Algebra

9. Factorising More Complicated Quadratic Expressions

Before looking at this resource on factorising, you might like to have a look at

- *Algebra – 6. Expanding Algebraic Expressions*
- *Algebra – 7. Factorising Pairs of Terms*
- *Algebra – 8. Factorising Quadratic Expressions*

INTRODUCTION

In *Algebra – 8. Factorising Quadratic Expressions*, we covered factorising quadratic trinomials such as $x^2 + 3x - 4$ where the coefficient of x^2 is 1. (The result of factorising $x^2 + 3x - 4$ is $(x + 4)(x - 1)$, by the way.) We are now going to look at expressions that are slightly more complicated because they have a coefficient of x^2 that is NOT 1.

There are several methods you can use to factorise these expressions. Firstly, we'll look at going back to the table we first used in *Algebra – 6. Expanding Algebraic Expressions*. Then we will cover using a formula and finally using pairs and grouping.

TABLE METHOD

We are going to factorise $3x^2 + 17x + 10$ using the table method.

$\times$	? Term in x	? Factor of 10
? Term in x	$3x^2$	
? Factor of 10		10

We can see that one of the "terms in x" in the table will have to be $3x$ and the other will have to be x as these are the only integer terms that multiply to $3x^2$. So our table can now become:

$\times$	$\boldsymbol{x}$	? Factor of 10
$\boldsymbol{3x}$	$3x^2$	
? Factor of 10		10

Now, the tricky bit! The factors of 10 are: 1 and 10, 2 and 5, -1 and -10, -2 and -5. But we need to know which one to multiply by $3x$ and which one to multiply by x in order to obtain the total of $17x$. After a little trial and error work, it can be found to work if we use $3x \times 5$ and $x \times 2$, so our table becomes:

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$\times$	x	5
$3x$	$3x^2$	
2		10

and our factors are $(x+5)(3x+2)$. You should check this by expanding, using the table method, or any other method.

USING A FORMULA

Due to the trial and error needed in performing the table method (and similar methods, such as the "cross" method), alternate methods are preferred. These may seem a little complicated at first, but they do work every time without the need for guesswork.

Let's try to factorise the same expression, $3x^2 + 17x + 10$.

The all-important figure in this method is the coefficient of x^2, which in this case is 3. We multiply the constant (the number on the end, in this case 10) by that coefficient and write the result above the 10. So we now have:

$$\begin{array}{r} +30 \\ 3x^2 + 17x + 10 \end{array}$$

In BOTH pairs of brackets, we insert $3x$ and then, to counter us putting it in twice, we divide the whole expression by 3. (I told you the 3 was the all-important figure in this method!). So we have:

$$\frac{(3x \quad)(3x \quad)}{3}$$

Instead of looking for the factors of 10, we look for the factors of the 30. These are 1 and 30, 2 and 15, 3 and 10, 5 and 6, -1 and -30, -2 and -15, -3 and -10, and -5 and -6. Because they need to add to a positive number, 17 (we still use 17 as our sum), they must also both be positive (so we can throw away the negative pairs). So, the factors that will work in this case are 2 and 15.

Now we write them in the brackets:

$$\frac{(3x+2)(3x+15)}{3}$$

The last step is to divide the denominator, 3, into the numerator factors so that it cancels completely (it will always do this). $(3x+2)$ cannot be divided by 3 (because 2 cannot be divided exactly by 3), however $(3x+15)$ CAN be divided by 3, and we obtain $(x+5)$. We now have

$$\frac{(3x+2)\overset{(x+5)}{\cancel{(3x+15)}}}{\underset{1}{\cancel{3}}}$$

and our factors are $(3x+2)(x+5)$, which agrees with the answer from our previous method (and you can check by expanding).

We will do a couple more examples.

Factorise $6x^2 + 11x + 4$.

Firstly, we multiply the 4 by 6. We write the result above 4. (This part is working only and so we don't write an equals beside the original expression.)

$$\begin{array}{r} +24 \\ 6x^2 + 11x + 4 \end{array}$$

Now we put $6x$ in both brackets and divide by 6

$$\frac{(6x\quad)(6x\quad)}{6}$$

Positive factors of 24 (since we will need them to add to a positive number we only need to consider the positive ones) are: 1 and 24, 2 and 12, 3 and 8, and 4 and 6. The pair that adds to 11 is 3 and 8.

So we get:

$$\frac{(6x+3)(6x+8)}{6}$$

This time, neither bracket will divide completely by 6; however, the first bracket divides by 3 and the second by 2, so we obtain:

$$\frac{\overset{(2x+1)}{\cancel{(6x+3)}}\overset{3x+4}{\cancel{(6x+8)}}}{\cancel{6}\ \cancel{2}\ 1}$$

So $\underline{6x^2+11x+4=(2x+1)(3x+4)}$ (Check by expanding)

Let's try one with a *negative number*!

Factorise $5x^2+33x-14$

This time I'll put the working only, without explanation. See if you can follow.

Factorise $5x^2+33x-14$

$$5x^2+33x\overset{-70}{-14}$$

$$\frac{(5x\quad)(5x\quad)}{5}$$

$$\frac{(5x+35)(5x-2)}{5}$$

$$\frac{\overset{(x+7)}{\cancel{(5x+35)}}(5x-2)}{\cancel{5}\ 1}$$

$\underline{\therefore 5x^2+33x-14=(x+7)(5x-2)}$ (Check by expanding, also, in mathematics we use the symbol ∴ to mean *therefore*)

PAIRS AND GROUPING METHOD

This last method is a variation on the above formula and uses the pairs and grouping factorising technique that we learnt in *Algebra – 7. Factorising Pairs of Terms*. It starts the same way as the formula method:

To factorise $16x^2-16x+3$ we calculate $16\times 3=48$ and write it above the constant term:

$$16x^2-16x\overset{+48}{+3}$$

However, instead of remembering a formula, we are going to split up the x term in the quadratic using the relevant factor pair. In this case the middle term is negative, so we only need to consider the negative factors of 48. These are -1 and -48, -2 and -24, -3 and -16, -4 and -12, and -6 and -8. So the relevant factor pair is -4 and -12. We now rewrite the quadratic as

$$16x^2-4x-12x+3$$

It does not matter which way around we write the factor pair. Next we note that the first two terms has a common factor of $4x$, while the last two terms have a common factor of 3, so our expression becomes

$$4x(4x-1)+3(-4x+1)$$

Now the two bracketed expressions are not the same, but rather the negative of each other, so we need to factor out a negative from the second term

$$4x(4x-1)-3(4x-1)$$

We can now factor the $(4x-1)$ out of both terms to obtain:

$\underline{16x^2-16x+3=(4x-1)(4x-3)}$ (Check by expanding)

Another example: Factorise $10x^2+11x-6$

$$\overset{-60}{10x^2+11x-6}$$

The factors of -60 are: 1 and -60, 2 and -30, 3 and -20, 4 and -15, 5 and -12, 6 and -10, -1 and 60, -2 and 30, -3 and 20, -4 and 15, -5 and 12, and -6 and 10. Therefore we can replace $11x$ with $-4x+15x$:

$$10x^2-4x+15x-6$$

$$2x(5x-2)+3(5x-2)$$

$$(2x+3)(5x-2)$$

$\underline{\therefore 10x^2+11x-6=(2x+3)(5x-2)}$ (Check by expanding)

A NEGATIVE COEFFICIENT OF x^2

What do we do when the coefficient of x^2 is negative? For example, $-x^2-2x+15$. All we need to do is factorise out -1 from all terms then proceed as before. So to start with we write

$$-x^2-2x+15=-(x^2+2x-15)$$

Now factorise the expression in the brackets:

$$\begin{aligned}-x^2-2x+15&=-(x^2+2x-15)\\&=-(x+5)(x-3)\end{aligned}$$

Now we could stop here, or we could multiply the -1 factor back into ONE of the other factors (it doesn't matter which since the order of multiplication doesn't matter). In this case we will choose the $(x-3)$ factor (since it already has a negative term in it). We work out that $-(x-3)=-x+3=3-x$ and so we replace the minus sign and the second bracket with $(3-x)$. So now we've got

$$-x^2-2x+15=(x+5)(3-x)$$

We also know that $x+5$ is the same as $5+x$ and so we can simply reverse this expression so that the numbers and the letters are lined up better. So we get

$-x^2-2x+15=(5+x)(3-x)$ or $-x^2-2x+15=(3-x)(5+x)$

whichever way you want to write it! All you have to be sure of is that the signs of the numbers and letters in the brackets are the same as mine.

Now factorise $10+3x-4x^2$

Firstly, rewrite so all the terms are in the correct order:

$$10+3x-4x^2=-4x^2+3x+10$$

Now, take the – out of brackets and proceed as usual (we will use the formula method):

$$\begin{aligned}10+3x-4x^2 &= -(4x^2-3x-10) \quad (-40)\\ &= -\frac{(4x\quad)(4x\quad)}{4}\\ &= -\frac{(4x-8)(4x+5)}{4}\\ &= -(x-2)(4x+5)\\ &= \underline{(2-x)(5+4x)}\end{aligned}$$

Here are some for you to try. You can check them by expanding, or with my solutions at the back.

EXERCISES

Factorise the following whichever method you prefer

1. $12x^2+7x+1$
2. $6x^2-11x+3$
3. $5x^2-13x-6$
4. $9x^2+80x-9$
5. $16x^2-16x-21$
6. $4x^2-12x+9$
7. $-x^2+x+2$
8. $3-x-2x^2$

If you need help with any of the maths covered in this resource (or any other maths topics), you can make an appointment with Learning Development through reception: phone (02) 4221 3977, or Level 2 (top floor), Building 11, or through your campus.

SOLUTIONS TO EXERCISES

Working

1. $12x^2 + 7x + 1 = (3x + 1)(4x + 1)$ ($+12$)   Working: $\frac{(12x+4)(12x+3)}{12}$

2. $6x^2 - 11x + 3 = (2x - 3)(3x - 1)$ ($+18$)   Working: $\frac{(6x-9)(6x-2)}{6}$

3. $5x^2 - 13x - 6 = (x - 3)(5x + 2)$ (-30)   Working: $\frac{(5x-15)(5x+2)}{15}$

4. $9x^2 + 80x - 9 = (x + 9)(9x - 1)$ (-81)   Working: $\frac{(9x+81)(9x-1)}{9}$

5. $16x^2 - 16x - 21 = (4x - 7)(4x + 3)$ (-336)   Working: $\frac{(16x-28)(16x+12)}{16}$

6. $4x^2 - 12x + 9 = (2x - 3)(2x - 3)$ (-36)   Working: $\frac{(4x-6)(4x-6)}{4}$

   or $= (2x - 3)^2$

7. $-x^2 + x + 2 = -(x^2 - x - 2)$

   $= -(x - 2)(x + 1)$

   $= (2 - x)(1 + x)$

8. $3 - x - 2x^2 = -(2x^2 + x - 3)$ (-6)   Working: $\frac{(2x+3)(2x-2)}{2}$

   $= -(2x + 3)(x - 1)$

   $= (3 + 2x)(1 - x)$