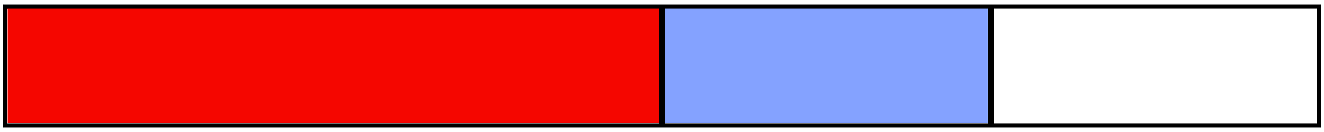
Fractions

3. Addition and Subtraction

If you have not worked with fractions for a while, you might like to begin by looking at *Fractions – 1. Manipulating Fractions*. For information on multiplying and dividing fractions, please refer to *Fractions – 2. Multiplication and Division*.

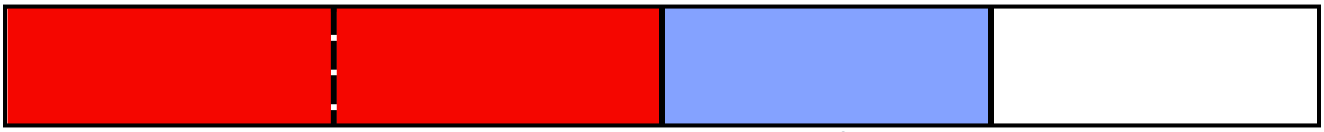
ADDITION

Imagine you have a chocolate bar and eat half, $\frac{1}{2}$, of it. Then you eat another quarter, $\frac{1}{4}$, of it and you wonder what fraction of the chocolate bar you've eaten. Let's look at our visualisation:



We can't just add these two pieces because they are different sizes! The size of the piece is dictated by the denominator of the fraction, the larger the denominator, the more pieces we break the whole into, so the smaller the pieces. To be able to add the fractions we need to make sure we are dealing with pieces of the same size, that is the fractions should have the same denominator. To do this we use *equivalent fractions*.

We can replace the $\frac{1}{2}$ with the equivalent fraction $\frac{2}{4}$, which we obtain by multiplying both top and bottom by 2:



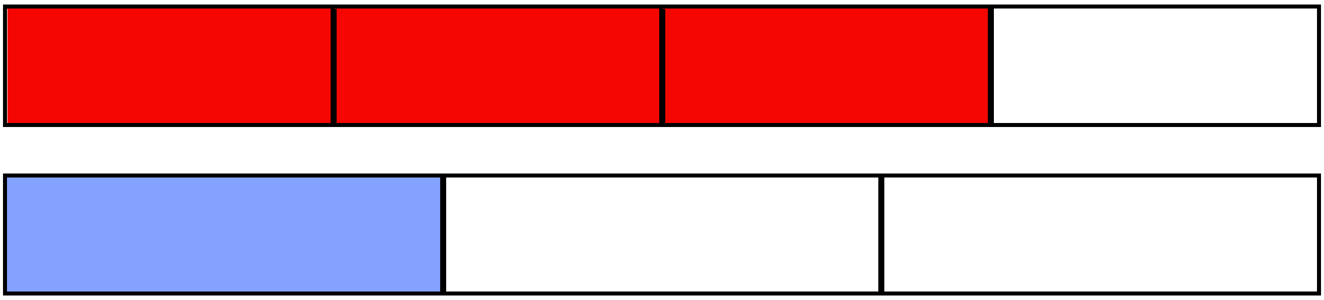
Now we can see that we have eaten 3 pieces out of 4. That is we have eaten $\frac{3}{4}$ of the chocolate bar. Let's look at what we have done using mathematics notation.

$$\begin{aligned}\frac{1}{2}+\frac{1}{4} &= \frac{1\times 2}{2\times 2}+\frac{1}{4}\\ &= \frac{2}{4}+\frac{1}{4}\\ &= \frac{3}{4}.\end{aligned}$$

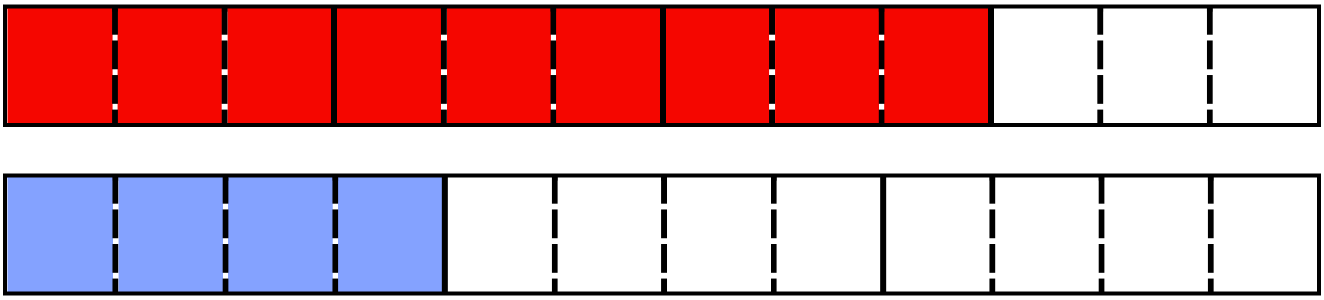
To summarise, when adding fractions, we need to make sure that we are combining the same kinds/sizes of pieces. With the model, we can do this by replacing at least one of the sizes with a smaller size. With the algorithm, we replace at least one of the fractions with an equivalent fraction, depending on denominators. We cannot add 2 fractions whose denominators are not the same and so we have to make them the same by choosing appropriate equivalent fractions.

To work out the equivalent fraction(s) involved, using a model, we need to think about the original denominators and try to picture if we could cut them up in any way to obtain the other denominator. This is reasonably straightforward with the example above (where the 2 quarter pieces fit neatly onto the 1 half piece - we are used to halves and quarters on the face of a clock, for example) but with other fractions perhaps it might not be so easy.

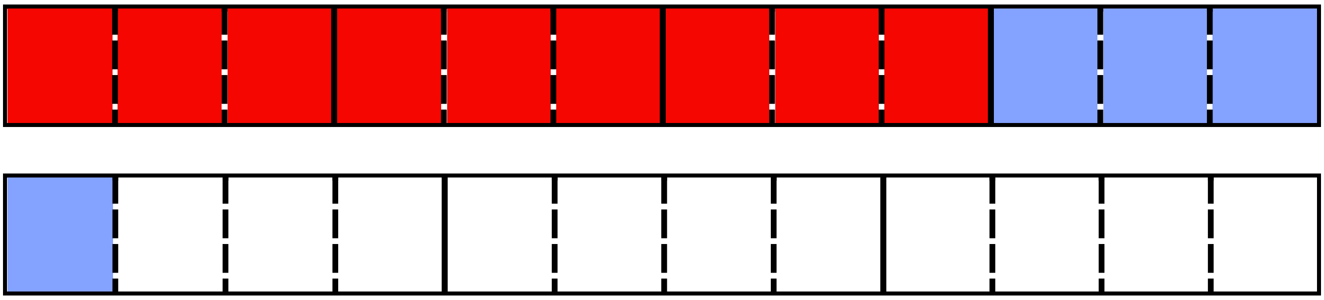
Consider $\frac{3}{4} + \frac{1}{3}$, which visually looks like:



It doesn't look like we can cut either fraction up to fit neatly into the other, so we need to work slightly differently. How about we cut the quarters into three pieces, and the thirds into four pieces:



We see here that our pieces are now the same size, we have broken each whole into 12 parts, so we can add the shaded parts together to get the result $\frac{13}{12}$, which is the same as $1\frac{1}{12}$. We could also rearrange the pieces to see this more clearly:



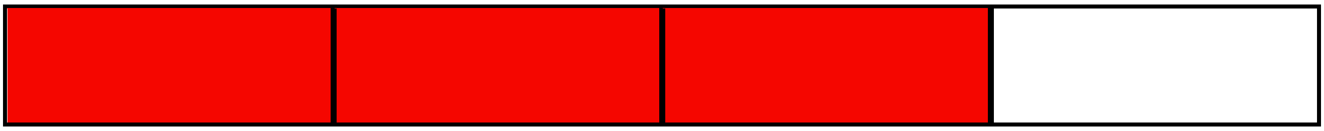
The algorithm we use to add fractions is to find a common number that is a multiple of both denominators. Ideally this would be the *lowest common multiple*, the smallest number divisible by both numbers, but if you are unsure on how to find this you can always use the product of the two numbers. In our example the denominators are 3 and 4, which have a lowest common multiple of 12 (in this case this is also their product).

Once a common multiple is decided on we then turn our fractions into two equivalent fractions that have the common multiple as the denominator. We can then complete the addition. Here is our example:

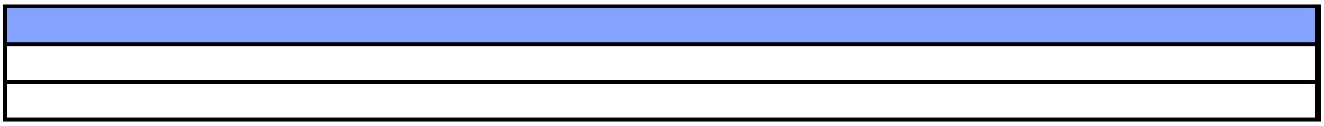
$$\begin{aligned}\frac{3}{4} + \frac{1}{3} &= \frac{\square}{12} + \frac{\square}{12} \\ &= \frac{3 \times 3}{4 \times 3} + \frac{1 \times 4}{3 \times 4} \\ &= \frac{9}{12} + \frac{4}{12} \\ &= \frac{13}{12} \\ &= 1\frac{1}{12}.\end{aligned}$$

Note that to work out the equivalent fraction for the first fraction we ask "how many times does 4 go into 12?", i.e. $12 \div 4 = 3$, and multiply the top and bottom by that number. We do the same for the second fraction.

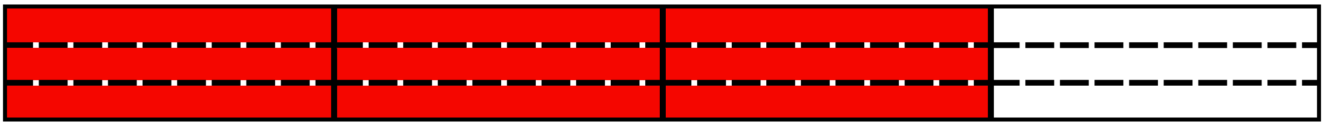
There is also a slightly different model we could use to visually represent our addition. Instead of breaking our pieces only vertically we break the first fraction, $\frac{3}{4}$, vertically:

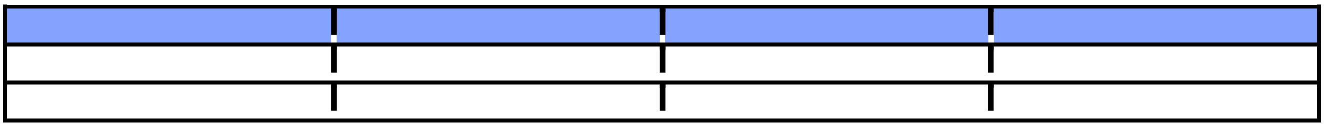


And the second fraction, $\frac{1}{3}$, horizontally:



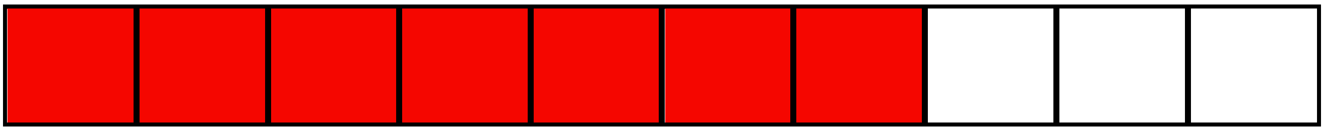
To make the pieces the same size we then split the first whole horizontally the same number of times (and similarly for the second whole but splitting it vertically).

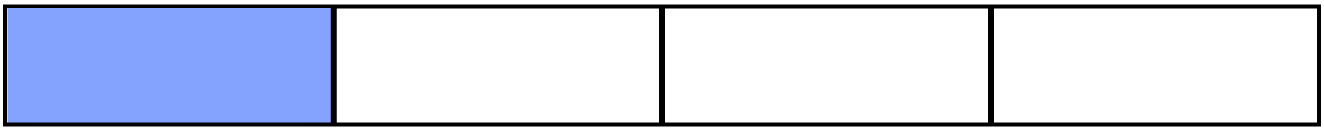




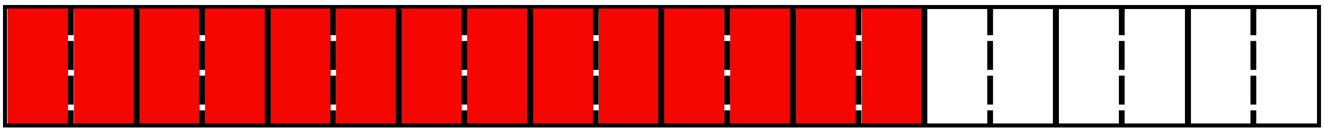
Now our pieces are the same size! So we can add them together to find we have 13 of them and that a whole piece is made up of 12 of them.

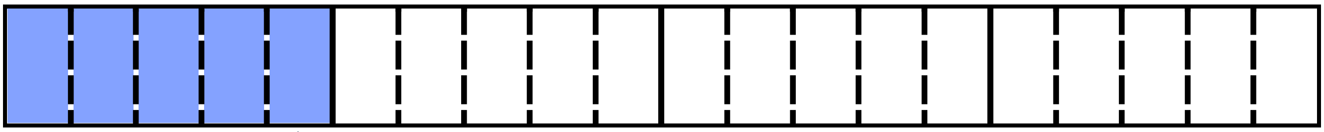
Let's try another example. Consider $\frac{7}{10} + \frac{1}{4}$, visually we have:



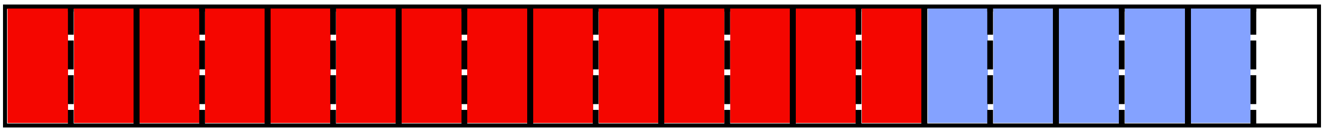


In this case to make the pieces have the same size we cut each of the tenths into 2 pieces and each of the quarters into 5 pieces. We could cut the tenths into 4 pieces and then quarters into 10 pieces, but this would be overkill and not give us the lowest common multiple (which in this case is 20):





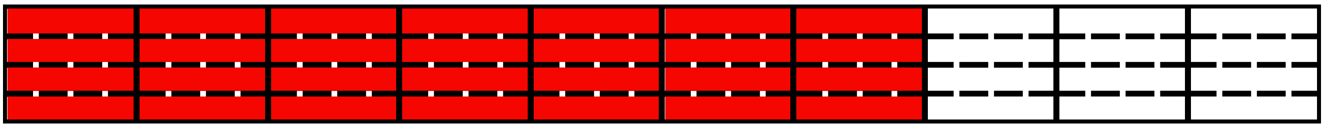
So we now have $\frac{14}{20}$ and $\frac{5}{20}$, which means altogether we have $\frac{19}{20}$. We can see this easily by moving the blue shaded area into the empty spots of the first fraction.

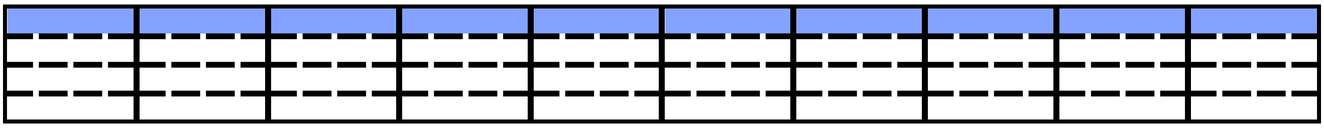


Now let's see this done mathematically. First we would need to notice that both 10 and 4 divide 20, so this will be our common denominator (we could use their product $10 \times 4 = 40$, but this is not the most efficient way).

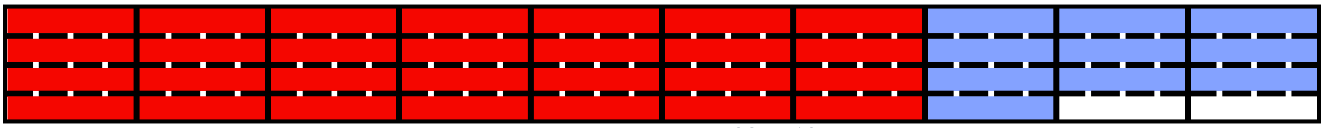
$$\begin{aligned} \frac{7}{10}+\frac{1}{4} &= \frac{\square}{20}+\frac{\square}{20} \\ &= \frac{7\times 2}{10\times 2}+\frac{1\times 5}{4\times 5} \\ &= \frac{14}{20}+\frac{5}{20} \\ &= \frac{19}{20}. \end{aligned}$$

Lastly, let's use our second modelling method, this is where we will break the whole into tenths horizontally and into quarters vertically:





Now the segments are the same size and we rearrange so that our shaded areas are together:



This shows we have 18 out of 40 pieces shaded and our answer is $\frac{38}{40}=\frac{19}{20}$. Note that by using this method you will always end up with a fraction that has the product of the two denominators as its denominator and you are more likely to need to simplify the fraction.

Remember: **Never add the denominators!**

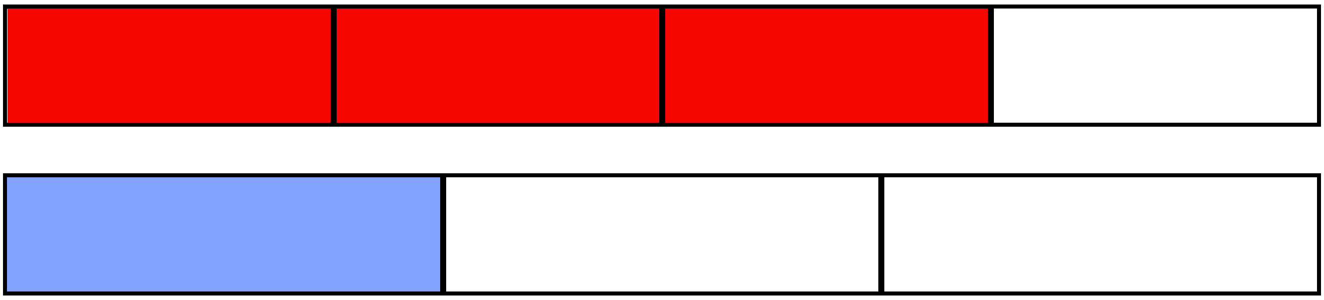
We only add the pieces we have together. The denominator just tells us the size of the pieces we have.

SUBTRACTION

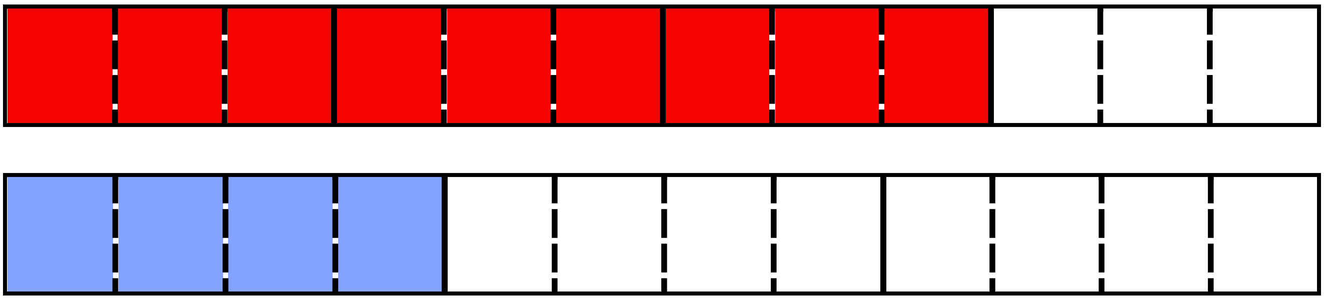
Subtraction of fractions is performed the same way as addition, except once the denominators are the same you subtract the numerators instead of adding them (but leave the new denominators alone!). Example:

$$\begin{aligned} \frac{3}{4}-\frac{1}{3} &= \frac{3\times 3}{4\times 3}-\frac{1\times 4}{3\times 4} \\ &= \frac{9}{12}-\frac{4}{12} \\ &= \frac{5}{12}. \end{aligned}$$

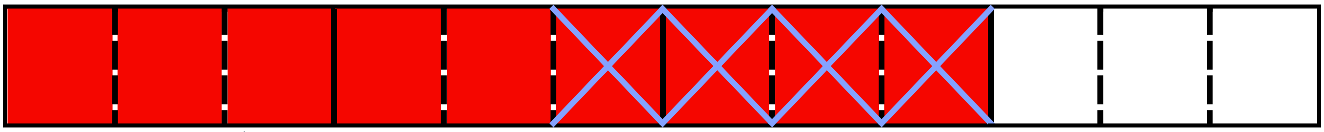
We see that we first create equivalent fractions so that the pieces we have and the pieces we want to take away are the same size (have the same denominator), then we remove some pieces from what we have. Here it is in visual form:



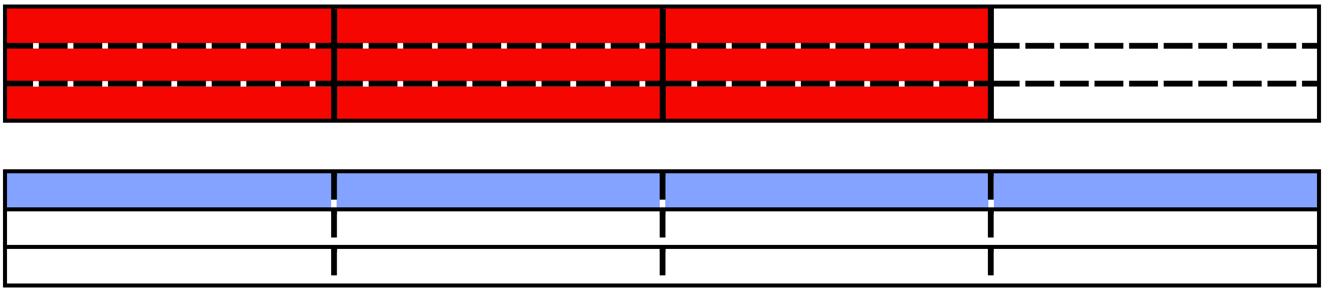
We break the pieces up further:



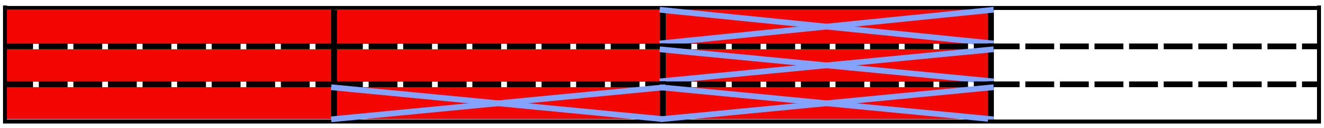
Now we remove the number of pieces in our bottom fraction from the top fraction:



So we are left with $\frac{5}{12}$. We could also use the second model where we break all pieces into four vertically and three horizontally:



Now we remove the number of shaded areas in the bottom from the shaded areas in the top:



Here are some for you to try; you can check your solutions with the answers at the end of the resource.

EXERCISES

1. $\frac{1}{2}+\frac{3}{4}$
2. $\frac{1}{4}+\frac{5}{8}$
3. $\frac{3}{7}+\frac{4}{7}$
4. $\frac{2}{9}+\frac{5}{6}$
5. $\frac{5}{9}-\frac{2}{9}$
6. $\frac{7}{8}-\frac{1}{2}$
7. $\frac{5}{6}-\frac{4}{5}$
8. $\frac{9}{10}-\frac{11}{15}$

UNIVERSITY OF WOLLONGONG AUSTRALIA

ADDING AND SUBTRACTING MIXED NUMBERS

The easiest way to deal with mixed numbers is to change them to improper fractions and perform the usual addition and subtraction steps. For example, if we want to add $1\,\frac{2}{3}$ and $2\,\frac{3}{4}$ we would perform the following:

$$\begin{aligned} 1\frac{2}{3}+2\frac{3}{4} &= \frac{5}{3}+\frac{11}{4} \\ &= \frac{5\times 4}{3\times 4}+\frac{11\times 3}{4\times 3} \\ &= \frac{20}{12}+\frac{33}{12} \\ &= \frac{53}{12}. \end{aligned}$$

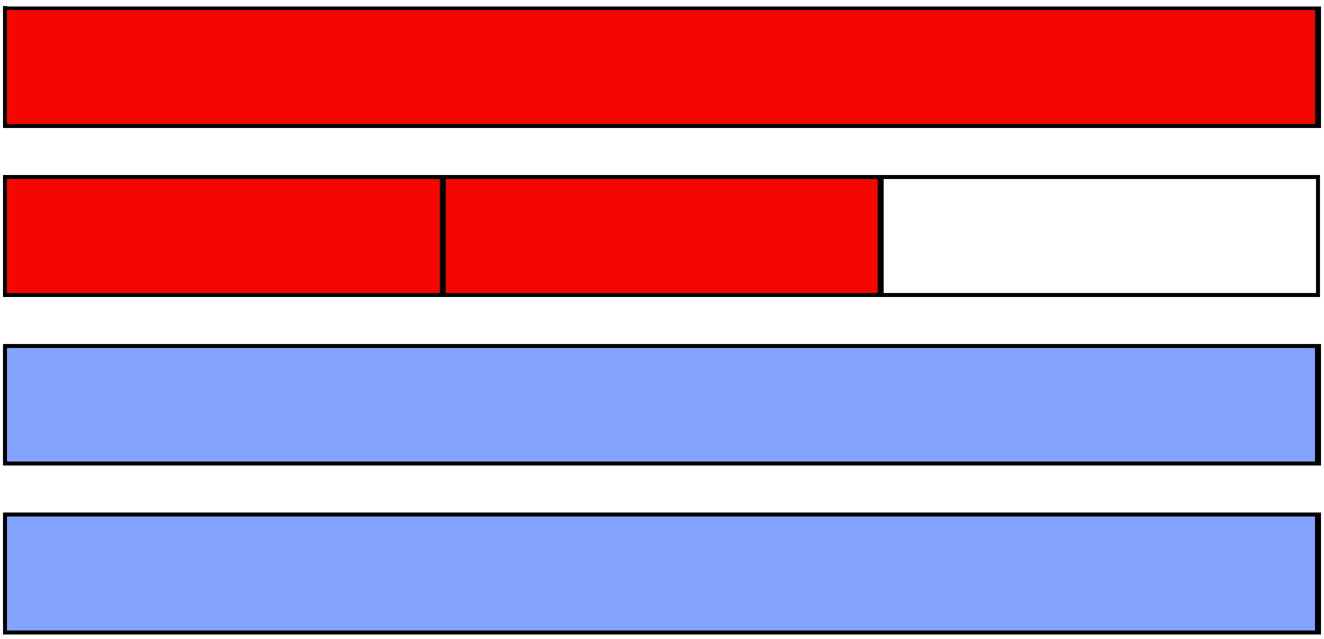
We could then write our answer as a mixed number $4\,\frac{5}{12}$.

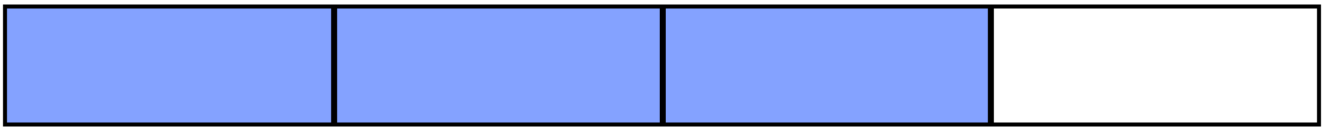
Here we recall that to move from a mixed number to an improper fraction we multiply the whole number part by the denominator and add it to the numerator. This is in fact performing addition of fractions! When we write $1\,\frac{2}{3}$ what we say is "one and two thirds", which is the same as $1+\frac{2}{3}$. We then perform our addition:

$$\begin{aligned} 1+\frac{2}{3} &= \frac{1}{1}+\frac{2}{3} \\ &= \frac{1\times 3}{1\times 3}+\frac{2}{3} \\ &= \frac{3}{3}+\frac{2}{3} \\ &= \frac{5}{3}. \end{aligned}$$

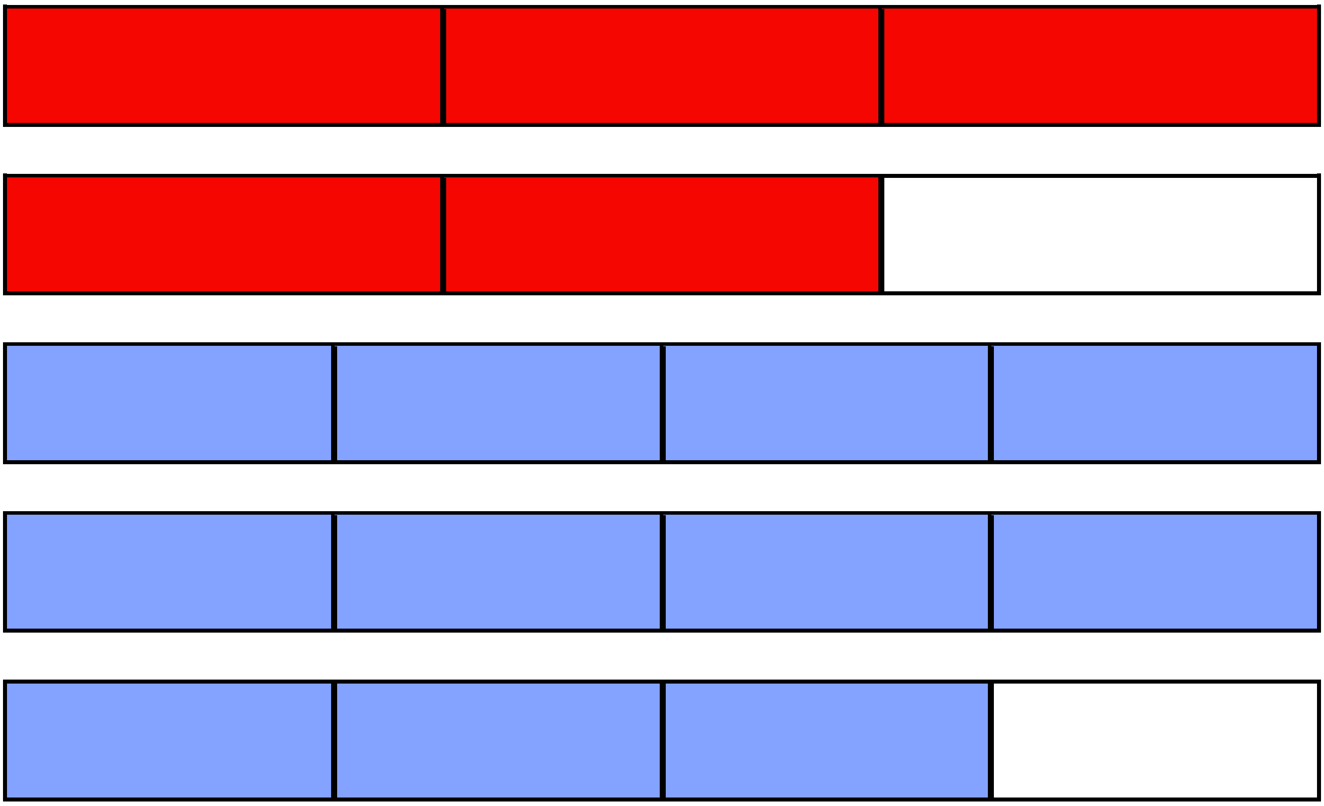
You can see here how the "multiply by the denominator and add it to the numerator" rule comes about.

The models discussed previously can also be used to perform the addition/subtraction, for example the visualisation of $1\,\frac{2}{3}$ and $2\,\frac{3}{4}$ is:

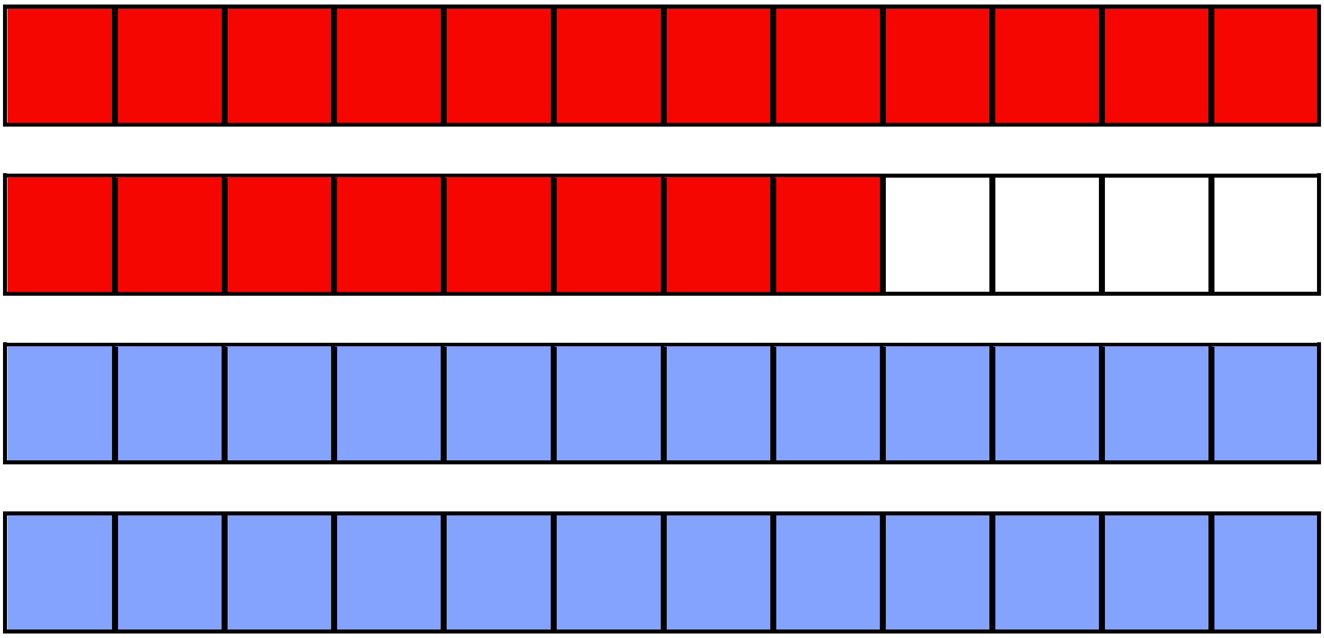


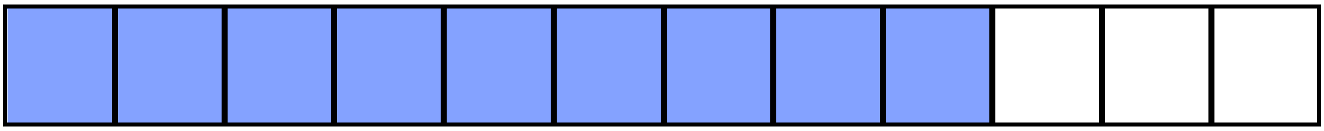


We can then represent them as improper fractions by dividing up the wholes:

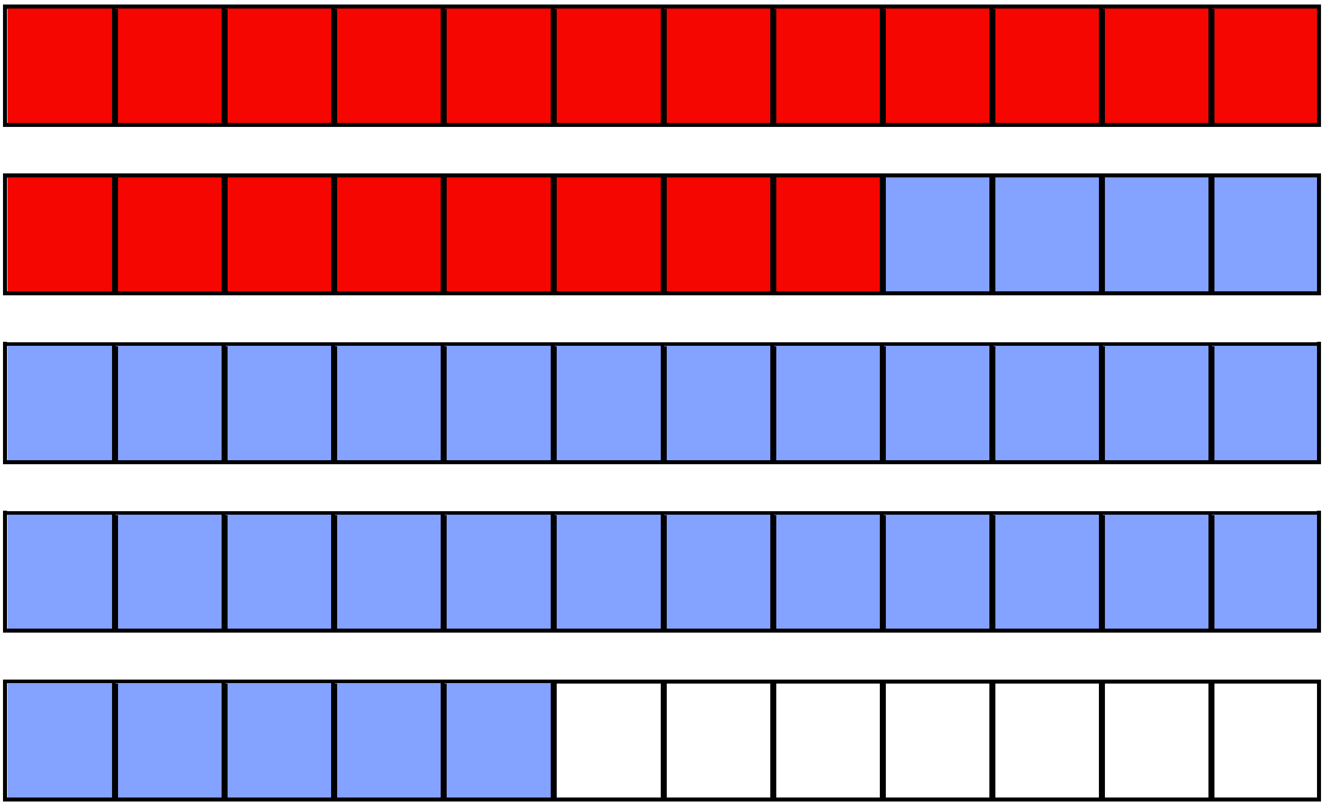


Now we cut up the pieces further so that they are all the same size (i.e. make the denominators the same):





Finally we see that we have 53 shaded pieces each of which is one twelth of a whole, so we have $\frac{53}{12}$, which is $4\,\frac{5}{12}$. The mixed number could also be seen by filling in the gaps with other shaded pieces:



The second visualisation method that we learnt would also work.

Another way to deal with mixed numbers is to add the whole parts and the fraction parts separately and then combine. For example in the above case of $1\,\frac{2}{3}$ and $2\,\frac{3}{4}$ we would first add the 1 and the 2 (to get 3) then separately add the $\frac{2}{3}$ and $\frac{3}{4}$:

$$\begin{aligned}\frac{2}{3}+\frac{3}{4} &= \frac{2\times 4}{3\times 4}+\frac{3\times 3}{4\times 3}\\ &= \frac{8}{12}+\frac{9}{12}\\ &= \frac{17}{12}\\ &= 1\,\frac{5}{12}.\end{aligned}$$

We now add back in the 3 to get $4\,\frac{5}{12}$. In general (and especially for subtraction), the first method is easier but you can use either.

EXERCISES

9. $1\frac{3}{8}+2\frac{1}{2}$

If you need help with any of the maths covered in this resource (or any other maths topic), you can make an appointment with Learning Development through reception: phone (02) 4221 3977, or Level 2 (top floor), Building 11, or through your campus.

SOLUTIONS TO EXERCISES

1. $\frac{1}{2}+\frac{3}{4}$

$$\begin{aligned}\frac{1}{2}+\frac{3}{4}&=\frac{1\times 2}{2\times 2}+\frac{3}{4}\\&=\frac{2}{4}+\frac{3}{4}\\&=\frac{5}{4}\\&=1\frac{1}{4}\end{aligned}$$

2. $\frac{1}{4}+\frac{5}{8}$

$$\begin{aligned}\frac{1}{4}+\frac{5}{8}&=\frac{1\times 2}{4\times 2}+\frac{5}{8}\\&=\frac{2}{8}+\frac{5}{8}\\&=\frac{7}{8}\end{aligned}$$

3. $\frac{3}{7}+\frac{4}{7}$

$$\begin{aligned}\frac{3}{7}+\frac{4}{7}&=\frac{7}{7}\\&=1\end{aligned}$$

4. $\frac{2}{9}+\frac{5}{6}$

$$\begin{aligned}\frac{2}{9}+\frac{5}{6}&=\frac{2\times 2}{9\times 2}+\frac{5\times 3}{6\times 3}\\&=\frac{4}{18}+\frac{15}{18}\\&=\frac{19}{18}\\&=1\frac{1}{18}\end{aligned}$$

5. $\frac{5}{9}-\frac{2}{9}$

$$\begin{aligned}\frac{5}{9}-\frac{2}{9}&=\frac{3}{9}\\&=\frac{3\div 3}{9\div 3}\\&=\frac{1}{3}\end{aligned}$$

6. $\frac{7}{8}-\frac{1}{2}$

$$\begin{aligned}\frac{7}{8}-\frac{1}{2}&=\frac{7}{8}-\frac{1\times 4}{2\times 4}\\&=\frac{7}{8}-\frac{4}{8}\\&=\frac{3}{8}\end{aligned}$$

7. $\frac{5}{6}-\frac{4}{5}$

$$\frac{5}{6}-\frac{4}{5}=\frac{5\times 5}{6\times 5}-\frac{4\times 6}{5\times 6}$$
$$=\frac{25}{30}-\frac{24}{30}$$
$$=\frac{1}{30}$$

8. $\frac{9}{10}-\frac{11}{15}$

$$\frac{9}{10}-\frac{11}{15}=\frac{9\times 3}{10\times 3}-\frac{11\times 2}{15\times 2}$$
$$=\frac{27}{30}-\frac{22}{30}$$
$$=\frac{5}{30}$$
$$=\frac{1}{6}$$

9. $1\frac{3}{8}+2\frac{1}{2}$

$$1\frac{3}{8}+2\frac{1}{2}=\frac{11}{8}+\frac{5}{2}$$
$$=\frac{11}{8}+\frac{5\times 4}{2\times 4}$$
$$=\frac{11}{8}+\frac{20}{8}$$
$$=\frac{31}{8}$$
$$=3\frac{7}{8}$$