Algebra

4. Rearranging Formulae

INTRODUCTION

Before you read this resource, you need to be familiar with how to solve equations. If you are not sure of the techniques involved in that topic, please refer to *Algebra – 3. Solving Equations.*

To give you an idea of what we mean by "rearranging formulae", think about going on a trip. You can work out any of the following if you know the other two: distance, speed, and time. We know that speed is measured in km/h, in other words you divide the number of kilometres by the number of hours to find speed. We can use the formula $v = \frac{d}{t}$, where v represents speed, d distance and t time. But how do we calculate d or t when we only know the other variables? If you can do it with numbers, you can do it with letters (*variables*). More about that later!

Rearranging a formula so that a different variable is in front of the "=" is the same as solving an equation for that variable. (We also call rearranging a formula "changing the subject of a formula".) You need to work out what has been done to the variable you want to isolate, then undo the procedure in the reverse order.

Here is a "normal, straightforward" equation to solve: $3x + 5 = 20$.

Let's think about what has been done to x. First it has been multiplied by 3, then 5 has been added, to get the result 20. Using the techniques of reversing the procedure, we would firstly subtract 5 from both sides to get:

$$3x + 5 - 5 = 20 - 5$$
$$3x = 15$$

and now divide both sides by 3 (to undo 3):

$$\frac{3x}{3} = \frac{15}{3}$$

So $\underline{x = 5}$.

Check the solution:

$$\begin{aligned} LHS &= 3x + 5 \\ &= 3 \times 5 + 5 \\ &= 20 = RHS \end{aligned}$$

So $x = 5$ is the correct solution.

Now, rather than use numbers in that equation, let's have some variables! Using the same techniques, we can solve the equation $3x + y = 20$, for x. The only difference is that we cannot find an actual numerical value when we subtract y from both sides:

$$3x + y - y = 20 - y$$

So we get:

$$3x = 20 - y$$

Now, divide both sides by 3:

$$\frac{3x}{3} = \frac{20 - y}{3}$$

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That is, $x = \frac{20-y}{3}$

We use exactly the same techniques in rearranging formulas as we do when solving equations. In other words, we apply inverse operations to undo what has been done to the variable we want to isolate.

This time, we will solve the same equation for x, but this time the equation contains only letters instead of numbers (*pronumerals*).

Solve for x: $ax + y = b$

What has been done to x? Firstly it has been multiplied by a, then y has been added, to get the result b. We can undo the process using the inverse operations:

First, subtract y from both sides:

$$ax + y - y = b - y$$

and get

$$ax = b - y.$$

Divide both sides by a:

$$\frac{ax}{a} = \frac{b-y}{a}$$

So, our solution is: $x = \frac{b-y}{a}$

EXAMPLES

Temperature Conversion

Solve for C: $F = \frac{9}{5}C + 32$.

(This is the formula that converts between temperature measurements in Celsius (°C) to Fahrenheit (°F).)

What has been done to C? We can think of this in two ways – we can say it has been multiplied by $\frac{9}{5}$, or that it has been multiplied by 9 and divided by 5. We will use the second way as it might be easier at this stage. After that, 32 has been added to the result to obtain F. Now let's undo in the reverse order: Subtract 32 from both sides:

$$F - 32 = \frac{9}{5}C + 32 - 32$$

So,

$$F - 32 = \frac{9}{5}C$$

Now, multiply both sides by 5. Notice that the <u>whole</u> of the LHS must be multiplied, so

$$(F - 32) \times 5 = \frac{9}{5}C \times 5$$

We can write the LHS in the other order, and so we get

$$5(F - 32) = 9C$$

Now divide both sides by 9

$$\frac{5(F-32)}{9} = \frac{9C}{9}$$

So, $\frac{5(F-32)}{9} = C$, which is simply the back-to-front way of saying

$$C = \frac{5(F - 32)}{9}.$$

We can also write this as $C = \frac{5}{9}(F - 32)$.

You can see if this works. We know that the boiling point of water, measured in °F is 212. Do you know what the boiling point measured in °C is? We can use our new formula to find out:

$$\begin{aligned} C &= \frac{5(F - 32)}{9} \\ &= \frac{5(212 - 32)}{9} \\ &= \frac{5 \times 180}{9} \\ &= 100. \end{aligned}$$

So the boiling point of water, measured in Celsius degrees is 100 °C. (This is correct!)

Speed

Now, back to our speed, distance and time formula: $v = \frac{d}{t}$. Use the techniques of inverse operations ("undoing") to solve the equation for d.

What has been done to d? It has only been divided by t, so that is all we need to undo. Multiply by t and we are finished!

$$v \times t = \frac{d}{\cancel{t}} \times \cancel{t}$$

So, $v \times t = d$, which is the same as $d = v \times t$.

(Check if it makes sense by thinking about a trip – say you travel for 2 hours at a constant speed of 100 km/h, you would have gone 200 km.)

Now we will solve $v = \frac{d}{t}$ for t (maybe a little harder?) What has been done to t? Unfortunately t has been the operator, this time, not the variable that has been operated on! To solve, we need to multiply by t, and then we can see how to get it.

$$\begin{aligned} v \times t &= \frac{d}{\cancel{t}} \times \cancel{t} \\ v \times t &= d \end{aligned}$$

Now we just need to undo the multiplying by v and we will have solved for t. Divide both sides by v and get:

$$\frac{v \times t}{v} = \frac{d}{v}$$

so $t = \frac{d}{v}$.

Circle Area

Let's try to solve $A = \pi r^2$, for r. (This formula finds the area, A, of a circle when we know the radius of the circle, represented by r). What has been done to r? First it has been squared, then it has been multiplied by π. (You may think, because π is written first, that the multiplication has been done first, but you must remember the order of operations here – squaring then multiplying.) To undo, first divide by π:

$$\frac{A}{\pi} = \frac{\cancel{\pi} r^2}{\cancel{\pi}}$$

So,

$$\frac{A}{\pi} = r^2$$

Undo the square by taking the square root of both sides, so

$\sqrt{\frac{A}{\pi}} = r$, which means that $r = \sqrt{\frac{A}{\pi}}$.

Pythagoras

Another well-known formula is for Pythagoras' theorem, which finds the length of the longest side of a right-angled triangle. But what if we know the longest side and need to find the length of one of the other sides?

Solve $a^2 + b^2 = c^2$ for b

What has been done to b? It has been squared, then a^2 has been added to it. So, reverse the procedure:

Subtract a^2 from both sides:

$$a^2 + b^2 - a^2 = c^2 - a^2$$

So

$$b^2 = c^2 - a^2$$

Take the square root of both sides:

$$b = \sqrt{c^2 - a^2}$$

Business Formulae

Here are two formulas used in business. Both use powers other than squaring. If you need to know how to rearrange a formula to isolate the power, please see *Algebra – 5. Rearranging Formulae to Isolate Powers.*

In the formula $S_n = P(1+i)^n$, the first n is simply an identifier, or a counter, so S_1 represents the first value of S, S_2 represents the second value of S, and so on.

First let's rearrange this formula (solve) for P. What has been done to P? It has only been multiplied by $(1+i)^n$, so we just need to divide by $(1+i)^n$.

$$\frac{S_n}{(1+i)^n} = \frac{P\cancel{(1+i)^n}}{\cancel{(1+i)^n}}$$

$\frac{S_n}{(1+i)^n} = P$, which is the same as $P = \frac{S_n}{(1+i)^n}$.

The second formula is

$$S_n = \frac{R[(1+i)^n - 1]}{i}$$

Solve this one for R. Undo the multiplication and division – it doesn't matter the order, so let's undo the division first by multiplying by i.

$$S_n \times i = \frac{R[(1+i)^n - 1]}{\cancel{i}} \times \cancel{i}$$
$$iS_n = R[(1+i)^n - 1]$$

Divide by $[(1+i)^n - 1]$

$$\frac{iS_n}{[(1+i)^n - 1]} = \frac{R[(1+i)^n - 1]}{[(1+i)^n - 1]}$$

$\frac{iS_n}{[(1+i)^n-1]} = R$, which is the same as $R = \frac{iS_n}{[(1+i)^n-1]}$

If you need help with any of the maths covered in this resource (or any other maths topics), you can make an appointment with Learning Development through reception: phone (02) 4221 3977, or Level 2 (top floor), Building 11, or through your campus.