been as many as 50000 (or more!) who voted or as few as 10.

CALCULATING WITH PERCENTAGES

This brings us to calculating with percentages and the use of proportions. By referring back to the original definition of percentages (out of 100), we see that 80% means 80 out of 100, which can be written as $\frac{80}{100}$. We can then express this fraction, or proportion, in lower terms by dividing both 80 and 100 by 20, so we get:

$$\frac{80}{100} = \frac{80 \div 20}{100 \div 20} = \frac{4}{5}.$$

So 80% represents the same proportion as $\frac{4}{5}$ does, that is, 4 out of 5. We could say that 4 out of every 5 people voted for the politician, (or 8 out of every 10 people did, or 12 out of 15, or 16 out of 20, or 4000 out of 5000, …), or that 80% of people voted for the politician.

If we want to express a percentage as a fraction, we just remember the definition and simplify it if possible. So, we might want to express 5% as a fraction. This is obviously a small fraction. It is only 5 out of 100, or $\frac{5}{100}$. This time we can divide both 5 and 100 by 5 and so we get $\frac{5\div5}{100\div5}=\frac{1}{20}$, a small proportion as expected.

To reverse this procedure, that is to go back from a fraction to a percentage, you multiply the fraction by 100. So, for example, let's do one where you will know the answer. Let's see what $\frac{1}{2}$ is as a percentage. We multiply it by 100, so:

$$\begin{aligned}\frac{1}{2}&=\frac{1}{2}\times 100\%\\&=\frac{1}{2}\times\frac{100}{1}\%\\&=\frac{100}{2}\%\\&=50\%.\end{aligned}$$

Another example: Express $\frac{2}{3}$ as a percentage.

$$\begin{aligned}\frac{2}{3}&=\frac{2}{3}\times 100\%\\&=\frac{2}{3}\times\frac{100}{1}\%\\&=\frac{200}{3}\%\\&=66.\dot{6}\%.\end{aligned}$$

Recall that a dot above a decimal point means that digit recurs indefinitely, i.e. $66.\dot{6}=66.666666$ ....

Here are some for you to try. You can check your results with the solutions at the end.

EXERCISES

2. Complete this table by filling in the blanks.

Fraction	Percent
	20%
$\frac{1}{4}$	
	70%
$\frac{9}{10}$	
	100%
$\frac{3}{4}$	
	$33\frac{1}{3}\%$

Often, percentages are expressed in decimal form. Again, to see how they are equivalent, we need to think about the original definition, that is, % means out of 100, or divided by 100. In this case, division is appropriate. So, for example, 80% is the same as 80 ÷ 100. Remembering place value, we obtain 0.8. (We do not need to put the extra 0 on the end – to make 0.80 – as it does not make any difference to the decimal.)

For example: 5% as a decimal is 5 ÷ 100, that is 0.05.

What about 23.5%? Again, just divide 23.5 by 100, and obtain 0.235. (Really, what is happening here is that 23.5 as a proportion of 100 is identical to 0.235 as a proportion of 1 whole unit.)

Reversing the procedure, that is, going from a decimal amount to a percentage, we multiply by 100. So, for example,

$$0.58 = 0.58 \times 100\% \\ = 58\%$$

And

$$1.91 = 1.91 \times 100\% \\ = 191\%$$

Note, there is nothing wrong with having percentages greater than 100.

Here are some for you to try. You can check your results with the solutions at the end.

EXERCISES

3. In the table, match the decimals on the left with their percentages on the right.

Decimal	Percent
0.48	201%
0.9	0.2%
0.67	100%
1	48%
0.09	9%
2.01	90%
0.002	67%

FINDING A PERCENTAGE OF A QUANTITY

We often need to calculate a percentage of a quantity. For example, we might want to know the value of a discount on something we are going to buy, or to know how much we will get back on our doctor's fee through Medicare, or to work out how much interest we have to pay or – hopefully – receive. The politician might want to know how many people voted for him or her, or you might read an article that discusses percentages and would like to know more details. Whatever is the case; there are several methods for calculating a % of some quantity.

For example: To find 30% of $480, you could:

- Multiply (because "of" means "multiply") 30 and 480 then divide by 100 ($30 \times 480 = 14400$, and $14400 \div 100 = 144$, so the answer is 144), or
- Change 30% to a fraction and multiply ($30\% = \frac{30}{100} = \frac{3}{10}$, and $\frac{3}{10} \times 480 = \frac{1440}{10} = 144$), or

- Change 30% to a decimal and multiply ($30\% = 0.3$, and $0.3 \times 480 = 144$), or
- Calculate 1% and multiply it by 30 (1% of 480 is 4.8, and $30 \times 4.8 = 144$), or
- Use a calculator! Enter 480×30 but before pressing equals, select the % symbol (it should be a secondary function of one the buttons, e.g. the open bracket or equals buttons, and is accessed via the *shift* button).

Notice that in every case you were multiplying the numbers and dividing by 100, just in different orders.

Second example: You want to buy a TV that is priced at $2999 but the store you go to has a sale on, which advertises 15% off store wide. How much money do you save?

We could use any of the methods above.

$$15 \times 2999 \div 100 = 44985 \div 100 = 449.85$$

So we save $449.85. Or:

$$\frac{15}{100} \times 2999 = \frac{3}{20} \times 2999 = \frac{8997}{20} = 449.85$$

Or, $15\% = 0.15$ so:

$$0.15 \times 2999 = 449.85$$

Or, 1% of 2999 is 29.99 so:

$$15 \times 29.99 = 449.85$$

Or, use a calculator.

There are always multiple ways to solve any problem and it is important that you choose one that you are comfortable. Some people may prefer not to deal with large numbers so use the decimal method. Others may not like decimals so put off bringing them in until the end, etc.

Here are some for you to try. You can check your results with the solutions at the end.

EXERCISES

4. A musician received a sponsorship on his musical instruments so that he only had to pay 35% of the cost. He wanted a new drum that originally cost $2000. How much would he need to pay?

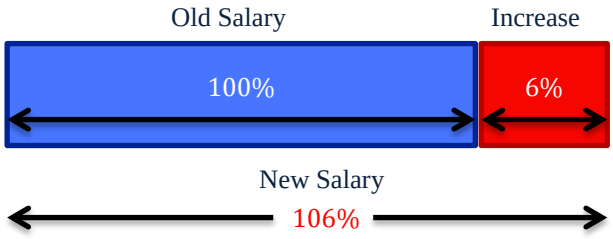
5. In a recent poll of 1500 voters, 59% of the people asked said they had confidence in the government and 37% said they were not confident.
   a. How many people said they were confident?
   b. How many people said they lacked confidence?
   c. How many people did not express their opinion?

6. A person's salary was $45000 in their first year of employment and increased by 6% in their second year. What was their pay increase, and how much was their new salary?

7. A couple buying a property for $270000 had to pay 3.5% stamp duty on $190000. How much did they have to pay in stamp duty?

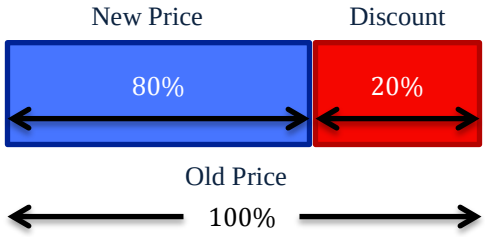
INCREASING AND DECREASING BY PERCENTAGE AMOUNTS

In problem six above we asked about the increase in a person's salary. Stating that there was a 6% increase we asked what their new salary is. You probably did this in two steps, working out 6% of their salary then adding it to what their current salary. However, we can actually do this in a single step. If we realise that their old salary is 100%, then their new salary would be 106% of their old one.



So we could calculate 106% of 45000: $\frac{106}{100} \times 45000 = 47700$, so their new salary would be \$47700.

In the same way we can do a discount problem and find the new cost. For example, a shopper finds a skirt with a sign saying 20% off, the original cost marker says \$59. To work out the new price we would work out 80% of \$59:



Using any of the methods we have discussed we find that 80% of \$59 is \$47.20, the new price of the skirt.

PERCENTAGE CHANGE

You have noticed that the supermarket has marked up an item that cost \$8.50 last week to \$9.35 this week. You think this is a big increase and want to work it out as a percentage. How would you do it?

We need to calculate the increase itself firstly, and then calculate it as a proportion of the original cost, as a percentage. So, the increase is:

$$9.35 - 8.50 = 0.85$$

Now put this as a proportion of 8.50: $\frac{0.85}{8.50}$ and multiply by 100 to get the %. So,

$$\frac{0.85}{8.50} \times 100\% = \frac{85}{8.5}\% = 10\%.$$

In other words there is a 10% increase.

We can also calculate a discount as a percentage. For example, an electrical appliance was advertised at \$1548 instead of the usual price of \$1798. What was the percentage discount?

First, find out the actual decrease:

$$1798 - 1548 = 250$$

As a percentage of the original cost, it would be:

$$\frac{250}{1798} \times 100\% \approx 14\%$$

(≈ means “is roughly equal to” and this calculation is best done on a calculator.)

One to think about:

A firm increased their staff of 1000 by 10%, but a while later, laid off 10% of the staff. Do you think there are now the same number of employees before the increase and decrease?

Let’s check. The total staff after the 10% increase is 110% of 1000

$$1.1 \times 1000 = 1100$$

Now the number of staff after the decrease is 90% of 1100

$$0.9 \times 1100 = 990$$

(The change depends on the amount you are calculating the percentage of, the 10% decrease is 10% of a larger number than the 10% increase.)

If you need help with any of the maths covered in this resource (or any other maths topic), you can make an appointment with Learning Development through reception: phone (02) 4221 3977, or Level 2 (top floor), Building 11, or through your campus.

SOLUTIONS TO EXERCISES

1. Match a percent to all these common phrases. For many of them, there is not an exact answer, but for some there are (you will recognize them!).

Phrase	%	Phrase	%
Just about all	About 90 to 99	About a third	About 33
Hardly any	About 1 to 5	The lot	100
A large proportion	About 80 to 90	Half	50
Most of	About 95	None	0
Twice as much	200	A quarter	25

Any of the answers that do not have "about" should be exact. All others can be around that number of range.

2. Complete this table by filling in the blanks.

Fraction	Percent
$\frac{1}{5}$	20%
$\frac{1}{4}$	25%
$\frac{7}{10}$	70%
$\frac{9}{10}$	90%
1	100%
$\frac{3}{4}$	75%
$\frac{1}{3}$	$33\frac{1}{3}\%$

3. In the table, match the decimals on the left with their percentages on the right.

Decimal	Percent
0.48	201%
0.9	0.2%
0.67	100%
1	48%
0.09	9%
2.01	90%
0.002	67%

That is, $0.48 = 48\%$, $0.9 = 90\%$, $0.67 = 67\%$, $1 = 100\%$, $0.09 = 9\%$, $2.01 = 201\%$, and $0.002 = 0.2\%$.

4. A musician received a sponsorship on his musical instruments so that he only had to pay 35% of the cost. He wanted a new drum that originally cost $2000. How much would he need to pay?

   We can do this any of the ways:

   $$35 \times 2000 \div 100 = 700$$

   Or:

   $$\frac{35}{100} \times 2000 = \frac{7}{20} \times 2000 = \frac{14000}{20} = 700$$

   Or:

   $$0.35 \times 2000 = 700$$

   Or:

   $$35 \times 20 = 700$$

   In whatever way you do it you should get $700.

5. In a recent poll of 1500 voters, 59% of the people asked said they had confidence in the government and 37% said they were not confident.

   d. How many people said they were confident?

   $$0.59 \times 1500 = 885$$

   e. How many people said they lacked confidence?

   $$0.37 \times 1500 = 555$$

   f. How many people did not express their opinion?

   $$1500 - (885 + 555) = 1500 - 1440 = 60$$

   Or: $59\% + 37\% = 96\%$, so 4% did not express their opinion, and

   $$0.04 \times 1500 = 60$$

6. A person's salary was $45000 in their first year of employment and increased by 6% in their second year. What was their pay increase, and how much was their new salary?

   $$0.06 \times 45000 = 2700$$

   So the salary increased by $2700 and the new salary is $\$45000 + \$2700 = \$47700$.

7. A couple buying a property for $270000 had to pay 3.5% stamp duty on $190000. How much did they have to pay in stamp duty?

   The stamp duty is 3.5% of $190000, so

   $$0.035 \times 190000 = 6650$$

   The stamp duty is $6650.