Graphs

1. Straight Lines

INTRODUCTION

Imagine you are in a supermarket. Bottled water is on sale for \$2 per bottle (*per* bottle means *for each* bottle). You grab one, then you see apples also on sale for \$2.98 per kg. Now, you only want one bottle of water, so you know that you have to pay \$2 for that. But you are not sure how much your apples weigh. If they weigh 1 kg, your total cost will be $\$2.98 + \$2$. What if they weigh 2 kg? Your total cost this time would be $\$2.98 \times 2 + \$2$. What if they weigh 3.5 kg? Your total cost would be $\$2.98 \times 3.5 + \$2$.

Here we have two *variables* (values that can vary/change). The first is the weight[1] of the apples and the second is the cost, which changes depending on that weight. There is also a *constant* (a number that doesn't change) in this relationship, the price of the bottled water[2].

We can use algebra to express this relationship. Think of appropriate letters to stand for each of the variables. We could use w for the weight of the apples and C for the total cost of the bottled water and apples. So the relationship is:

$$C = 2.98 \times w + 2$$

This is an example of a *linear relationship between two variables*. We will see in the next section why it is called this. Note that the multiplication sign between 2.98 and w is often omitted but the multiplication is still implied, that is:

$$C = 2.98w + 2.$$

GRAPHING AN EQUATION

Whenever we have a relationship (or equation) linking two variables, we can graph the relationship. One way to do this is by first creating a table with some weight and corresponding cost values in it (called *data points*). To draw up a table we will choose the values of w to use to find C, this is because the cost depends on the weight of apples, that is, C depends on w. We call w the *independent* variable and C the *dependent* variable. We can choose any value we like for w, although in this case we won't have negative values because we can't have negative weights of apples! As you complete the table, think about what your answers mean:

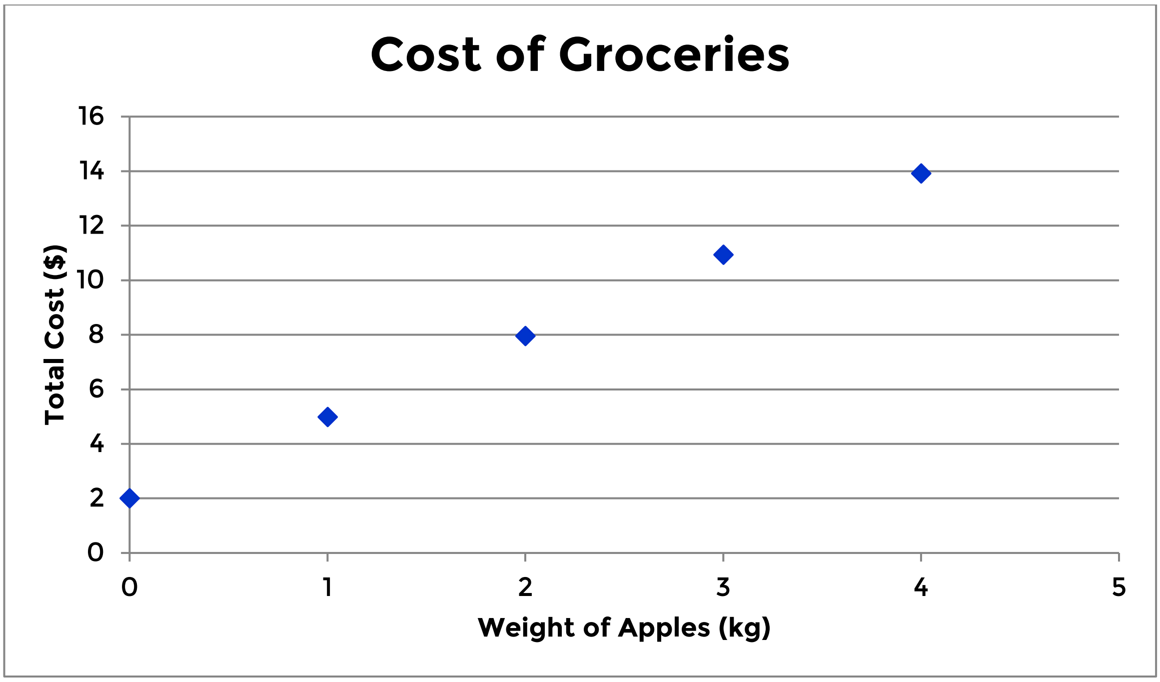
w (weight kg)	0	1	2	3	4
C (cost \$)			7.96		13.92

When we transfer this table data to the graph we will put the weight (w) on the horizontal axis, which is the convention for the independent variable, and the cost (C) on the vertical axis, since it is the dependent variable.

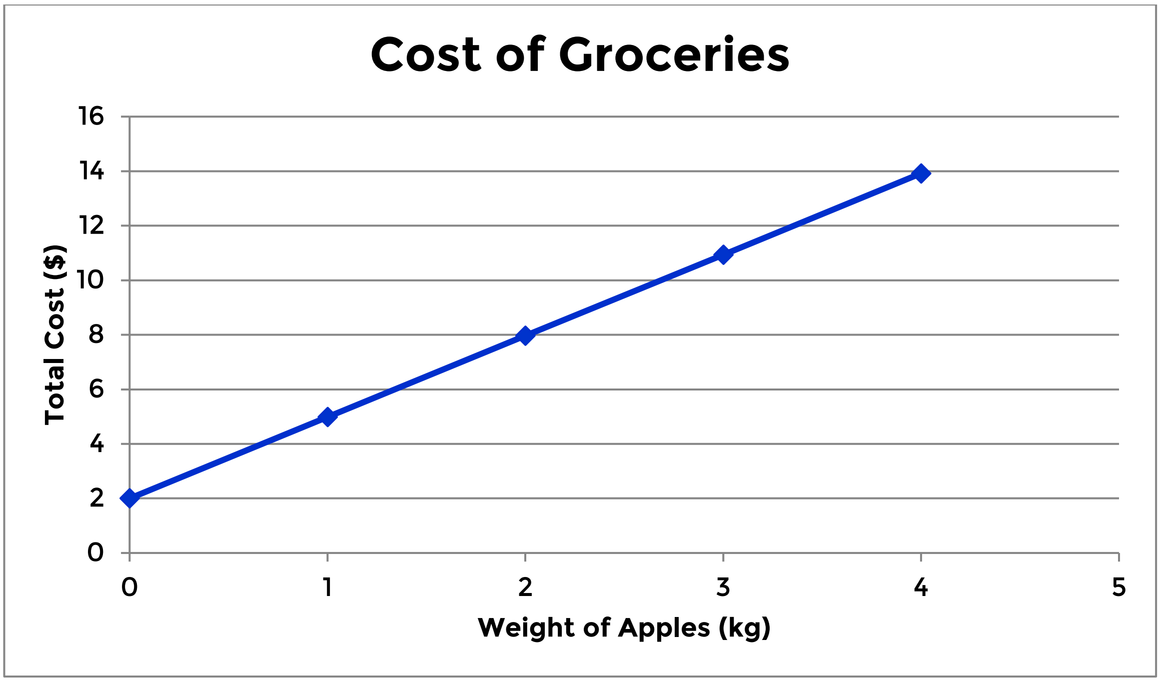
If you look at the figure below, you will see that we mark the values of weights of apples evenly along the horizontal axis, using an appropriate distance between whole numbers of kilograms. Then we mark the costs evenly along the vertical axis, using an appropriate distance between whole numbers of dollars. We also have labelled the axes with what each represents (*weight of apples,* and *total cost*) and have given the graph a title.

Each pair of values have been marked on the figure by a blue diamond. For example, for 2 kg of apples, the cost is \$7.96 so we put a point (a dot or a cross or whatever you choose) above the 2 on the horizontal axis and just under the 8 in line with the vertical axis. The figure itself was created in Excel.

[1] Technically we should be referring to their *mass* but we'll stick with using the common usage of *weight*.
[2] In fact the number 2.98 is a second constant as it also doesn't change, however since it is being multiplied by a variable it has another more important name that we will talk about later



Now we can see that the dots are all in a straight line! This is why we called it a *linear* relationship. We can now join the dots to obtain the graph of our relationship.



When graphing the important part is the line, rather than the data points we used to guide us. For this reason they are often not present in graphs, however if you are plotting by hand it is a good idea to keep them there.

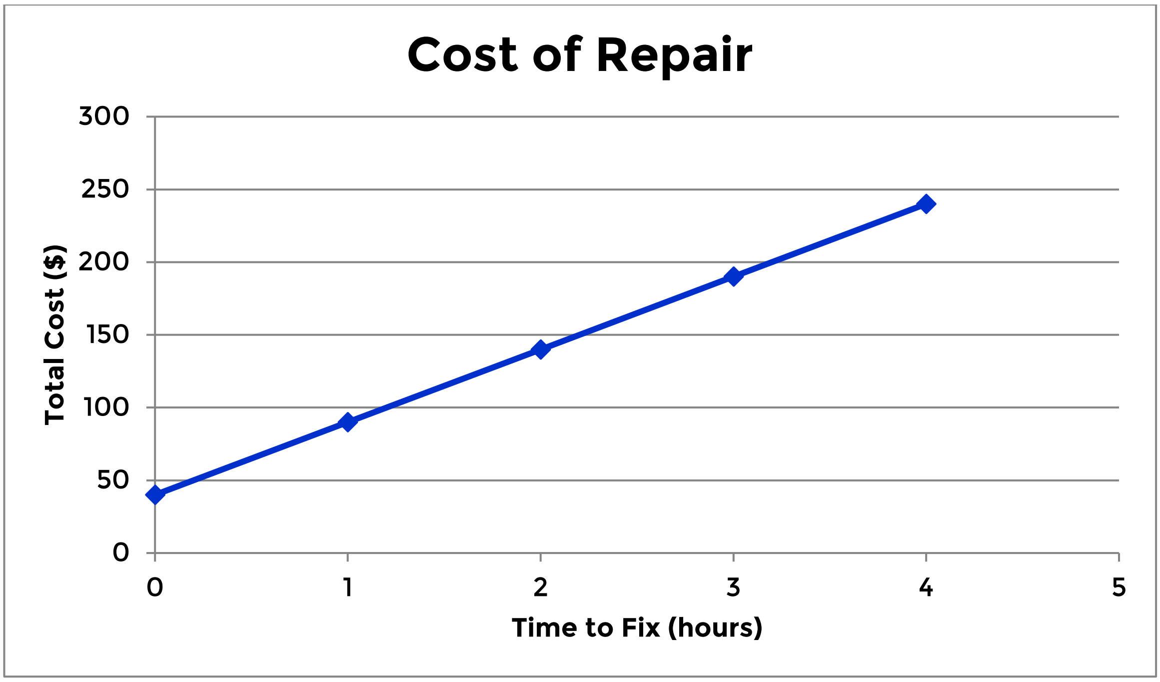
You can use the graph to work out many different scenarios. For example, you can work out how much it would cost for the water and $\frac{1}{2}$ kg of apples by tracing from $\frac{1}{2}$, or 0.5, on the horizontal axis up to the straight line, then across to the vertical axis. You should find about $3.49, though it's hard to be so accurate on this graph! You can also do this by substituting $w = \frac{1}{2}$, or 0.5, into the equation, which gives the more accurate answer.

The graph also lets us work backwards. We could see how many apples we can buy for $10 by going across from the $10 on the vertical axis and seeing that w would be about 2.7. So we could get our bottle of water and about 2.7 kg of apples for $10.

Another example of a linear relationship, also using cost as the dependent variable would be when you need to get someone to fix your fridge. The repair company charges $40 for the repair person just to come to your house, then $50 for each hour they are there fixing the fridge. This means if they are there for 1 hour your charge would be $90, 2 hours $140, $\frac{1}{2}$ an hour $65 and so on. The relationship is $C = 50h + 40$, where h is the number of hours the repair man is there and C is the total cost. We could draw up the following table:

h **(hours)**	0	1	2	3	4
C **(cost $)**	40	90	140	190	240

And obtain the following graph:



CHARACTERISTICS OF LINEAR GRAPHS

In general, we place the independent variable on the horizontal axis, and will often give it the label x, while the dependent variable goes on the vertical axis and is often designated y. So we could have called the cost in the fridge problem y instead of C and the hours x instead of h. The equation would have been written as $y = 50x + 40$. We have already noted that x and y are referred to as the variables, 40 is referred to as the constant, but now we note that 40 is called the <u>x coefficient</u> (or <u>coefficient of x</u>). In the earlier example when we were grocery shopping, 2.98 was the coefficient of w.

If you look at the table, or the graph, you can see that the values of C increase by $50 each time. This means that every time h increases by 1, C increases by $50. This is called the *gradient* of the relationship (or the gradient of the straight line). It is also called <u>the rate of change of C with respect to h</u>.

The gradient is a very important characteristic of graphs. It is given by the following relationship

$$\text{gradient} = \frac{\text{change in } y}{\text{change in } x}.$$

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We have seen that in the fridge example that as x (or h if you prefer) changes by 1 then y (or C) changes by 50. So the gradient is $\frac{50}{1}$, or just 50. The gradient measure how *steep* the line is.

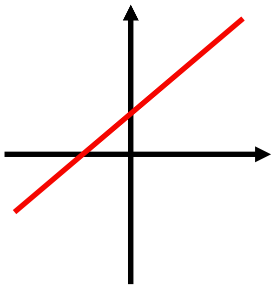
Other important characteristics are the axes intercepts. That is, the values where the graph crosses the vertical axis, the y-intercept, and any value where graph crosses the horizontal axis, the x-intercept. In the fridge example, the value of the y-intercept is 40. This is the amount the repair company charges if the repair person comes to your house but leaves immediately – the fridge is already fixed, or you had just forgotten to turn it on!

To calculate the y-intercept you can just set $x = 0$ in the relationship, and similarly to work out an x-intercept you set $y = 0$. If we write down the general form of a linear equation it will have the form:

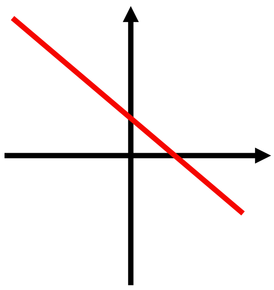
$$y = mx + b,$$

Here m is the gradient and b is the y-intercept. The x-intercept is a little trickier to get but it is equal to $\frac{-b}{m}$.

The gradient, m, tells us the slope of the line, or how steep it is. It can be *positive*, where the line *increases* from left to right:



Or it can be *negative*, where the line *decreases* from left to right:



What is the gradient and the y-intercept of the grocery shopping (for apples and water) example?

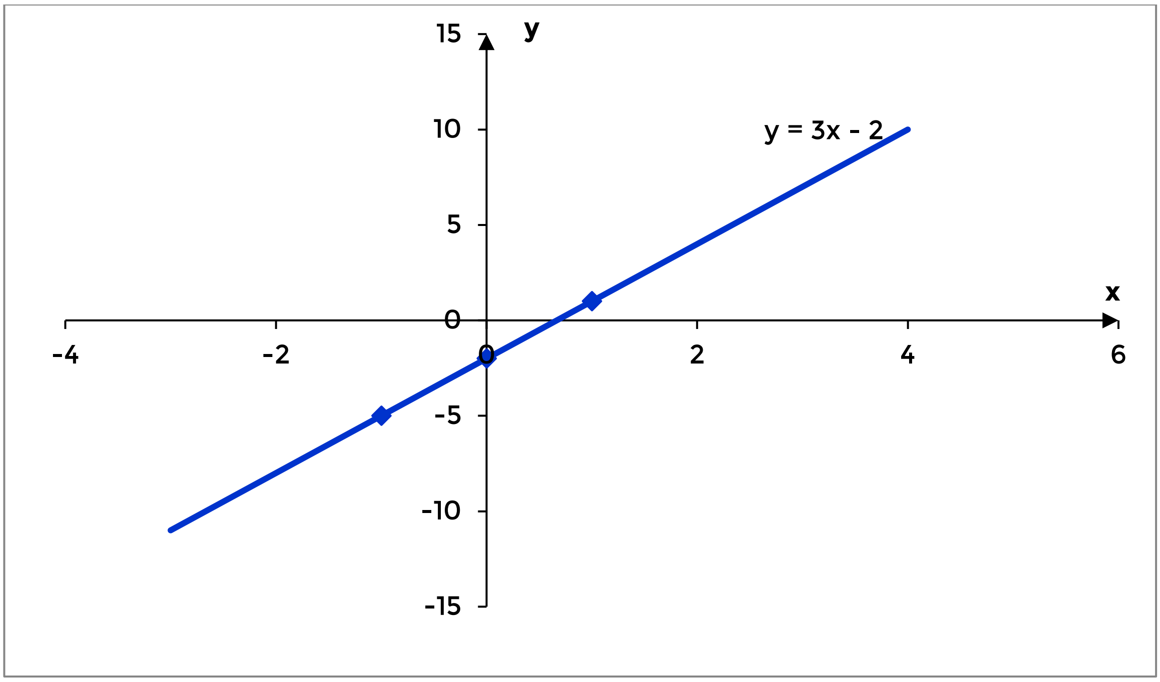
Answer: Gradient of 2.98, y-intercept of 2

One last example. Draw the graph of $y = 3x - 2$ and write down the gradient and y-intercept of the line.

To draw a line we only need to know two points, but it is a good idea to calculate at least three in case you have made an error with one of them (if you can't draw a straight line through all of your points, you have made an error!). We will use three points:

x	-1	0	1
y	-5	-2	1

This time we don't have context for our graph, so we don't need a title and we only label the axes with x and y. We might also like to label the line with its equation and continue the plot past the data points that we have used (we know the relationship is linear, so there is no harm in doing this).



From the relationship we can also see that the gradient of our graph is 3 and the y-intercept is -2.

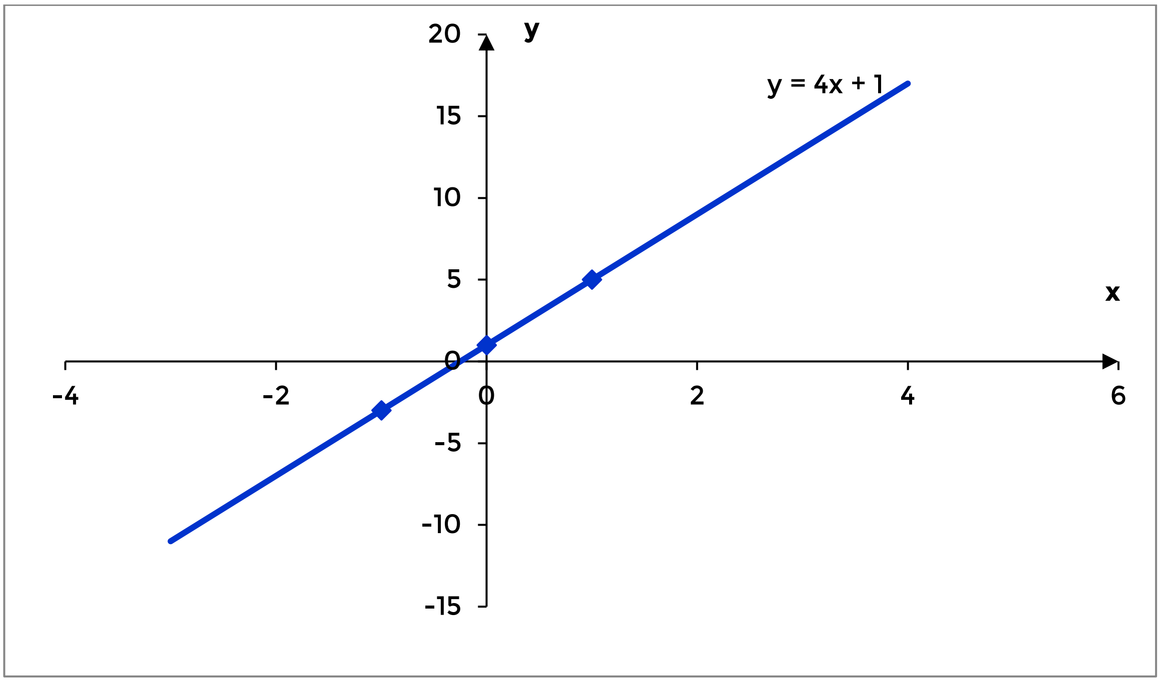
Here's one for you to try. You can check your answer with the solution on the next page.

EXERCISES

Draw the graph of $y = 4x + 1$ and write down its gradient and y-intercept.

If you need help with any of the maths covered in this resource (or any other maths topic), you can make an appointment with Learning Development through reception: phone (02) 4221 3977, or Level 2 (top floor), Building 11, or through your campus.

SOLUTIONS TO EXERCISES



The gradient is 4 and the *y*-intercept is 1.