Algebra

8. Factorising Quadratic Expressions

Before looking at this resource on factorising, you might like to have a look at:

- *Algebra – 6. Expanding Algebraic Expressions*
- *Algebra – 7. Factorising Pairs of Terms*

INTRODUCING QUADRATICS

A *quadratic expression* is any expression where the highest power of the variable (often x) is 2. Examples:

- x^2
- $4x^2 + 7$
- $2x^2 - 3x + 4$
- $-2x^2 + x - 1$

A *quadratic trinomial* is an expression such as $3x^2 + x - 5$. Quadratic trinomials have the highest power of the variable (x) being 2 (so x^2) and there are three terms in the expression. It may be possible to factorise a quadratics.

REVERSING AN EXPANSION

In the handout on *Algebra – 6. Expanding Algebraic Expressions*, we worked through expanding pairs of brackets, the results of which were quadratic trinomials. This means that, going backwards and starting with the trinomial, we could factorise it and obtain the original pairs of brackets.

So, for example, if we expand $(x + 3)(x + 2)$, we obtain $x^2 + 5x + 6$. This means that we can factorise $x^2 + 5x + 6$ to obtain $(x + 3)(x + 2)$. We will go through the expansion and then work backwards to obtain the factors. Using the table method for expanding:

$$(\boldsymbol{x + 3})(\boldsymbol{x + 2})$$

×	x	3
x	x^2	$3x$
2	$2x$	6

Adding like terms we obtain the result $x^2 + 5x + 6$ (as above).

Notice that the 6 is the result of multiplying the two numbers 2 and 3. Notice also that the term $5x$ is the result of adding the two terms in x, that is $2x$ and $3x$. Without knowing the original factors, our table to factorise $x^2 + 5x + 6$ might look like this:

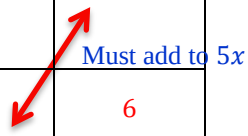
×		
	x^2	
		6

We know that, to obtain x^2 our factors are usually x and x so we are pretty safe in putting those in the table:

×	x	
x	x^2	
		6

Now think of the factors that might multiply to give 6. They might be 1 and 6, 2 and 3, or even -1 and -6, or -2 and -3 (Note we now do have to include the negative factors). We also know that the two numbers involved, when multiplied by x, MUST add to $5x$. The only pair of factors to do that are 2 and 3. We can now complete the rest of the table:

×	x	3
x	x^2	
2		6



and that means that our factors must be $(x + 3)(x + 2)$.

When factorising a quadratic trinomial, the major things to look for are:

- all factors of the constant (the number on its own without any x), paying particular attention to the sign of that constant as this dictates what signs the factors can take
- which pair of factors we just found add together to give the coefficient of the term in x (the coefficient is the number multiplying with x)

Let's try an example without expanding first.

Factorise $x^2 + 7x + 12$

- Firstly, look for all the factors of 12. Because it is positive, the factors must have the same sign, that is, both positive or both negative. There are six pairs of factors of 12: 1 and 12, 2 and 6, 3 and 4, -1 and -12, -2 and -6, and -3 and -4
- Now we look at the term in x and see that its coefficient is 7. Which pair of factors add to 7? Only 3 and 4.
- So $x^2 + 7x + 12 = (x + 3)(x + 4)$. (It doesn't matter which bracket you write first.) You should of course check by expanding.

Now try this one: Factorise $x^2 - 8x + 15$

- The factors of 15 are: 1 and 15, 3 and 5, -1 and -15, and -3 and -5.
- Now they need to add to -8 this time, which means both must be negative and the only pair that fits that is -5 and -3.
- So $x^2 - 8x + 15 = (x - 3)(x - 5)$. Again, you should check by expanding.

Now let's try with a negative number on the end: Factorise $x^2 + 3x - 10$.

- We still look for the factors of 10, but because it is -10, one of the factors has to be positive, and one negative (because they multiply together to give a negative number). The positive pairs of factors of 10 are: 1 and 10, and 2 and 5. So the pairs of factors of -10 are: 1 and -10, 2 and -5, -1 and 10, and -2 and 5
- We need them to add together to give 3. We see that the pairs add up to give: -9, -3, 9, and 3 respectively. So the pair that works is -2 and 5!
- So, our factors are $(x + 5)(x - 2)$. Again, it doesn't matter what order we write the brackets in, as long as the signs are correctly placed with the numbers, so we could also write $(x - 2)(x + 5)$. Please check by expanding!

Another example: Factorise $x^2 - x - 20$.

- The positive factors of 20 are: 1 and 20, 2 and 10, and 4 and 5. So the factors of -20 are: 1 and -20, 2 and -10, 4 and -5, -1 and 20, -2 and 10, and -4 and 5
- The term in x is $-x$, which stands for $-1x$, so we need our numbers to add to -1. The pair of factors that do this are 4 and -5
- So, $x^2 - x - 20 = (x - 5)(x + 4)$. Again, expand this answer out to check.

One last example. Factorise $x^2 - 5x - 6$

- Again, the number on the end is negative so our factors will have different signs. The positive factors of 6 are 1 and 6, and 2 and 3. So the factors of -6 are: 1 and -6, 2 and -3, -1 and 6, and -2 and 3
- We need them to add to -5, so the correct factors are 1 and -6
- So $x^2 - 5x - 6 = (x - 6)(x + 1)$. Please check!

Note: we have not yet considered factorising trinomials when the coefficient of x^2 is different from 1. For the methods of factorising such expressions, please see *Algebra – 9. Factorising More Complicated Quadratic Expressions.*

Here are some for you to try. You can check your answers by expanding (and also with the solutions at the end of this handout).

EXERCISES

Factorise each of the following quadratic expressions:

1. $x^2 + 6x + 8$
2. $x^2 - 9x + 18$
3. $x^2 + 5x + 4$
4. $x^2 - 17x + 16$
5. $x^2 + 4x - 21$
6. $x^2 - 2x - 24$
7. $x^2 + x - 6$
8. $x^2 - 6x - 55$

If you need help with any of the maths covered in this resource (or any other maths topics), you can make an appointment with Learning Development through reception: phone (02) 4221 3977, or Level 2 (top floor), Building 11, or through your campus.

SOLUTIONS TO EXERCISES

1. $x^2 + 6x + 8 = (x + 4)(x + 2)$

2. $x^2 - 9x + 18 = (x - 6)(x - 3)$

3. $x^2 + 5x + 4 = (x + 4)(x + 1)$

4. $x^2 - 17x + 16 = (x - 16)(x - 1)$

5. $x^2 + 4x - 21 = (x + 7)(x - 3)$

6. $x^2 - 2x - 24 = (x - 6)(x + 4)$

7. $x^2 + x - 6 = (x + 3)(x - 2)$

8. $x^2 - 6x - 55 = (x + 5)(x - 11)$