Algebra

6. Expanding Algebraic Expressions

INTRODUCTION

Before we start with expanding algebra, let's recall some aspects of multiplication.

When needing to multiply 53 by 2, say, you can double the 50, double the 3, and add the results together, to get 106. That is, you separate the 53 into its components of $50 + 3$, and actually perform 53×2 as $(50 + 3) \times 2$, which is the same as $50 \times 2 + 3 \times 2$.

The written method you were probably taught looks like this:

$$\begin{array}{r} 53 \\ \underline{\times \;\; 2} \\ \underline{106} \end{array}$$

What about 56×2? We can still do this as $50 \times 2 + 6 \times 2$, to obtain $100 + 12$, so the result is 112. This time, the written method looks like this:

$$\begin{array}{r} {}^{1}56 \\ \underline{\times \;\; 2} \\ \underline{112} \end{array}$$

We can describe this method in full, say for 23×4:

$$\begin{array}{r} 23 \\ \underline{\times \;\; 4} \end{array}$$

Start from the right and calculate 3×4 (or 4×3 – it doesn't matter) and obtain 12. Instead of writing down 12, we write down the 2 in the units column (underneath the 3) and need to remember to include the 1 when we have done the multiplying for the next (tens) column. We then work out 4×2, which is 8, and adding that extra 1 we obtain 9, and our answer is 92. The working is as follows:

$$\begin{array}{r} {}^{1}23 \\ \underline{\times \;\; 4} \\ \underline{92} \end{array}$$

We would have obtained the same answer if we had separated the original 23 into $20 + 3$ and multiplied both 20 and 3 by 4, then added the results together. That is, 23×4 is the same as $(20 + 3) \times 4$, which is the same as $20 \times 4 + 3 \times 4$, which is $80 + 12$, which is 92. This might be easier to show in a table:

23×4

$\times$	20	3
4	80	12

Now add $80 + 12$ to obtain the answer 92. (This method allows us to calculate without having to remember the extra ten.)

Now let's try 35×14. We can think of this as $(30 + 5) \times (10 + 4)$ and use the following table:

35×14

$\times$	30	5
10	300	50
4	120	20

Now we add 300, 50, 120, and 20 and obtain the answer 490 (you can of course check this using the other procedure).

I've used this method especially to illustrate that when we are multiplying numbers that contain more than one digit, all digits of one number need to be multiplied by all digits of the other. For example, we might need to multiply 357×246. We could set up our multiplication as follows:

$$357 \times 246 = (300 + 50 + 7) \times (200 + 40 + 6).$$

We then need to multiply

- 300×200, 300×40, 300×6;
- 50×200, 50×40, 50×6
- 7×200, 7×40, 7×6

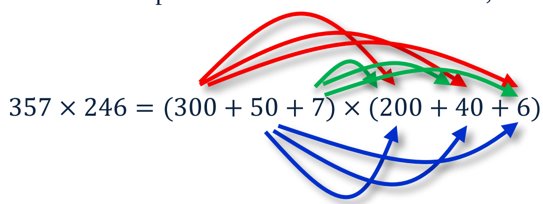
It doesn't matter what order we do these multiplications in but we do need to multiply all pairs of numbers together. Our table would look like this:

357×246

$\times$	300	50	7
200	60000	10000	1400
40	12000	2000	280
6	1800	300	42

Add all the red figures to obtain the answer 87822.

Without using the table, we could use the pairs and illustrate with arrows, like this:



which can start to look a little complicated!

EXPANDING ALGEBRA

So how does this help us expand algebraic expressions? Well expanding brackets in algebra is exactly the same procedure. For example, $2(x+1)$ represents multiplying the sum of x and 1 (which can only be written as $x+1$) by 2, just as multiplying 56 by 2 means multiplying the sum of 50 and 6 by 2. That is, both x and 1 have to be multiplied by 2. (Note: When there is no symbol between the 2 and the open bracket, it indicates multiplication; in fact we usually don't put the × sign in).

So

$$\begin{aligned} 2(x+1) &= 2 \times x + 2 \times 1 \\ &= 2x + 2 \end{aligned}$$

Because these terms are unlike, they can't be added further than this result.

We could also use the table method here:

$2(x+1)$

×	x	1
2	$2x$	2

Now add all results together to obtain $2x+2$. We would obtain the same result if the 2 was written after the brackets: $(x+1)2 = x \times 2 + 1 \times 2$ which is $2x+2$.

Now let's try $(5x+2)(3x+1)$, where each term in each set of brackets has to be multiplied by each term in the other brackets, and remembering that when we multiply x by x, we obtain x^2, and so $5x \times 3x = 15x^2$.

$(5x+2)(3x+1)$

×	$3x$	1
$5x$	$15x^2$	$5x$
2	$6x$	2

Now, adding together $15x^2 + 5x + 6x + 2$, we see that the only like terms are $5x$ and $6x$ so our complete result is $15x^2 + 11x + 2$.

We need to be careful with negative terms! For example, when calculating $-5(2x+3)$, using the table method gives

$-5(2x+3)$

×	$2x$	3
– 5	$-10x$	– 15

There are no like terms, so our result is $-10x - 15$.

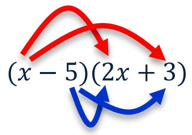
Try $(x - 5)(2x + 3)$. Using the table method,

$$(\boldsymbol{x - 5})(\boldsymbol{2x + 3})$$

$\times$	x	-5
$2x$	$2x^2$	$-10x$
3	$3x$	-15

Adding all the terms, we have $2x^2 - 10x + 3x - 15$, which gives $2x^2 - 7x - 15$.

Rather than using the table, we could again realise that each term in a pair of brackets[1] needs to be multiplied by each term in the other pair of brackets. So, for the last example, we could use the following method:



So x has to be multiplied by $2x$ (which gives $2x^2$), and x has to be multiplied by 3 (which gives $3x$); and – 5 has to be multiplied by $2x$ (which gives $-10x$) and – 5 has to be multiplied by 3 (which gives – 15). It doesn't matter what order we multiply in as long as we obtain the correct signs for our answers. This time we get $2x^2 + 3x - 10x - 15$, and end up with $2x^2 - 7x - 15$, as before.

It is very important that you understand the process of expanding brackets before trying to factorise algebraic expressions (the next unit), so here are some for you to try. The solutions are at the back of this resource.

EXERCICES

1. $3(x + 5)$

2. $(4x - 1)5$

3. $7(3 - 2x)$

4. $-2(5x + 3)$

5. $(x + 3)(x + 2)$

6. $(2x - 1)(x + 1)$

7. $(3x - 5)(2x - 1)$

8. $(x + 5)^2$

9. $(5 - x)(3x + 1)$

10. $(2x + 1)(2x - 1)$

If you need help with any of the maths covered in this resource (or any other maths topics), you can make an appointment with Learning Development through reception: phone (02) 4221 3977, or Level 2 (top floor), Building 11, or through your campus.

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1 Strictly speaking, pairs of bracket like "( )" are actually called "parentheses".

SOLUTIONS TO EXERCISES

1. $3(x+5) = 3x + 15$

2. $(4x-1)5 = 20x - 5$

3. $7(3-2x) = 21 - 14x$

4. $-2(5x+3) = -10x - 6$

5. $(x+3)(x+2) = x^2 + 3x + 2x + 6$

$= x^2 + 5x + 6$

6. $(2x-1)(x+1) = 2x^2 - x + 2x - 1$

$= 2x^2 + x - 1$

7. $(3x-5)(2x-1) = 6x^2 - 10x - 3x + 5$

$= 6x^2 - 13x + 5$

8. $(x+5)^2 = (x+5)(x+5)$

$= x^2 + 5x + 5x + 25$

$= x^2 + 10x + 25$

9. $(5-x)(3x+1) = 15x - 3x^2 + 5 - x$

$= 14x - 3x^2 + 5$

10. $(2x+1)(2x-1) = 4x^2 + 2x - 2x - 1$

$= 4x^2 - 1$

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