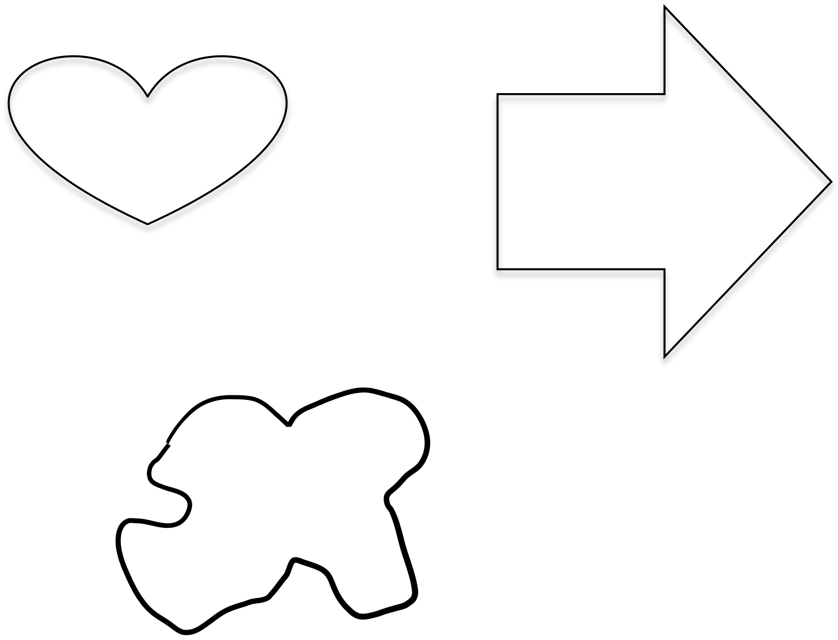
Area and Perimeter

1. Areas of Plane Shapes

INTRODUCTION

A plane figure or shape is a two-dimensional, flat shape. Here are 3 plane shapes:



All of them have two dimensions that we usually call length and width (or sometimes height). Plane shapes do not have thickness, which means we can draw them on paper. The amount of space inside each shape – or the amount of space each figure occupies – is called the *area* of that shape.

UNITS TO MEASURE AREA

When you hear the word *area*, you may think of school maths when you had to find the area of different shapes, mostly rectangles and squares. But we can actually find the area of ANY shape, even the strange one above. To do this we firstly need to think about the basic unit of area.

Try to think of a small shape that, if it was repeated over and over again, perhaps turned upside down or back-to-front, would cover each of the shapes above. Using a shape like this is called *tessellating* – and examples can be found in patchwork, tiling, mosaics and mosques. Designs range from simple to ornate and can be very attractive. There are some examples over the page.

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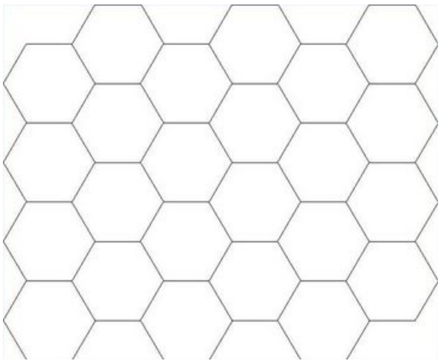
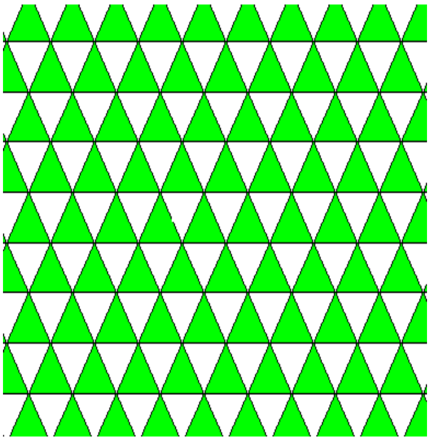


Retrieved January 22, 2013 from http://www.ehow.com/facts_5034966_definition-hexagon-tessellations.html and http://www.ehow.com/how_8562307_design-tessellations.html

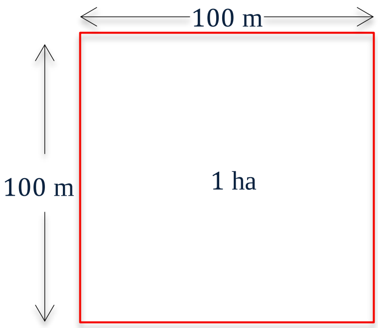


Retrieved January 22, 2013 from http://www.orchidpalms.com/polyhedra/tessellations/infinite.htm and http://britton.disted.camosun.bc.ca/escher/jbescher.htm

But what if we need to compare the areas of our shapes on the first page? Say we want to cover them all perhaps – or, even worse – these are the shapes of your late great aunt's farm paddocks and she has left them to the family – who is going to get the biggest share?!

The small shape that has been accepted throughout the world for tessellating to determine areas is the square. In the *metric system* of measurement, the unit of length is the *metre* and so for area, the unit is the *square metre* – written as m^2. Smaller measures are given in *square centimetres* (cm^2) or – even smaller – *square millimetres* (mm^2) and large measures are given in *hectares (ha).*

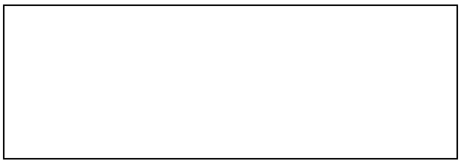
Note that *ha* doesn't have the superscript "^{2}" as it is already a square unit – 1 ha measures an area 100 m by 100 m, that is, 1 ha = 10 000 m^2.



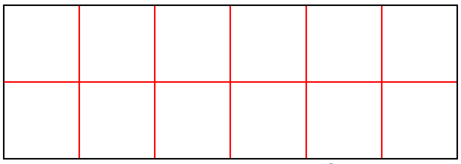
To be able to work out the areas of the shapes above, we could draw a grid of squares on each shape and count them. (We would need to approximate for the irregular shapes.)

THE AREA OF ANY RECTANGLE

With “regular” shapes, the area is more straightforward.



To find the area of this rectangle, we can cover it with a grid of squares with length 1 cm and width 1 cm, that is they are each square centimetres (1 cm^2), and count them.



In this rectangle, there are 12 square centimetres, so its area is 12 cm^2.

Is there a faster way to find the area of this rectangle? Yes! We know that the rectangle is 6 cm long and 2 cm wide, so we can fit two rows each of 6 centimetre squares in the complete rectangle. So, its area is 12 cm^2. We can also calculate this as $6\text{ cm} \times 2\text{ cm} = 12\text{ cm}^2$.

We can generalise this with a formula. If we let the length of the rectangle be l and the width of the rectangle be w, then to calculate the area (A) we multiply the length by the width and so we have:

$$A = l \times w$$

We can apply this formula to <u>any</u> rectangle as long as we know its dimensions (the size of its length and width).

You may have seen this formula before, using b (represents *breadth*) instead of w – width and breadth are the same – and in this case, the area would be $A = l \times b$. Both formulas give the same result.

To find the area of the rectangle given above, we know that $l = 6$ cm and $w = 2$ cm, so $A = 6\text{ cm} \times 2\text{ cm} = 12\text{ cm}^2$.

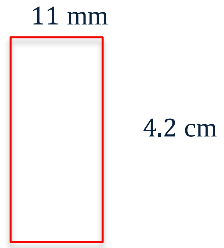
To find the area of any rectangle, the dimensions must be measured using the same unit such as centimetres, kilometres and so on. We cannot find the area of a rectangle where the length is given in kilometres and the width is in metres, for example, we must convert one of those measures to the other.

The following diagrams are not drawn to scale. See if you can find the area of each rectangle. You can check your results with the solutions at the end of this resource.

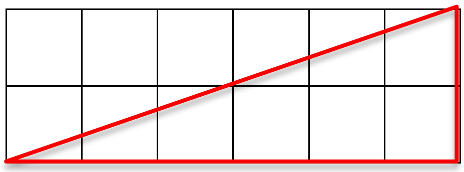
EXERCISES

1.

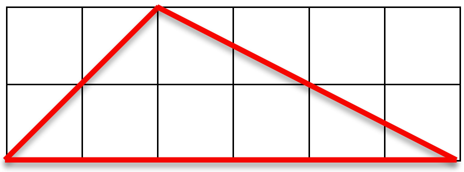
2.



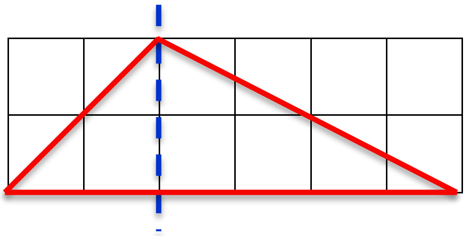
What about triangles?



Can you see that the area of one of the triangles formed is half the area of the whole rectangle? So we can say its area is 6 cm^2 straight away. But what about this one?

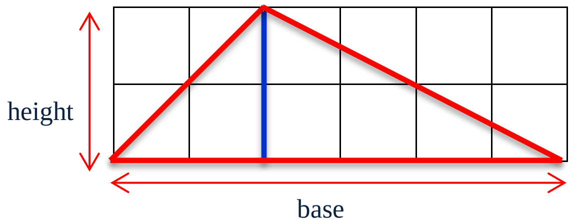


We could do this:



Now we have 2 triangles that are both half their respective rectangles in area. So the area of the complete triangle is 2 cm^2 (for the one on the left) + 4 cm^2 (for the one on the right) which makes 6 cm^2.

Here is the formula for the area of a triangle – you might remember it from school. With a triangle, we'll refer to the length (of the rectangle) as the *base* of the triangle and the width (of the rectangle) as the *height* of the triangle.



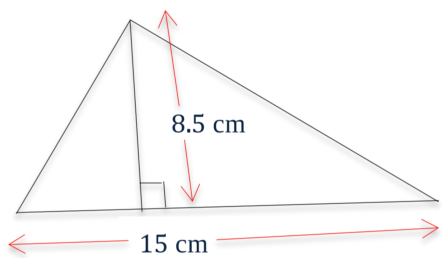
The area of a triangle is given by half the area of a rectangle, so the area is given by

$$A = \frac{1}{2} \times b \times h,$$

where A represents the area of the triangle, b represents the length of the base of the triangle and h represents the height of the triangle. The height of the triangle must be at right angles to the base of the triangle and the units of measure must be the same.

Example:

Find the area of this triangle.



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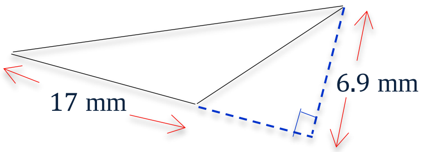
The area is $\frac{1}{2} \times 15 \text{ cm} \times 8.5 \text{ cm} = 63.75 \text{ cm}^2$.

Find the area of this triangle:



Notice that in this triangle, the base has to be extended so that we can draw a height at right angles to it. The area can still be calculated in the same way, though.

So,

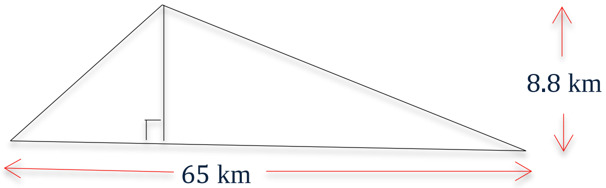
$$\begin{aligned}\text{Area} &= \frac{1}{2} \times 17 \text{ mm} \times 6.9 \text{ mm} \\ &= 58.65 \text{ mm}^2\end{aligned}$$

Here are some for you to try. You can check your results with the solutions at the end.

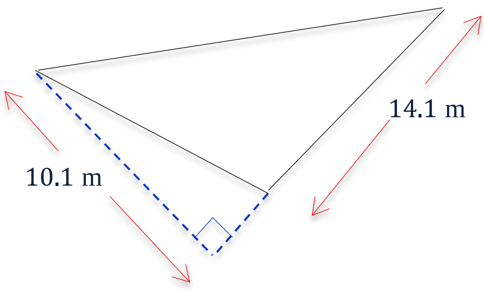
EXERCISES

Calculate the areas of these triangles (the diagrams are not drawn to scale):

3.



4.



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What about circles?

If we know the size of the *radius* of a circle, we can work out the area.

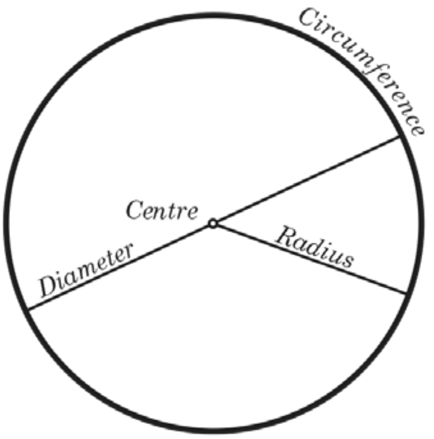


Diagram retrieved 22 January 2013 from http://en.wikipedia.org/wiki/Circle

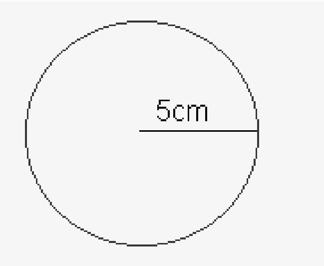
The *radius* is any line joining the *centre* of the circle to the circle itself; it is half of a *diameter*, which is any line connecting two points on the circle that goes through the centre. If we let r be the length of the radius, then the area (given by A) is:

$$A = \pi r^2$$

π is a Greek letter (pronounced "pie" in Australia) and is a constant. It is the answer when you divide the *circumference* of a circle, which is the distance around a circle, by the length of the diameter of that circle. It is equal to approximately 3.141592654 and can be calculated correctly to many decimal places. If you use a calculator when you work with π, your answer will always be accurate and you will need to round it as it will contain many decimal places.

When you want to calculate using $A = \pi r^2$, you type π in your calculator (often you need to use the [Shift] key) followed by [$\times$], then the radius followed by [x^2], and [$=$].

Example: Calculate the area of this circle, correct to 3 decimal places.



$$Area = \pi r^2$$

In this circle, the radius is 5 cm long, so $r = 5$ cm, and the area is

$$\begin{aligned} A &= \pi \times 5^2 \text{ cm}^2 \\ &= 78.53981634 \ldots \text{ cm}^2 \\ &\approx 78.540 \text{ cm}^2 \end{aligned}$$

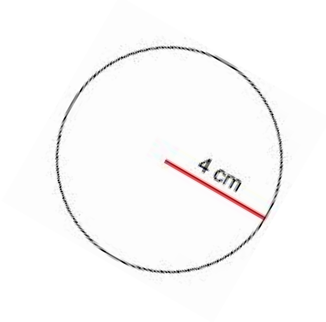
Where we rounded to 3 decimal places

Here are two for you to try. Remember, if you are given a diameter instead of a radius, you need to calculate half of it (to work out the radius) before calculating the area. (The diagrams are not to scale.)

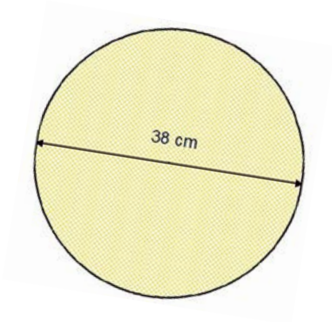
EXERCISES

Calculate the area of each of the following circles:

5.



6.



CONVERTING BETWEEN SQUARE UNITS

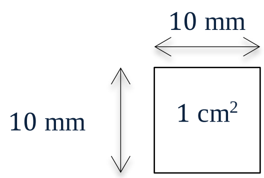
Consider the following rectangle:

This rectangle was exercise 2 in this resource, where you were required to find its area. In the solutions, the side length given in centimetres is changed to millimetres and the area is calculated in mm^2. The area is 462 mm^2. Now we are going to calculate the area using cm^2 and compare our answers.

First, change 11 mm to cm, by dividing by 10, so $11 \text{ mm} = (11 \div 10) \text{ cm} = 1.1 \text{ cm}$. Therefore:

$$\begin{aligned} \text{Area} &= 4.2 \text{ cm} \times 1.1 \text{ cm} \\ &= 4.62 \text{ cm}^2 \end{aligned}$$

Is this what you expected? The area in mm^2 is **100** times as big as the area in cm^2. This is because we are no longer dealing with one dimension (length) where 1 cm = 10 mm, but with **2** dimensions (length and width):



So, $1 \text{ cm}^2 = 100 \text{ mm}^2$

In the same way,

$$\begin{aligned} 1 \text{ m}^2 &= 100 \text{ cm} \times 100 \text{ cm} \\ &= 10\,000 \text{ cm}^2 \end{aligned}$$

and $1 \text{ km}^2 = 1\,000 \text{ m} \times 1\,000 \text{ m} = 1\,000\,000 \text{ m}^2$

Here is an example: The measured space for a small window is given in cm^2, but the builder needs the measurement in mm^2. The measurement was 3 600 cm^2. What measurement should we tell the builder?

1 cm^2 = 100 mm^2, so

$$\begin{aligned} 3\,600 \text{ cm}^2 &= 3\,600 \times 100 \text{ mm}^2 \\ &= 360\,000 \text{ mm}^2 \end{aligned}$$

Try one yourself.

EXERCISE

7. We have measured a block of land in m^2 but would like to know how many hectares this is. The measurement is 42 500m^2.

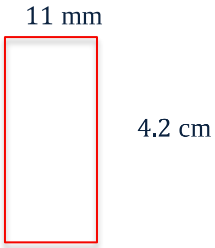
If you need help with any of the maths covered in this resource (or any other maths topics), you can make an appointment with Learning Development through reception: phone (02) 4221 3977, or Level 2 (top floor), Building 11, or through your campus.

SOLUTIONS TO EXERCISES

1.

$$\begin{aligned}\text{Area} &= 3.7\text{ cm} \times 1.5\text{ cm}\\ &= 5.55\text{ cm}^2\end{aligned}$$

2.



To deal with this rectangle, we must make sure both measurements are in the same unit. We will make them both mm, so:

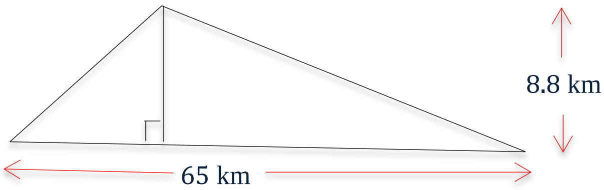
$$4.2\text{ cm} = (4.2 \times 10)\text{ mm} = 42\text{ mm}$$

Now the area is found using the formula.

$$\text{Area} = 42\text{ mm} \times 11\text{ mm} = 462\text{ mm}^2$$

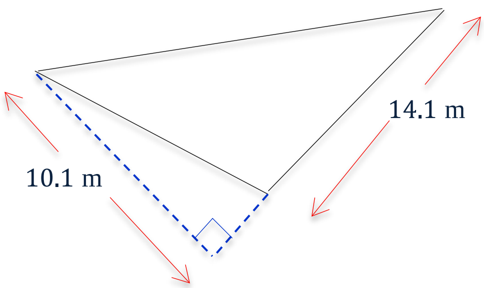
(If you calculated the area in cm^2, see the section on converting units.)

3.



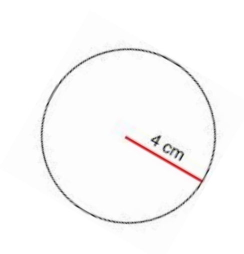
$$\text{Area} = \frac{1}{2} \times 65\text{ km} \times 8.8\text{ km} = 286\text{ km}^2$$

4.



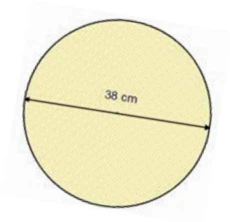
$$\text{Area} = \frac{1}{2} \times 14.1\text{ m} \times 10.1\text{ m} = 71.205\text{ m}^2$$

5.



$$\begin{aligned}\text{Area} &= \pi \times 4^2\text{ cm}^2 = 50.26548246\ldots\text{ cm}^2\\ &\approx 50.265\text{ cm}^2\end{aligned}$$

6.



Firstly, we are given the diameter instead of the radius, so we must find half of 38 cm to get the length of the radius. $(38 \div 2 = 19)$.

So the area is

$$\begin{aligned}\text{Area} &= \pi \times 19^2 \text{ cm}^2 \\ &= 1\,134.114948\ldots \text{ cm}^2 \\ &\approx 1\,134.115 \text{ cm}^2\end{aligned}$$

7. We know that 1 ha = 10 000 m^2. The measurement is 42 500 m^2. So how many lots of 10 000 m^2 are in 42 500 m^2? We need to divide 42 500 by 10 000 to find the number of ha, and obtain 4.25 ha.