Algebra

10. Factorising Special Quadratic Expressions

Before looking at this resource on factorising, you might like to have a look at

- *Algebra – 6. Expanding Algebraic Expressions*
- *Algebra – 7. Factorising Pairs of Terms*
- *Algebra – 8. Factorising Quadratic Expressions*
- *Algebra – 9. Factorising More Complicated Quadratic Expressions*

PERFECT SQUARES

Let's start by going back to expanding. Using any method you like, expand $(x+5)^2$, remembering that this means $(x+5)(x+5)$.

Answer: $x^2 + 10x + 25$

Now expand $(2x-3)^2$, again remembering that squaring means multiplying the expression to be squared by itself.

Answer: $4x^2 - 12x + 9$

There is a pattern here. Let's use the table method to investigate $(x+y)^2$ and see if we can work out the pattern.

$(\boldsymbol{x}+\boldsymbol{y})^2$

$\times$	x	y
x	x^2	xy
y	xy	y^2

So the table tells us that $(x+y)^2$ is ALWAYS $= x^2 + 2xy + y^2$.

Now, we can apply the pattern to the example above, and see that

$$\begin{aligned}(x+5)^2 &= x^2 + 2 \times x \times 5 + (5)^2 \\ &= x^2 + 10x + 25\end{aligned}$$

Next, let's look at $(x-y)^2$

$(\boldsymbol{x}-\boldsymbol{y})^2$

$\times$	x	$-y$
x	x^2	$-xy$
$-y$	$-xy$	y^2

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And so, $(x-y)^2$ is ALWAYS $= x^2 - 2xy + y^2$. Note that we could have also obtained this by changing every y to $-y$ in the formula for $(x+y)^2$.

Again, using the example from before, we can now see that

$$\begin{aligned}(2x-3)^2 &= (2x)^2 - 2\times 2x \times 3 + (3)^2 \\ &= 4x^2 - 12x + 9\end{aligned}$$

The formulas

$$\boldsymbol{(x+y)^2 = x^2 + 2xy + y^2}$$

and

$$\boldsymbol{(x-y)^2 = x^2 - 2xy + y^2}$$

are called the squares of binomials and are perfect squares.

DIFFERENCE OF TWO SQUARES

A third useful expansion to know is that of $(x+y)(x-y)$. Let's use the table method to expand this expression.

$$\boldsymbol{(x+y)(x-y)}$$

$\times$	x	y
x	x^2	xy
$-y$	$-xy$	$-y^2$

This time the terms in xy add to 0, and so

$$\boldsymbol{(x+y)(x-y) = x^2 - y^2}$$

This is called the Difference of Two Squares. (Notice that in the Difference of Two Squares, the factors $(x+y)$ and $(x-y)$ have different signs, but when squaring binomials the factors are exactly the same as each other.)

FACTORISING

When dealing with the square of a binomial it can be difficult to recognise it as such. However, normal quadratic factorisation can always still be used.

Example: Factorise $25x^2 + 30x + 9$

The clues that this could be a perfect square (square of a binomial) are the $25x^2$ and the 9. $25x^2 = (5x)^2$, and $9 = (\pm 3)^2$. We now check whether $2 \times 5x \times 3 = 30x$, is the remaining term and since it is we have the perfect square $(5x+3)^2$.

But we can also use the methods of factorising trinomials:

$$+225$$

$$25x^2 + 30x + 9$$

$$\frac{(25x\quad)(25x\quad)}{25}$$

$$(5x+3) \qquad (5x+3)$$

$$\frac{(25x+15)(25x+15)}{25 \;\; 5 \;\; 1}$$

$$\therefore 25x^2 + 30x + 9 = (5x+3)(5x+3)$$

The Difference of Two Squares is an important relationship to learn, especially when it comes to recognising the factors. This formula can be used when there are only two terms in the expression and they have opposite signs.

Examples:

$$36 - 25a^2 = (6 - 5a)(6 + 5a)$$

$$81 - 16y^2 = (9 - 4y)(9 + 4y)$$

$$25x^4 - 9y^2 = (5x^2 - 3y)(5x^2 + 3y)$$

$$\begin{aligned} 36x^2 - 4y^2 &= (6x - 2y)(6x + 2y) \\ &= 2(3x - y)2(3x + y) \\ &= 4(3x - y)(3x + y) \end{aligned}$$

OR

$$\begin{aligned} 36x^2 - 4y^2 &= 4(9x^2 - y^2) \\ &= 4(3x - y)(3x + y) \end{aligned}$$

Here are some for you to try. You can check your results with the solutions at the back.

EXERCISES

1. Expand each of the following expressions:

a) $(2x+1)^2$

b) $(3x-5)^2$

c) $(a+2b)^2$

(d) $(5x+2y)(5x-2y)$

(e) $(2a-b)(2a+b)$

2. Factorise each of these expressions:

(a) $144a^2 - 9b^2$

(b) $9x^2 - 24x + 16$

(c) $1 - 36x^2y^2$

(d) $x^2y^2 - 2xy + 1$

(e) $(a+2b)^2 - c^2$

If you need help with any of the maths covered in this resource (or any other maths topics), you can make an appointment with Learning Development through reception: phone (02) 4221 3977, or Level 2 (top floor), Building 11, or through your campus.

SOLUTIONS TO EXERCISES

1.

(a) $(2x+1)^2 = 4x^2 + 4x + 1$

(b) $(3x-5)^2 = 9x^2 - 30x + 25$

(c) $(a+2b)^2 = a^2 + 4ab + 4b^2$

(d) $(5x+2y)(5x-2y) = 25x^2 - 4y^2$

(e) $(2a-b)(2a+b) = 4a^2 - b^2$

2.

(a) $144a^2 - 9b^2 = 9(4a-b)(4a+b)$

(b) $9x^2 - 24x + 16 = (3x-4)^2$

(c) $1 - 36x^2y^2 = (1-6xy)(1+6xy)$

(d) $x^2y^2 - 2xy + 1 = (xy-1)^2$

(e) $(a+2b)^2 - c^2 = (a+2b-c)(a+2b+c)$