Algebra

5. Rearranging Formulae to Isolate Powers

Often we need to isolate a variable which is a power in a given formula. For example, we might need to know the value of x given $3^x = 9$. This one is fairly straightforward as we know that $3^2 = 9$, so $x = 2$. However, we may need to know the value of x when $3^x = 19$, which is not straightforward at all!

METHOD 1

We need to rearrange the power form of this equation into its logarithm form. The background to this technique is covered in *Powers and Logarithms – 2. Logarithms.*

$$3^x = 19 \leftrightarrow \log_3 19 = x.$$

(The symbol "→" means "implies that" and a double headed arrow means it goes both ways, so $3^x = 19$ implies that $\log_3 19 = x$ and vice versa.)

Now, we need to work out $\log_3 19$. Here we can use the change of base formula:

$$\log_a b = \frac{\log_c b}{\log_c a}$$

So $\log_3 19 = \frac{\log_{10} 19}{\log_{10} 3}$, for which we can use a calculator and get $\log_3 19 \approx 2.68$, so $x \approx 2.68$.

Check this answer by raising 3 to the power 2.68 on your calculator.

A second example: Solve for t: $2 = 5.4^t$. (This type of equation is very useful in many disciplines.)

We need to find the value of the power, t. We rearrange the equation into its logarithm form: $\log_{5.4} 2 = t$. Using the change of base, this gives us:

$$\log_{5.4} 2 = \frac{\log_{10} 2}{\log_{10} 5.4}$$

and working this out on the calculator we get $\log_{5.4} 2 \approx 0.411$, so $t \approx 0.411$. You can check this on the calculator by finding $5.4^{0.411}$.

Let's apply this technique to rearranging a formula to isolate a power.

Solve $S = P(1+i)^n$, for n.

First we need to divide both sides by P:

$$\frac{S}{P} = \frac{P(1+i)^n}{P},$$

that is,

$$\frac{S}{P} = (1+i)^n.$$

Next, put the equation into its logarithm form:

$$\log_{(1+i)} \left(\frac{S}{P}\right) = n,$$

where $(1 + i)$ is the base. To calculate the value of n, we would use the change of base as above, so

$$n = \frac{\log_{10}\left(\frac{S}{P}\right)}{\log_{10}(1+i)}$$

EXERCISE

Use this new formula to calculate the value of n, given $S = 1338.22$, $P = 1000$, and $i = 0.06$.

Answer: 5

METHOD 2

Example 1

Solve $3^x = 19$

Rather than putting the equation into its logarithmic form, we can simply take logs of both sides. (It is up to you to choose whether you take logs to base 10 (log) or logs using base e (ln), as you will obtain the same result.)

$$3^x = 19$$

So taking the log of both sides gives:

$$\log 3^x = \log 19$$

Now we need to use the logarithm law that says logs turn powers to multiplication:

$$\log_a b^c = c \log_a b$$

Applying this law gives:

$$x \log 3 = \log 19$$

Divide both sides by $\log 3$ (which is just a number, after all):

$$x = \frac{\log 19}{\log 3}$$

Use your calculator to find this value: $x \approx 2.68$ (correct to 2 decimal places), (the same as in Method 1).

Notice that you would get the same answer if you used ln instead of log – try it!

Example 2

Solve for t: $2 = 5.4^t$.

Take logs of both sides. (We will use ln this time.)

$$\ln 2 = \ln 5.4^t$$

Use the log law to bring the power out the front:

$$\ln 2 = t \ln 5.4$$

Divide both sides of the equation by $\ln 5.4$:

$$\frac{\ln 2}{\ln 5.4} = t$$

Use your calculator to evaluate, and $t \approx 0.411$, as in Method 1.

Example 3

Solve $S = P(1+i)^n$, for n.

Before taking logs of both sides, we need to get rid of the P from the right hand side of the equation, so divide both sides by P.

$$\frac{S}{P} = (1+i)^n$$

Now take logs of both sides:

$$\ln\left(\frac{S}{P}\right) = \ln(1+i)^n$$

Use the log law to bring the power out the front:

$$\ln\left(\frac{S}{P}\right) = n\ln(1+i)$$

Divide both sides by $\ln(1+i)$:

$$\frac{\ln\left(\frac{S}{P}\right)}{\ln(1+i)} = n$$

That is,

$$n = \frac{\ln\left(\frac{S}{P}\right)}{\ln(1+i)}$$

This will give the same value as using log (to base 10) as we did in Method 1. Try the example using this method.

If you need help with any of the maths covered in this resource (or any other maths topics), you can make an appointment with Learning Development through reception: phone (02) 4221 3977, or Level 2 (top floor), Building 11, or through your campus.