Directed Numbers

Sorting the Positives from the Negatives

INTRODUCTION

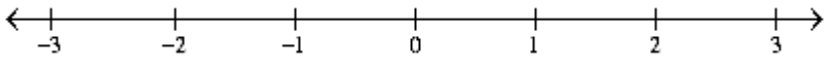
Firstly, why do we need negative numbers? Well, let's say you owe someone money – you are in debt to them. How would we write this if we didn't have negative numbers? If you owe $2, then rather than writing "I owe you $2", we simply write "−2".

Here are some further examples of where negatives occur in our everyday life:

NEGATIVE	"ZERO"	POSITIVE
Downstairs	The landing between stairs	Upstairs
Below freezing (Brrr!)	0°C or 32°F	Above freezing
You owe money!	You have no money but don't owe any either!	You have money
You failed the subject	You got the pass mark (e.g. 50%)	You passed the subject
You feel lousy! ☹	You feel Ok 😐	You feel great! ☺
You dislike someone	You feel neutral towards someone	You really like someone
Left	Zero/0/Centre/Neutral	Right
Down	Steady	Up

DIRECTION

When we talk about positives and negatives, direction is implied. Positives and negatives go in opposite directions from each other. You may remember from school that there is an object call the *number line*. The number line is an object that puts numbers in order. In the middle of the number line is zero 0, positive numbers then go on the right and increase in size (*magnitude*) as you go further along the line, and negative numbers go on the left and increase in magnitude the further along you go:



The arrows at each end of the number line tell us that the line will continue indefinitely in both directions. So, to the right of the number line shown, we would continue with 4, 5, 6,… and to the left with −4, −5, −6,…. We can also use a vertical number line, with the positive numbers increasing upwards and the negatives increasing downwards.

We can also talk about the opposite of a number (also called the *additive inverse*, since if you add the two numbers we get zero, nothing, zilch). The opposite of 3 is −3 and the opposite of −516 is 516. (Note, we could also write +516 but generally we leave out the + sign.) In a pair of opposites each number is exactly the same distance from 0. For example, both 3 and −3 are 3 units from 0. Both 516 and −516 are 516 units from 0, but in opposite directions. So their size, or magnitude, is the same, but their direction is opposite.

Here is a very quick exercise for you. You can check your answers with those at the end of this resource.

EXERCISES

Write down the opposite of each of the following numbers:

1. -5
2. 13
3. -49
4. 2560
5. 3.87
6. $\frac{13}{4}$
7. 0

ENTERING A NEGATIVE NUMBER OF A CALCULATOR

To enter a negative number into a calculator, use the following key:

So to enter -2, for example, you would press (-) 2.

CALCULATING WITH POSITIVE AND NEGATIVE NUMBERS

To calculate using positives and negatives, it may be easiest to think of banking transactions. Positive numbers would represent money in the bank, negatives would represent debt.

ADDITION

Keeping the idea of banking transaction in mind, adding a positive number would mean you are depositing money into your account, while adding a negative number would mean you are withdrawing.

Example: $5 + 7$.

We know this would be 12 (using banking: we start with a balance of \$5 and deposit \$7 to end up with \$12).

Now, what about $-6 + -3$?

This means that we start with a debt of \$6 but want to withdraw a further \$3 so we would be a total of \$9 in debt, that is, $-6 + -3 = -9$.

(Notice that whether we add two positive numbers or two negative numbers, we end up with a number that is greater in size/magnitude than the individual numbers we started with. If the numbers are positive then end result will be positive and if the numbers are negative we get a negative result.)

How about $-6 + 2$?

We are starting with a debt of \$6 but depositing \$2, so our debt will be lessened. We will go up \$2 to a debt of \$4, so $-6 + 2 = -4$. You can think of this in terms of the number line also, you start at -6 and move 2 units to the right (the positive direction) to arrive at -4.

Final example: $5 + -7$.

This would mean we would be trying to withdraw \$7 when we only have \$5 in the account. If our bank lets us do this, we would end up with a debt of \$2, that is, $5 + -7 = -2$. Again you can also think of this in terms of the number line. You start at 5 and move 7 units to the left (the negative direction) to arrive at -2.

(Notice that when we add two numbers the sign will be the same as the sign of the number with the *larger* size/magnitude.)

Here are some for you to try. Remember you can check your answers using your calculator (this gives you good practice with your calculator) and also by looking at the answers at the end of this resource.

EXERCISES

8. $-5+3=$ 9. $6+4=$ 10. $-5+-7=$ 11. $-4+-2=$

12. $5+-2=$ 13. $4+-8=$ 14. $-5+5=$ 15. $-3+4+-2=$

SUBTRACTION

Let's consider subtraction of positive numbers to start with, for example, $6-4$. In banking terms subtracting is the same as withdrawing (essentially, subtraction is the same as addition of a negative number). So we know $6-4=2$, but what about when the second number has a greater magnitude? For example: $6-8$. We are trying to withdraw \$8 but we only have a balance of \$6 to start with. So we would end up in debt \$2, in other words, $6-8=-2$. Try these on your calculator. You would enter: [6] [−] [8] [=].

What about if we start with a negative number (that is we are already in debt)? For example: $-6-8$. This would mean we go further into debt. We start with a debt of \$6, then withdraw a further \$8, so we end up with a debt of \$14, that is $-6-8=-14$. Subtraction can also be performed using the number line, but this time when subtracting a positive number we move to the left!

Now let's look at something a bit more tricky, subtracting a negative number. For example: $5--2$. What happens here? One way to think of it is that the two minus signs cancel out and we end up with a positive, just like a "double negative" in English. For example, you might say "he is *not unlike* my brother"; which would mean he is *like* your brother. That is *not unlike = like*. The negatives cancel! In the same way "he is not not guilty" means "he is guilty"! It is the same when we subtract a negative. So $5--2$ is the same as $5+2$, which gives 7.

We can also think back at our banking transactions. In this instance $5--2$ might mean that we have \$5 but then we cancel a previous withdrawal of \$2. This would mean that we end up \$2 better off! That is we end up with \$7. In terms of the number line We have started at 5 and move to the right 2 spots (remember when subtracting a positive number we moved to the left, so here we do the opposite).

Try this calculation on your calculator. You need to be careful with minus sign you enter. To enter $5--2$, use the following: [5] [−] [(-)] [2] [=].

Here are some subtractions for you to try. Check them on your calculator and with the answers at the end of the handout.

EXERCISES

16. $3-2=$ 17. $-3-2=$ 18. $3--2=$ 19. $-3--2=$

20. $4-6=$ 21. $-4-6=$ 22. $4--6=$ 23. $-4--6=$

MULTIPLICATION

Perhaps the easiest way to think about what is happening with multiplication is to consider it as repeated addition. For example we know that 3×2 is 6 because it says "three lots of two" which is the same as $2+2+2$. In terms of banking this is like making three deposits of \$2.

What about 3×-2? Remember from the addition section that -2 is like a withdrawal of \$2. So we are making three withdrawals of \$2 and we end up with a debt of \$6. That is, $3\times-2=-6$.

How about -3×2? In this case we can think of the -3 as three cancellations, and the 2 as a deposit of \$2. So this statement reads "three cancellations of a \$2 deposit". When we cancel a deposit we end up with less money, so cancelling three \$2 deposits means we would end up \$6 in debt. That is $-3\times2=-6$. Another way to think of this is to use that multiplication is symmetric (it doesn't matter which way around we write the two numbers), so $-3\times2=2\times-3$, which is two withdrawals of \$3.

Now a trickier one. How about -3×-2? The result of this is $+6$. Thinking as above we can see this as three cancellations of \$2 withdrawals. Remember if we cancel a withdrawal we end up with more money. So if we cancel three withdrawals each worth \$2 then we would end up \$6 better off!

A general rule for multiplication is: if the signs are the same we get a positive number, but if the signs are different we get a negative number. So:

$$+ \times + = +; \qquad - \times - = +; \qquad + \times - = -; \qquad - \times + = -.$$

Here are some for you to try. Please check them with your calculator, as well as with the answers at the end of the resource.

EXERCISES

24. $3 \times 5 =$ 25. $-3 \times 5 =$ 26. $4 \times -3 =$ 27. $-5 \times -2 =$

DIVISION

Division is much like multiplication. In fact the general rule is the same!

A general rule for division is: if the signs are the same we get a positive number, but if the signs are different we get a negative number. So:

$$+ \div + = +; \qquad - \div - = +; \qquad + \div - = -; \qquad - \div + = -.$$

We can also think about this in our banking terms. Division is a "how many" operation. For example $6 \div 2$ asks "how many deposits of \$2 are there in a balance of \$6?" We know there are 3 deposits, so $6 \div 2 = 3$.

Likewise, $-6 \div -2$ asks "how many withdrawals of \$2 in a debt of \$6?" Again there would be 3 withdrawals, so $-6 \div -2 = 3$.

For $6 \div -2$ we need to be a bit trickier. The statement asks "how many withdrawals of \$2 in a balance of \$6?" Well we can't get money by withdrawing it, so these must be cancelled withdrawals!! So in the above we must have cancelled three withdrawals of \$2 to end up with a balance of \$6. Hence, $6 \div -2 = -3$.

Similarly, $-6 \div 2$ asks "how many deposits of \$2 is there in a debt of \$6?" Again we must have cancelled these deposits to end up in debt. So, $-6 \div 2 = -3$ (three cancelled deposits).

Here are some for you to try. Again, please check them with the calculator and with the answers at the end.

EXERCISES

28. $12 \div 2 =$ 29. $-8 \div 4 =$ 30. $10 \div -5 =$ 31. $-15 \div -3 =$

RAISING A NEGATIVE NUMBER TO A POWER

When we raise a number to a power, we are simply multiplying it by itself a certain number of times, which is indicated by the power. So, for example, $3^2 = 3 \times 3 = 9$ and $(-3)^2 = -3 \times -3 = 9$. Notice that the brackets here are essential. If we do not have brackets around the -3, then we would get -9 because -3^2 means $-(3^2)$ as the squaring takes precedence over the negation. You must also use brackets when entering such a calculation into the calculator.

Let's look at some examples:

- $(-5)^2 = -5 \times -5 = 25$
- $(-2)^3 = -2 \times -2 \times -2 = 4 \times -2 = -8$
- $(-3)^4 = -3 \times -3 \times -3 \times -3 = 9 \times -3 \times -3 = -27 \times -3 = 81.$

Notice how raising a negative number to an even power gives a positive number, while raising a negative number to an odd power gives a negative number.

For more information on powers you can refer to the handouts: *Calculators – Getting to Know Your Scientific Calculator* and *Powers and Logarithms – 1. Powers, Indices, Exponents*.

Here are some for you to try, please check your answers with a calculator and with the solutions at the end of the resource.

EXERCISES

32. $(-5)^3 =$ 33. $(-4)^2 =$ 34. $(-1)^5 =$ 35. $2^3 - 3^2 =$

If you need help with any of the maths covered in this resource (or any other maths topic), you can make an appointment with Learning Development through reception: phone (02) 4221 3977, or Level 2 (top floor), Building 11, or through your campus.

SOLUTIONS TO EXERCISES

1. -5 has opposite 5
2. 13 has opposite -13
3. -49 has opposite 49
4. 2560 has opposite -2560
5. 3.87 has opposite -3.87
6. $\frac{13}{4}$ has opposite $-\frac{13}{4}$
7. 0 has opposite 0
8. $-5+3=-2$
9. $6+4=10$
10. $-5+-7=-12$
11. $-4+-2=-6$
12. $5+-2=3$
13. $4+-8=-4$
14. $-5+5=0$
15. $-3+4+-2=-1$
16. $3-2=1$
17. $-3-2=-5$
18. $3--2=5$
19. $-3--2=-1$
20. $4-6=-2$
21. $-4-6=-10$
22. $4--6=10$
23. $-4--6=2$
24. $3\times 5=15$
25. $-3\times 5=-15$
26. $4\times -3=-12$
27. $-5\times -2=10$
28. $12\div 2=6$
29. $-8\div 4=-2$
30. $10\div -5=-2$
31. $-15\div -3=5$
32. $(-5)^3=-125$
33. $(-4)^2=16$
34. $(-1)^5=-1$
35. $2^3-3^2=8-9=-1$