

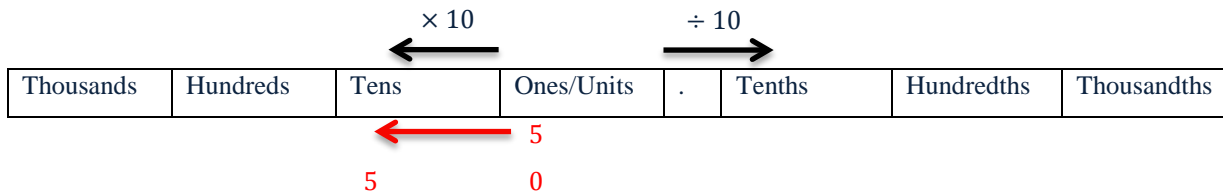
Decimals

2. Multiplication and Division

INTRODUCTION

To be able to perform the usual operations (+, −, × and ÷) using decimals, we need to remember what decimals are. To review this, please refer to *Fractions – 4. Fractions, Decimals, and Percentages*. You might also like to look at *Fractions – 2. Multiplication and Division*.

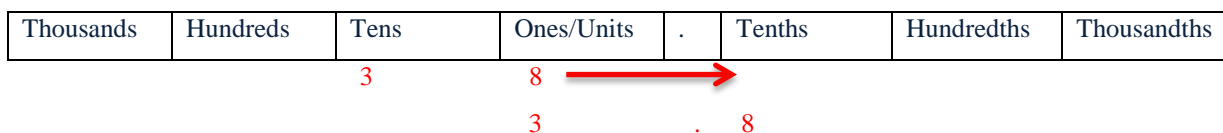
Before looking at multiplication and division with decimal numbers in general, we need to be familiar with multiplication and division by powers of 10 (i.e. 10, 100, 1000). Let's think about 5×10 . We know that the answer to this problem is 50. This is because if we look at our decimal system of numbers, each column to the left is obtained from the one on its right by multiplying by 10. To show that we have moved over a column, we need to attach a 0 in the units column.



If there is a decimal number involved, we still pick up the number and move it across to the left if multiplying by 10, 100, or 1000. This is the same as moving the decimal point in the number to the right the number of times that there are 0s involved (this method is probably the one you learned at school).

For example, to calculate 5.9×100 , we pick up 5.9 and move it to the left 2 columns (because we are multiplying by 100) which is equivalent to moving the decimal point to the right 2 places. The result would be 590.

With division, we go the other way. This means that to divide 38 by 10, for example, we need to move the number across to the right by one column (the decimal point section does not count as a column), which means we no longer have a whole number, and we need to go into the tenths column.



Moving the number to the right one column is the equivalent of moving the decimal point one place to the left. Moving the decimal point is probably easier than drawing up the columns and picking up numbers, so in practice we use this method. However seeing the other method gives you a better understanding how the methods are based on our decimal system of numbers.

When multiplying by 10, 100, 1000, and so on, move the decimal point to the right the same number of places that there are 0s (if there is no decimal point, just add the same number of 0s to the end of the number).

When dividing by 10, 100, 1000, and so on, move the decimal point to the left as many places as there are 0s.

So when multiplying/dividing by 10 move the decimal point one place right/left, by 100 two places, by 1000 three places, and so on. If there is no decimal point in the original number, then imagine it at the end of the number.

Examples

$$5.3 \times 10 = 53$$

The decimal point is moved one place to the right (equivalently we pick up the digits and move them all one place to the left)



$$23.1 \div 1000 = 0.0231$$

The decimal point is moved three places to the left

$$78 \times 100 = 7800$$

We imagine a decimal point on the end of the 78 and move it two places to the right (equivalently we just add the two zeros in 100 to the end of 78)

$$593400 \div 10 = 59340$$

Again we imagine a decimal point on the end of 593400 and move it to the left one place

Here are some for you to try. You can check your answers with the solutions at the end of this resource.

EXERCISES

1. $4100 \div 100 =$

2. $58 \times 1000 =$

3. $4.9 \times 100 =$

4. $0.05 \times 10 =$

5. $0.5 \div 10 =$

6. $1.05 \div 10 =$

7. $23.1 \times 100 =$

8. $0.003 \times 1000 =$

9. $580 \div 100 =$

10. $0.581423 \times 100000 =$

MULTIPLYING DECIMAL NUMBERS

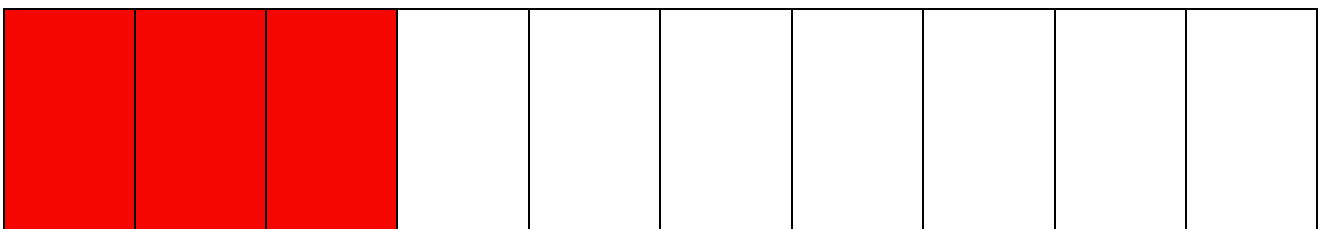
When we multiply 2 by 3 we get 6, we arrive at this because we think “2 lots of 3”. It follows that if we multiply 2.4 by 3 we should think “2.4 lots of 3” and that we should get a number between 2 lots of 3 (6) and 3 lots of 3 (9). For the moment let’s ignore the decimal point and just multiply 24 by 3, $24 \times 3 = 72$. From here you might notice that if we put the decimal point back in we end up 7.2, which is between 6 and 9. This is precisely what we do.

In general, *the combined number of decimal places before the multiplication is the number of decimal places needed after the multiplication*. That is the decimal places on each side of the equals sign need to balance.

Let’s do the calculation 0.2×0.6 . We first ignore the decimal points and calculate $2 \times 3 = 6$, next we put the decimal point back in. We started with two decimal places (one in each number), so we need two decimal places on the right hand side. Therefore:

$$0.2 \times 0.3 = 0.06.$$

We can check this result by converting our numbers to fractions. $0.2 = \frac{2}{10}$ (two tenths) and $0.3 = \frac{3}{10}$ (three tenths). So 0.2×0.3 can be read as “two tenths of three tenths”. Here are three tenths:



We need two tenths of them, so would get the purple region of intersection below:

All up in this last picture we have 100 cells, and 6 are in the shaded intersection. This means the result is $\frac{6}{100}$, or six hundredths, which is equal to 0.06. This agrees with our method of balancing the number of decimal places before and after the multiplication.

Examples

$$4.91 \times 0.01$$

Step 1: Ignore the decimal point and multiply: $491 \times 1 = 491$

Step 2: Count the total decimal places before the multiplication: $2 + 2 = 4$ decimal places

Step 3: Insert a decimal point so the decimal places balance (using zeros at the start of the number if necessary: 0.0491

So:

$$4.91 \times 0.01 = 0.0491$$

$$0.58 \times 2.3$$

Step 1: Ignore the decimal point and multiply: 58×23

$$\begin{array}{r} 58 \\ \times 23 \\ \hline 174 \\ 1160 \\ \hline 1334 \end{array}$$

Step 2: Count the total decimal places before the multiplication: $2 + 1 = 3$ decimal places

Step 3: Inset a decimal point so the decimal places balance: 1.334

So:

$$0.58 \times 2.3 = 1.334$$

$$3.8 \times 40$$

Step 1: $38 \times 40 = 1520$, note that here we just had to multiply 38 by 4 (which is 152) then add an extra 0 at the end

Step 2: 1 decimal place

Step 3: 152.0

So

$$3.8 \times 40 = 152$$

Note that when the only digits after the decimal point are zeros, we don't write them in our final answer, as shown above.



Here are some for you to try. You can check your results with the solutions at the end.

EXERCISES

11. $14.9 \times 3 =$

12. $2.01 \times 0.4 =$

13. $0.08 \times 0.09 =$

14. $2.03 \times 2000 =$

DIVIDING DECIMAL NUMBERS

By Whole Numbers

For division we'll first talk about dividing a decimal by a whole number. This is because when you are dividing by a whole number you can do your ordinary division and just make sure the decimal point aligns with where it is in the number you are dividing. For example, $10.6 \div 2 = 5.3$ (which you can check is between $10 \div 2 = 5$ and $11 \div 2 = 5.5$). To see this properly let's use the short/long division set up:

$$\begin{array}{r} 5.3 \\ 2 \overline{) 10.6} \end{array}$$

Notice how the decimal points align. If there is a remainder, such as with $10.7 \div 2$, we can attach as many 0s as needed to the end of the number, i.e. $10.7 = 10.70$:

$$\begin{array}{r} 5.35 \\ 2 \overline{) 10.70} \end{array}$$

We can divide any decimal number by a whole number using this method.

Here are some for you to try. You can check your results by looking up the solutions at the end of the resource.

EXERCISES

15. $28.5 \div 3 =$

16. $9.17 \div 7 =$

17. $1351.2 \div 16 =$

By Decimal Numbers

When dividing decimal numbers, we use a nice trick to turn the divisor into a whole number. Consider $16 \div 0.2$, this expression asks the question "how many 0.2s in 16?" We know that there are 8 twos in sixteen ($16 \div 2 = 8$) and since 0.2 is much smaller (in fact ten times smaller) than 2 there should be a lot more 0.2s in 16. In fact there are ten times as many, so the answer is 80.

How can we turn this into a procedure? The trick here is to realise that if we multiply both numbers by 10 the amount of times one goes into the other *doesn't change*. That is $16 \div 0.2 = 160 \div 2 = 80$, and we have turned the divisor (the 0.2) into a whole number.

In practice we talk about moving the decimal point the same number of places so that the equation remains balanced. To turn 0.2 into a whole number we need to move the decimal place one spot to the right, and to balance the equation we need to do the same thing to the dividend (the 16).

Examples



$$0.58 \div 0.029$$

Step 1: Move the decimal points the same number of spots until the divisor is a whole number: 0.029 has three decimal places, so we need to move the decimal point three places

$$\overset{\text{wavy}}{0.029}$$

$$\overset{\text{wavy}}{0.580}$$

Step 2: Perform the division on the new numbers: $580 \div 29$

$$\begin{array}{r} 20 \\ 29 \overline{) 580} \end{array}$$

So:

$$0.58 \div 0.029 = 20.$$

Again, if there is a remainder, we can add 0s to the decimal in the number we are dividing into (the dividend).

$$153.013 \div 0.03$$

Step 1: Make the divisor a whole number by moving the decimal point

$$\overset{\text{wavy}}{153.013} \div \overset{\text{wavy}}{0.03} = 15301.3 \div 3$$

Step 2: Divide

$$\begin{array}{r} 5100.433\dots \\ 3 \overline{) 15301.300\dots} \end{array}$$

In this case our division will continue indefinitely, the result can be written by placing a marker on top of the repeating part:

$$153.013 \div 0.03 = 5100.\overline{43}$$

Here are some for you to try. The solutions are at the end.

EXERCISES

18. $18.64 \div 0.4 =$

19. $0.00582 \div 0.03 =$

20. $1984 \div 0.005 =$

21. $0.003 \div 1.5 =$

If you need help with any of the maths covered in this resource (or any other maths topic), you can make an appointment with Learning Development through reception: phone (02) 4221 3977, or Level 2 (top floor), Building 11, or through your campus.



SOLUTIONS TO EXERCISES

- $4100 \div 100 = 41$ (we move the decimal place, which is at the end of the number, two spots to the left)
- $58 \times 1000 = 58000$ (we move the decimal place three spots to the right, or equivalently, attach three zeros)
- $4.9 \times 100 = 490$
- $0.05 \times 10 = 0.5$
- $0.5 \div 10 = 0.05$
- $1.05 \div 10 = 0.105$
- $23.1 \times 100 = 2310$
- $0.003 \times 1000 = 3$
- $580 \div 100 = 5.8$
- $0.581423 \times 100000 = 58142.3$
- 14.9×3

$$149 \times 3 = 447$$

The numbers have a combined one decimal place, hence the answer must also have one decimal place, so

$$14.9 \times 3 = 44.7$$

- 2.01×0.4

$$201 \times 4 = 804$$

The numbers have a combined three decimal places, hence the answer must also have three decimal places, so

$$2.01 \times 0.4 = 0.804$$

- 0.08×0.09

$$8 \times 9 = 72$$

Four decimal places are required, so $0.08 \times 0.09 = 0.0072$

- 2.03×2000

$$203 \times 2000 = 406000$$

Two decimal places are required, so $2.03 \times 2000 = 4060.00 = 4060$

- $28.5 \div 3$

$$\begin{array}{r} 9.5 \\ 3 \overline{) 28.15} \end{array}$$

So $28.5 \div 3 = 9.5$



16. $9.17 \div 7$

So $9.17 \div 7 = 1.31$

$$\begin{array}{r} 1.31 \\ 7 \overline{) 9.17} \end{array}$$

17. $1351.2 \div 16$

So $1351.2 \div 16 = 84.45$

$$\begin{array}{r} 84.45 \\ 16 \overline{) 1351.20} \\ \underline{-128} \\ 71 \\ \underline{-64} \\ 72 \\ \underline{-64} \\ 80 \\ \underline{-80} \\ 0 \end{array}$$

18. $18.64 \div 0.4$

0.4 has one decimal place, so we move the decimal point to the left one place in both numbers to obtain $186.4 \div 4$.

So $18.64 \div 0.4 = 46.6$

$$\begin{array}{r} 46.6 \\ 4 \overline{) 186.4} \end{array}$$

19. $0.00582 \div 0.03$

0.03 has two decimal places, so we move the decimal point to the left two places in both numbers to obtain $0.582 \div 3$.

So $0.00582 \div 0.03 = 0.194$

$$\begin{array}{r} 0.194 \\ 3 \overline{) 0.582} \end{array}$$

20. $1984 \div 0.005$

0.005 has three decimal places, so we move the decimal point to the left three places in both numbers. The first number has no decimal point but $1984 = 1984.000$ so it becomes $1984000 \div 5$.

So $1984 \div 0.005 = 396800$

$$\begin{array}{r} 396800 \\ 5 \overline{) 1984000} \end{array}$$

21. $0.003 \div 1.5$

1.5 has one decimal place, so this becomes $0.03 \div 15$

So $0.003 \div 1.5 = 0.002$

$$\begin{array}{r} 0.002 \\ 15 \overline{) 0.030} \end{array}$$

