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Optimal Control of Broadcasting Spectrum with Variety-Reception Trade-off and Consumers' Income Sensitivity

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Abstract

This paper uses optimal control theory to derive a desirable trajectory of the number of royalties-paying users of state-owned spectrum for broadcasting. The spectrum royalties are set by the public planner to maximize the consumers' utilities over an infinite planning horizon. The consumers' utilities are generated from the quality of the service of the broadcasting industry, from the consumption of other goods, and from the public services financed by the spectrum royalties. The number of broadcasters adjusts to above-normal profits. The quality of the broadcasting industry's service is determined by variety and reception. The trade-off between the benefits from higher variety and royalties' revenues and the costs of intensified interferences associated with the number of broadcasters is considered. The positive information-dissemination effect and the negative effort-diversion effect of the quality of the broadcasting industry's service on the consumers' aggregate income are also considered. The possibility of convergence of the derived trajectory of the number of broadcasters to steady state and the comparative statics of the steady state are analyzed.

Keywords: Over-The-Air Broadcasts; Variety; Interferences; Spectrum Royalties; Optimal Control

JEL Classification : C61, C62, D61, K23, L52

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1. Introduction

Satisfaction from over-the-air (OTA) broadcasts increases with contents' variety and reception's clarity. An expansion of the number of OTA broadcasters increases programs' variety, but the increased spectral congestion intensifies interferences. The variety-congestion trade-off is the focal theme of our proposed optimal control of the broadcasting spectrum. Our focus on this trade-off is motivated by a brief background description of the OTA broadcasting industries in technologically advanced countries.

Due to the public good nature of OTA broadcasts and their educational, cultural and political impacts and because of scarce bandwidth and high sunk costs, broadcasts and entry rules have been tightly regulated in all major OECD countries (cf., Webbink 1973). Until the late 1970s the television broadcasting industries in OECD countries comprised only a handful of licensed and highly protected public and commercial firms. Since 1980 alternative transmission techniques, such as satellite and cable, have created a more favorable environment for entry into the television broadcasting industry and localization of broadcasters and broadcasts. Yet the focus of the veteran incumbent OTA broadcasters on nation-wide audience and market shares have remained very high (cf., Motta and Polo, 1997; Caves, 2006). The radio broadcasting industries have been less concentrated and more localized, but entry to these industries has also remained highly regulated due to sunk costs and tight spectrum constraint. In many European metropolitan areas, OTA radio broadcasts are provided by twenty to eighty FM stations and by similar numbers of AM stations. Metropolitan areas in Italy offer the largest FM variety. With only fifty kHz separation between stations, their FM broadcasting spectrum is the most congested in Europe.

The intensified trade-off between variety and reception has been recently stressed by the Electronic Communication Committee within the European Conference of Postal and Telecommunications Administrations (CEPT, 2010).[1] The recent adoption of digital transmission technologies has expanded the scope for program variety in both the television and radio broadcasting industries. Digital technology is spectrally efficient – it can increase the number of television channels six-fold (cf., Adda and

[1] "The available [FM] spectrum (20.5 MHz) constitutes a limited resource that is used intensively in Europe. In many countries the introduction of new FM services is difficult and may lead to an unacceptable degradation of existing services." (CEPT, 2010, Section 1, P. 5) … "The FM spectrum is in many areas overcrowded and may be reaching saturation if the high quality of reception and existing coverages must be retained. This results in FM services increasingly being interference-limited by design or otherwise and these higher interference levels may have to be accepted to allow the introduction of many more additional services". (CEPT, 2010, Section 2, P. 5)

Ottaviani, 2005). The adoption of digital technology by American OTA broadcasters in 2007 and the subsequent turning off of analogue signals in 2008 have freed a significant UHF space. However, the spectral gains have not relaxed the US television broadcasting industry's spectrum constraint. The spectral dividend from the digital switchover was mainly auctioned off to large telecommunications carriers for accommodating the deployment of 4G mobile-phone networks. Similar diversion of the digital switchover's spectral dividend is expected in other OECD countries. The situation is more complex for radio broadcasting as only a few countries have successfully adopted and rolled out digital platforms for radio transmissions, and even fewer have clear digital switchover plans for analogue radio broadcasting. Still, buffer zones between broadcasters' bands have to be reduced in order to accommodate new entrants to the OTA broadcasting industry.

Despite the digital switchover, expansion of the OTA broadcasting industry can be expected to intensify the trade-off between variety and reception in the pursuit of the overall quality of the industry's service. The variety-reception trade-off is likely to be most prominent under a deregulatory scheme that allows free entry and exit. In view of the recent broadcasting spectrum deregulatory trends (cf., De Vany, 1998; Hazlett, 2008) and the variety-reception trade-off, our theoretical analysis explores the possibility of convergence of the number of ATO broadcasters to steady state and the comparative statics of the steady state when entry and exit are motivated by above-normal profits and moderated by sunk costs. We treat the broadcasting spectrum as a state-owned, time-invariant, scarce natural resource. As in the case of any other state-owned natural resource, governments are entitled to royalties on the use of spectrum. Hence, in addition to the direct benefits from the service provided by the broadcasting industry, there are indirect benefits – the public services financed by the states' royalties on this natural resource.

We construct a conceptual framework where the state's royalties are allowed to vary over time so as to maximize the sum of the discounted direct and indirect benefits stemming from the use of the broadcasting spectrum. The number of broadcasters is allowed to adjust to the above-normal profit from broadcasting at a rate moderated by sunk costs. The broadcasting industry's above-normal profit is assumed to rise with the quality of the broadcasting industry's service. On the one hand, entry increases variety, heightens competition and, in turn, raises the quality of the industry’s service. On the other hand, entry increases spectral congestion and the intensified interferences lower the quality of the industry’s service.

The possible sensitivity of the consumers’ incomes to the quality of the industry’s service is another theme in our analysis. In constructing the optimal control model, we take into account two opposing effects of the quality of the service on consumers’ incomes. On the one hand, the information disseminated by broadcasts enhances knowledge, forms standards of performance, and generates transactions. On the other hand, broadcasts divert time from work and other modes of investment in human and social capitals. The higher the overall quality (variety and reception) of the industry’s broadcasts is, the stronger these opposing effects. The net effect of quality on the consumers’ aggregate income is not clear.

We identify the conditions that facilitate a stable path along which the number of broadcasters gradually converges to the steady state of a royalties-based optimally controlled industry and present its comparative statics. In contrast to the observed consolidation and return to concentration in the aftermath of deregulatory reforms in the United States, Italy, Germany and Japan (cf., Noam, 1992; Motta and Polo, 1997; Hazlet, 2005), our analysis of royalties-based optimal control of the OTA broadcasting industry reveals a possible convergence to a steady state with a larger number of broadcasters.

Our optimal control analysis of the OTA broadcasting industry is structured as follows. Section 2 presents the dynamics of the number of broadcasters. Section 3 argues that the broadcasting industry have a multifaceted effect on consumers’ utility: a quality enhanced positive direct effect of the industry service, a negative indirect effect of the industry service through diverting budget from other goods, an unclear indirect effect stemming from the aforementioned opposing effects of the quality of the broadcasting industry’s service on the consumers’ aggregate income and budget, and a positive effect of the spectrum royalties paid by the industry on the provision of public goods. These utility aspects and the OTA broadcasting industry’s dynamics described in section 2 are incorporated into the construction of the public planner’s optimal control problem in section 4. Section 5 analyzes the possibility of convergence of the industry’s trajectory to steady state. Section 6 presents the comparative statics of the steady-state number of OTA broadcasters.

2. Industry Dynamics

Let n(t) denote the number of suppliers (broadcasters) of OTA transmitted programs (broadcasts) at time t. At every instance t each broadcaster uses a single channel and delivers a single program. Let the broadcasters be technologically and location-wise identical and paying royalties, r(t), to the government for using a channel (spectrum band) at t. Also let the width of each band be technologically determined and fixed, ω, and the bands evenly spread along a fixed homogeneous spectrum space set aside for the broadcasting industry, $\hat{S}$. In which case, the bands shift, and the buffer zones between them is evenly reduced, as the number of broadcasters increases. For tractability, let us further assume that the consumers of the OTA broadcasting industry's service are located at an identical, physically unobstructed, distance from the broadcasters; i.e., along a flat circle with the broadcasters at its centre. Then all the broadcasts are equally receivable by all the consumers. We assume that the consumers of the broadcasting industry's service are also users of broadcast time. In addition to watching and/or listening to programs, they advertise their services during the programs. The consideration of consuming-using agents simplifies the analysis which, more generally, could have considered the demands of three types of agents: consumers only, users only and consumer-users.

Broadcasters enter (exit) the industry as long as the above-normal profit (ANP) per broadcaster from broadcasting is positive (negative). That is,

$$\dot{n}(t) = \phi ANP(t), \qquad (1)$$

where $\phi > 0$ reflects the speed of adjustment (ease of entry and exit). Sunk costs (i.e., the fixed costs associated with facilities, equipment and knowledge that are not transferable to other industries) deter entry and exit. The larger the broadcasters' sunk costs are, the lower the speed of adjustment of the number of broadcasters to the above-normal profit from broadcasting. For simplicity, the sunk costs are assumed to be time-invariant and, consequently, ϕ is taken to be a scalar. With $\hat{S}$ denoting the fixed amount of spectrum available to the broadcasting industry, $n(t) \in (0, \hat{S}/\omega)$.

From the perspective of the consumers, the overall quality of the aerially transmitted programs, Q(t), rises with variety and reception. While the variety of programs is increasing with the number of channels, interferences intensify as the buffer zones between the channels diminish. As the buffer zones are evenly reduced, reception is positively related to the size of the unused spectrum (S), which is:

$$S(t) = \hat{S} - \omega n(t)\,. \tag{2}$$

Consequently, the quality of the aerially transmitted broadcasts is taken to be given by an implicit form

$$Q(t) = q(n(t), S(t)) = q(n(t), (\hat{S} - \omega n(t))) \tag{3}$$

that has the following properties:

i. $q(0, \hat{S}) = 0$;

ii. the direct, variety, effect of the number of channels on quality, is positive but not increasing, $q_n > 0$ and $q_{nn} \leq 0$;

iii. the indirect effect of the number of channels on quality (through deteriorating reception) is negative, as $q_s > 0$ and (for simplicity) unchanged ($q_{ss} = 0$), and is equal to $-\omega q_s$; and

iv. up to a critical number of channels, $\tilde{n}$ ($0 < \tilde{n} < \hat{S}/\omega$), the positive variety effect dominates the negative interference effect: $Q_n = q_n - \omega q_s \overset{>}{\underset{<}{=}} 0$ for $n \overset{<}{\underset{>}{=}} \tilde{n}$.

The overall demand for broadcasts increases with quality. Consequently, the broadcasting industry's aggregate revenue from advertisements and subscription fees (in the case of pay TV) is, at any t, $R(Q(t))$ with $R(0) = 0$, $R_Q > 0$ and, for tractability, $R_{QQ} = 0$. Assuming that consumers do not have favourite channels, the industry's aggregate revenue is equally distributed.

We consider some computationally convenient specifications of q and R. In particular, we specify the overall quality of the service of the OTA broadcasting industry for the consumers to be given by:

$$Q(t) = n(t)[\hat{S} - \omega n(t)] \tag{4}$$

where, as indicated by equation (2), $\hat{S} - \omega n$ is the size of the unused spectrum set aside for broadcasting. This parabolic specification of Q reflects properties that can be resulted from the opposite effects of n on variety and reception: $Q_n = \hat{S} - 2\omega n \overset{>}{\underset{<}{=}} 0$ as $n \overset{<}{\underset{>}{=}} \hat{S}/2\omega$ and $Q_{nn} = -2\omega < 0$. With this convenient specification $\tilde{n} = 0.5(\hat{S}/\omega)$. That is, the marginal effect of the number of OTA broadcasters on the overall quality of the service of the OTA broadcasting industry is initially positive, but when more than half the spectrum's carrying capacity is used the negative congestion effect dominates the positive variety effect on the quality of the industry's service.

We assume that the marginal revenue (i.e., increment in revenues from subscription fees and commercial advertisements) from quality of service is constant (p) and, for simplicity, also time-invariant. That is, p can be interpreted as the price of broadcasts' quality for consumers. In which case, the OTA broadcasting industry revenue function is:

$$R(t) = pQ(t)\,. \tag{5}$$

By substituting the right-hand side of Eq. (4) for Q into this specification of R, the industry's revenue function is:

$$R(t) = p\hat{S}n(t) - p\omega n(t)^2\,. \tag{6}$$

For simplicity, the instantaneous operational cost (c) of each channel and the (foregone) normal profit per agent attainable in other industries (π) are taken to be time-invariant. Consequently, the industry's above-normal profit per broadcaster is

$$ANP(t) = R(t)/n(t) - (c + \pi + r(t)) \tag{7}$$

and, in recalling (1) and (6), the change in the number of broadcasters at t is

$$\dot{n}(t) = \phi[(p\hat{S} - p\omega n(t)) - (c + \pi + r(t))]\,. \tag{8}$$

This motion equation suggests that a one and for all increase of the royalties charged on bands used by an initially stable industry ($\dot{n}$=0) reduces the above-normal profit and, subsequently, the number of broadcasters. In turn, the variety of programs is reduced, but the reception of each broadcast is improved. If the former (latter) effect dominates the latter (former), the industry's overall revenue decreases (increases), the number of broadcasters decreases (increases), and so on and so forth until the above normal profit is completely diminished. The following section considers the consumers' utilities and let the royalties charged on bands be chosen by the public planner so as to maximise the sum of the discounted consumers' utilities over an infinite planning horizon, subject to the number of broadcasters' motion equation (8).

3. Consumers' utility

The consumers are taken to have an aggregate income, Y(t), of which R(Q(t)) is spent on access to, and advertisements in, the OTA broadcasts and the remainder, Y(t)-R(Q(t)), on private goods. Broadcasts have two opposing effects on aggregate income. On the one hand, they disseminate information that enhances knowledge, forms standards of performance and generates transactions. On the other hand, they divert time from work and active investment in human and social capitals. These opposing effects are intensified by the quality of the broadcasts. With $\varphi_1 > 0$ indicating the information dissemination effect, $\varphi_2 > 0$ the production-effort diversion effect of broadcasts, and $\hat{Y} > 0$ the aggregate income attainable when the said effects offset one another (or nil), the consumers' aggregate income is:

$$Y(t) = \hat{Y} + (\varphi_1 - \varphi_2)Q(t) . \tag{9}$$

With (9) and the explicit specifications (4) and (5) of the broadcasting industry's quality of service and revenue, and with $\alpha > 0$ indicating the consumers' direct marginal instantaneous utility from the quality of the broadcasts and $\beta > 0$ their marginal instantaneous utility from the private goods, the consumers' instantaneous utilities from the quality of the broadcasts (u_1) and from consuming the private good (u_2) are:

$$u_1(t) = \alpha[\hat{S}n(t) - \omega n(t)^2] \tag{10}$$

and

$$u_2(t) = \beta\{\hat{Y} + (\varphi_1 - \varphi_2 - p)[\hat{S}n(t) - \omega n(t)^2]\} . \tag{11}$$

The royalties paid by the broadcasters at t, $n(t)r(t)$, are immediately directed to finance public services. The consumers derive instantaneous utility (u_3) from the investment of the spectrum's royalties in the provision of public goods. The following explicit form ensures diminishing positive marginal utility from the public goods and also concavity in the control variable g of the Hamiltonian associated with the public planner's optimal control problem described in the following section:

$$u_3(t) = \gamma \boldsymbol{\ell n}(n(t)r(t)) \tag{12}$$

where $\gamma > 0$.

In sum, the consumers' instantaneous utility can be expressed as:

$$\begin{aligned} u(t) &= u_1(t) + u_2(t) + u_3(t) \\ &= \beta\hat{Y} + [\alpha + \beta(\varphi_1 - \varphi_2 - p)][\hat{S}n(t) - \omega n(t)^2] + \gamma \boldsymbol{\ell n}(n(t)r(t)). \end{aligned} \tag{13}$$

As there is no strong interaction between public goods' consumption and private goods' consumption, the assumed separability of the utilities derived from these consumptions is sensible. In justifying the assumed separability between the utilities generated from the use of the broadcasting service and from the rest of the private good consumption, we stress that these service and private good consumption are aggregates and that the sign of the cross derivatives of a utility function defined on these aggregates is not clear. Some components of the broadcasting service are substitute to some components of the consumer's aggregate consumption of the rest of the private goods, but complementing others. For example, watching OTA broadcast sport competitions complements the use of some household's facilities and utilities and the consumption of home-made food's ingredients, but substitutes attendance of sport competitions, use of transportation and stadium related services and consumption of fast food. For this reason, and for tractability, the cross derivative is assumed to be nil. Under the said specification, the consumers' marginal instantaneous utility from the number of broadcasting channels is:

$$u_n(t) = [\alpha + \beta(\varphi_1 - \varphi_2 - p)][\hat{S} - 2\omega n(t)] + \gamma / (n(t) . \qquad (14)$$

Recalling that $\hat{S} - 2\omega\tilde{n} = 0$ and $\gamma / n > 0$, $u_n = 0$ for $\hat{n} = \tilde{n} + \varepsilon > \tilde{n}$ that satisfies $[\alpha + \beta(\varphi_1 - \varphi_2 - p)]2\omega\varepsilon = \gamma / (\tilde{n} + \varepsilon)$. The only positive root for this quadratic equation is $\varepsilon = -0.5\tilde{n} + 0.5\sqrt{\tilde{n}^2 + 2\gamma / \{\omega[\alpha + \beta(\varphi_1 - \varphi_2 - p)]\}}$. In turn, u_n is positive for $0 < n < \hat{n}$, and negative for $n > \hat{n}$, where

$$\hat{n} = 0.5\tilde{n} + 0.5\sqrt{\tilde{n}^2 + 2\gamma / \{\omega[\alpha + \beta(\varphi_1 - \varphi_2 - p)]\}} . \qquad (15)$$

This property is recalled in the construction of the phase-plane diagram in section 5.

4. Royalties-based optimal control

In the proposed framework, the number of broadcasters is indirectly controlled by the royalties per band set by the state to maximise the consumers' utility over an infinite planning horizon. The public planner's decision-problem is postulated as choosing the trajectory of g that maximizes $\int_0^\infty e^{-\rho t} u(t) dt$ subject to the state-equation (8) and the initial condition $n(t = 0) = n_0$, where $\rho > 0$ indicates the public planner's time-preference rate. The current-value Hamiltonian associated with this problem is:

$$\mathcal{H} = \beta\hat{Y} + [\alpha + \beta(\varphi_1 - \varphi_2 - p)](\hat{S}n - \omega n^2) + \gamma \boldsymbol{ln}(nr) + \lambda\phi[p(\hat{S} - \omega n) - c - r - \pi] \qquad (16)$$

where the time index is omitted for compactness and the co-state variable, λ, reflects the public planner's current shadow value of the number of channels (i.e., broadcasts' variety and competition).

The second-order (curvature) condition for maximum requires the Hessian matrix of $\mathcal{H}$ with respect to (n,r) to be negative semidefinite. This Hessian matrix is

$$H = \begin{bmatrix} \mathcal{H}_{nn} & \mathcal{H}_{nr} \\ \mathcal{H}_{rn} & \mathcal{H}_{rr} \end{bmatrix} = \begin{bmatrix} -2\omega[\alpha+\beta(\varphi_1-\varphi_2-p)]-\gamma/n^2 & 0 \\ 0 & -\gamma/r^2 \end{bmatrix}. \tag{17}$$

Since $\mathcal{H}_{rr} < 0$ and $\mathcal{H}_{nr} = 0 = \mathcal{H}_{rn}$, $\boldsymbol{det}\mathrm{H}$ is positive and H is negative semidefinite as long as

$$\mathcal{H}_{nn} = u_{nn} = -2\omega[\alpha+\beta(\varphi_1-\varphi_2-p)]-\gamma/n^2 < 0. \tag{18}$$

This inequality requires the consumers' marginal instantaneous utility from the number of channels (i.e., broadcasts) to be diminishing. This curvature condition sets a lower bound on the number of broadcasters in the said optimally controlled industry:

$$n > \sqrt{\gamma/\{2\omega[\alpha+\beta(\varphi_1-\varphi_2-p)]\}} \equiv \underline{n}. \tag{19}$$

This lower bound rises with the ratio of the royalties-financed public good's utility coefficient to the negative of the instantaneous utility loss from the decline in the marginal improvement in the quality of the broadcasting industry's service induced by a rise in the number of broadcasters. As shown in the next sections, inequality (18) also helps identify the nature of the steady state and its comparative static properties.

If inequality (18) holds, the following conditions are sufficient:

$$\dot{\lambda} = -\mathcal{H}_n + \rho\lambda = -u_n + (\rho+\phi p\omega)\lambda \tag{20}$$

$$\mathcal{H}_r = \gamma/r - \lambda\phi = 0 \tag{21}$$

$$\dot{n} = \phi[p(\hat{S}-\omega n) - c - r - \pi] \tag{22}$$

$$\lim_{t\to\infty} e^{-\rho t}\lambda(t)n(t) = 0. \tag{23}$$

By differentiating (21) with respect to time and substituting (20) for $\dot{\lambda}$, (21) for λ and (14) for u_n, the Euler equation for the public planner's optimal change in the royalties charged on bands is:

$$\begin{aligned} \dot{r} &= (\phi/\gamma)u_n r^2 - (\rho+\phi p\omega)r \\ &= (\phi/\gamma)\{[\alpha+\beta(\varphi_1-\varphi_2-p)](\hat{S}-2\omega n)+\gamma/n\}r^2 - (\rho+\phi p\omega)r. \end{aligned} \tag{24}$$

The multiplication of both sides of this Euler equation by 1/r suggests that the optimal rate of change in the spectrum royalties is moderated by the public planner's rate of time preference and the price of broadcasts' quality. The optimal rate of change in the royalties rises with the consumers' marginal direct utility from the broadcasts and with the net effect of the broadcasts on the consumers' aggregate income ($\varphi_1 - \varphi_2$), proportionally to the product of the consumers' marginal utility from the private goods and the available broadcasting spectrum.

5. Convergence to steady state

In conjunction with the initial and tranversality conditions, the differential equation-system (22) and (24) constitutes the solution to the aforesaid optimal control problem. The steady state (n^*, r^*) of this system satisfies the following equalities:

$$r^* = p(\hat{S} - \omega n^*) - c - \pi \tag{25}$$

$$r^* = \frac{\gamma[(\rho/\phi) + p\omega]}{u_n(n^*)}. \tag{26}$$

In turn, the steady-state number of broadcasters under royalties-based optimal control satisfies the following equality:

$$n^* = \frac{\hat{S}}{\omega} - \left[\frac{\gamma[1 + \rho/(\phi p\omega)]}{u_n(n^*)} + \frac{c + \pi}{p\omega}\right]. \tag{27}$$

where $\hat{S}/\omega$ is the upper-bound number of channels. As the system's probability of being in steady state at the beginning of the planning horizon is zero, it is sensible to investigate firstly the possibility of convergence to steady state, particularly from an industry with an initially small number of broadcasters. Once convergence to steady state is proven to be possible, the effects of the model parameters on the steady-state industry size indicated in (27) would be explored.

Evaluated in steady state, the Jacobian matrix of the equation-system (22) and (24) is:[2]

$$\begin{bmatrix} \dot{n}_n(n^*, r^*) & \dot{n}_r(n^*, r^*) \\ \dot{r}_n(n^*, r^*) & \dot{r}_r(n^*, r^*) \end{bmatrix} = \begin{bmatrix} -\phi p\omega & -\phi \\ (\phi/\gamma)u_{nn}(n^*)r^{*2} & (\rho + \phi p\omega) \end{bmatrix}. \tag{28}$$

The eigenvalues of the Jacobian matrix are:

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[2] By differentiation,

$$\dot{r}_r(n_{ss}, r_{ss}) = 2(\phi/\gamma)\{[\alpha + \beta(\varphi_1 - \varphi_2 - p)](\hat{S} - 2\omega n_{ss}) + \gamma/n_{ss}\}r_{ss} - (\rho + \phi p\omega).$$

By setting $\dot{r} = 0$ in (22), in steady state

$$(\phi/\gamma)\{[\alpha + \beta(\varphi_1 - \varphi_2 - p)](\hat{S} - 2\omega n_{ss}) + \gamma/n_{ss}\}r_{ss} = (\rho + \phi p\omega).$$

Hence,

$$\dot{r}_r(n_{ss}, r_{ss}) = \rho + \phi p\omega.$$

$$\mu_{1,2} = 0.5\{\rho \pm \underbrace{\sqrt{\rho^2 + 4[\phi p\omega(\rho + \phi p\omega) - (\phi^2 / \gamma)u_{nn}(n^*)]r^{*2}}}_{\Delta}\}. \qquad (29)$$

In view of (18), $\Delta > \rho$ and, as $\mu_1 > 0$ and $\mu_2 < 0$, (n^*, r^*) is a saddle point. The phase-diagram in the plane surrounding this steady state is constructed as follows.

By setting the right-hand side of equation (22) to zero and rearranging terms, the isocline $\dot{n} = 0$ is linear and downwardly sloped $(dr / dn = -p\omega)$. The isocline $\dot{r} = 0$ is given by:

$$r = \frac{(\gamma / \phi)(\rho + \phi p\omega)}{u_n}. \qquad (30)$$

Recalling (18), the slope of this isocline is positive:

$$\frac{dr}{dn}\Big|_{\dot{r}=0} = -\frac{(\gamma / \phi)(\rho + \phi p\omega)u_{nn}}{u_n^{\,2}} > 0. \qquad (31)$$

It is also variable. In consideration of (14),

$$\frac{d^2r}{dn^2}\Big|_{\dot{r}=0} = \frac{2(\gamma / \phi)[u_{nn}^{\,2} - (\gamma / n^3)u_n](\rho + \phi p\omega)}{u_n^{\,3}}. \qquad (32)$$

Recalling that u_n is negative for $n > \hat{n}$, the numerator on the right-hand side of (32) is positive, whereas the denominator is negative, for $n > \hat{n}$. Hence, the isocline $\dot{r} = 0$ is concave for $n > \hat{n}$. Recalling that u_n is positive for $0 < n < \hat{n}$, the isocline $\dot{r} = 0$ can be convex for sufficiently small values of n, as depicted in Figure 1. As $\dot{n}_r(n^*, r^*) = -\phi < 0$, the horizontal arrows in the region above (below) the isocline $\dot{n} = 0$ are pointed leftward (rightward). As the curvature condition requires that $u_{nn}(n^*) < 0$, $\dot{r}_n(n^*, r^*) = (\phi / \gamma)u_{nn}(n^*)r^{*2} < 0$. Therefore, the vertical arrows in the regions to the right (left) hand side of the isocline $\dot{r} = 0$ are pointed downward (upward).

As can be seen from the phase-plane diagram (n^*, r^*) is the only interior steady state. Convergence to (n^*, r^*) can be along one of the two arms of the stable manifold. Starting from a highly regulated industry with a small number of broadcasters (i.e., $\underline{n} < n_0 < n^*$), the left upward sloped converging arm (indicated by solid arrows) is relevant for the public planner. Along this arm the number of broadcasters increases gradually despite the rising royalties. The public planner can let the broadcasting industry gradually approach the steady state along this arm by appropriate setting of the initial royalties followed by gradual increments.

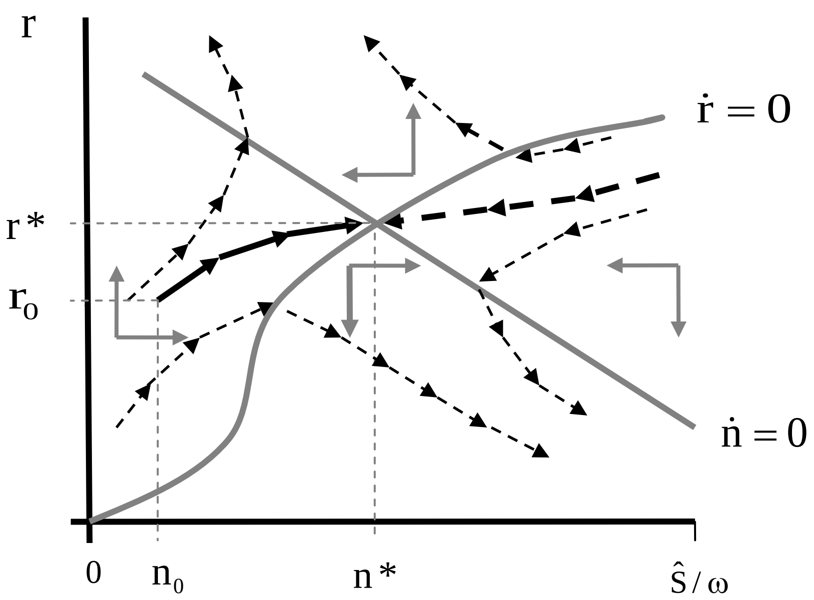

Figure 1. Phase-plane diagram

6. Comparative statics of the steady state

The effects of the model parameters on the steady-state number of broadcasters are obtained by total differentiation of the equality obtained from (25) and (26) and in consideration of (14):

$$V \equiv p(\hat{S} - \omega n^*) - (c + \pi) - \frac{\gamma[(\rho/\phi) + p\omega]}{\underbrace{[\alpha + \beta(\varphi_1 - \varphi_2 - p)](\hat{S} - 2\omega n^*) + \gamma / n^*}_{u_n(n^*)}} = 0\,. \tag{33}$$

In studying these effects we recall inequality (18) and note that

$$\frac{dV}{dn^*} = \frac{\gamma[(\rho/\phi) + p\omega]u_{nn}(n^*)}{[u_n(n^*)]^2} - p\omega < 0\,. \tag{34}$$

The directions, but not the size, of the effects of some of the model parameters on the steady-state number of broadcasters are found to be independent of the model's two central features: the opposite effects of variety and congestion on the quality of the broadcasting industry's service and the opposite information-dissemination and effort-diversion effects of the quality of the industry service on the consumers' income. Hence, within the confine of the outlined analytical framework, a change in any of these structural parameters affects the steady-state number of broadcasters in one, easily identifiable, direction. In particular, $dn^*/dc = dn^*/d\pi = (dV/dn^*)^{-1} < 0$ and $dn^*/dc = dn^*/d\pi = (dV/dn^*)^{-1} < 0$. That is, the steady-state number of broadcasters decreases with the instantaneous operational cost of channels and with the normal profit attainable in other industries and increases with the size of the spectrum available to the broadcasting industry.

The directions of the effects of the rest of the model parameters on the steady-state number of broadcasters are found to be dependent on at least one of the aforesaid pairs of opposite effects. A rise in the effect of broadcasting industry's service on aggregate output increases (reduces) the number of broadcasters in the new steady state if in the old steady state less (more) than half the spectrum carrying capacity was used so that the positive variety effect on the quality of the industry's service dominated (was dominated by) the negative congestion effect:

$$\frac{dn^*}{d(\varphi_1-\varphi_2)}=\left[\frac{\dfrac{-\beta(\hat{S}-2\omega n^*)\gamma[(\rho/\phi)+p\omega]}{[u_n(n^*)]^2}}{dV/dn^*}\right]\overset{>}{\underset{<}{=}}0 \tag{35}$$

as $n^*\overset{<}{\underset{>}{=}}0.5(\hat{S}/\omega)=\tilde{n}$.

Also a rise in the consumers' direct marginal instantaneous utility from the quality of the broadcasts increases (reduces) the number of broadcasters in the new steady state if in the old steady state the variety effect on the quality of the industry's service dominated (was dominated by) the congestion effect:

$$\frac{dn^*}{d\alpha}=\left[\frac{\dfrac{-(\hat{S}-2\omega n^*)\gamma[(\rho/\phi)+p\omega]}{[u_n(n^*)]^2}}{dV/dn^*}\right]\overset{>}{\underset{<}{=}}0 \tag{36}$$

as $n^*\overset{<}{\underset{>}{=}}0.5(\hat{S}/\omega)=\tilde{n}$.

The effect of the public planner's rate of time preference on the steady-state number of broadcasters is negative (positive) as long as the consumers' sum of direct and indirect marginal instantaneous utilities from the number of channels is positive (negative):

$$\frac{dn^*}{d\rho}=\left[\frac{\dfrac{\gamma}{\phi u_n(n^*)}}{dV/dn^*}\right]\overset{<}{\underset{>}{=}}0 \tag{37}$$

as $u_n(n^*)\overset{>}{\underset{<}{=}}0$ (i.e., as $n^*\overset{<}{\underset{>}{=}}\hat{n}$, where $\hat{n}$ is given by (15)).

The effect of the speed of adjustment to above-normal profit in the broadcasting industry on the steady-state number of broadcasters is positive (negative) as long as the consumers' sum of direct and indirect marginal instantaneous utilities from the number of available channels is positive (negative):

$$\frac{dn^*}{d\phi} = \left[\frac{\dfrac{-\gamma\rho}{\phi^2 u_n(n^*)}}{dV/dn^*} \right] \overset{>}{\underset{<}{=}} 0 \tag{38}$$

as $u_n(n^*) \overset{>}{\underset{<}{=}} 0$ (i.e., as $n^* \overset{<}{\underset{>}{=}} \hat{n}$).

Narrowing the channels' spectral bands does not necessarily increase the steady-state number of channels. It is only clear that when less than half the spectrum's carrying capacity is utilized ($\hat{S} > 2\omega n^*$) and the marginal effect of broadcasts on aggregate output is positive and exceeding the marginal revenue from the broadcasts' quality ($\varphi_1 - \varphi_2 > p$), a reduction in the channels' spectral width leads to a rise in the steady-state number of channels. To see this formally, note that

$$\frac{dn^*}{d\omega} = \left[\frac{pn^* + \gamma \dfrac{[\alpha + \beta(\varphi_1 - \varphi_2 - p)][(\hat{S} - 2\omega n^*)p + ((\rho/\phi) + p\omega)2n^*] + \gamma p / n^*}{[u_n(n^*)]^2}}{dV/dn^*} \right] \overset{<}{\underset{>}{=}} 0 \tag{39}$$

as $\left\{ pn^* + \gamma \dfrac{[\alpha + \beta(\varphi_1 - \varphi_2 - p)][(\hat{S} - 2\omega n^*)p + ((\rho/\phi) + p\omega)2n^*] + \gamma p / n^*}{[u_n(n^*)]^2} \right\} \overset{>}{\underset{<}{=}} 0$.

With regard to the effect of the broadcast-quality price, it can only be clearly shown that the steady-state number of channels decreases with the broadcast quality price when the congestion effect dominates the variety effect (i.e., more than half the spectrum's carrying capacity is utilized, $\hat{S} - 2\omega n^* < 0$) and the information-dissemination effect dominates the effort-diversion effect on aggregate output ($\varphi_1 - \varphi_2 > 0$):

$$\frac{dn^*}{dp} = \left[\frac{\gamma \dfrac{(\hat{S} - 2\omega n^*)\{\beta[(\rho/\phi) + \omega(\varphi_1 - \varphi_2)] + \omega\alpha\} + \omega\gamma / n^*}{[u_n(n^*)]^2} - (\hat{S} - \omega n^*)}{dV/dn^*} \right] \overset{<}{\underset{>}{=}} 0 \tag{40}$$

as $\left\{\gamma \frac{(\hat{S}-2\omega n^*)\{\beta[(\rho/\phi)+\omega(\varphi_1-\varphi_2)]+\omega\alpha\}+\omega\gamma/n^*}{[u_n(n^*)]^2}-(\hat{S}-\omega n^*)\right\} \overset{>}{\underset{<}{=}} 0$.

The steady-state number of channels increases (decreases) with the consumers' marginal instantaneous utility from the private goods as long as $\varphi_1-\varphi_2-p$ and $\hat{S}-2\omega n^*$ have identical (different) signs. Formally,

$$\frac{dn^*}{d\beta}=\left[\frac{\frac{-(\varphi_1-\varphi_2-p)(\hat{S}-2\omega n^*)\gamma[(\rho/\phi)+p\omega]}{[u_n(n^*)]^2}}{dV/dn^*}\right] \overset{>}{\underset{<}{=}} 0 \tag{41}$$

as $(\varphi_1-\varphi_2-p)(\hat{S}-2\omega n^*) \overset{>}{\underset{<}{=}} 0$. That is, if in the initial steady state the variety effect dominated (was dominated by) the congestion effect and also the difference between the information-dissemination effect and effort-diversion effect of the broadcasts' quality on the consumers' income is larger (smaller) than the quality price of broadcasts, then an increase in the marginal utility from the consumption of private goods increases (decreases) the number of broadcasters in the new steady state.

Almost in contrast to the claimed previous effect, the steady-state number of channels increases (decreases) with the consumers' marginal instantaneous utility from the public good financed by the spectrum royalties as long as $[\alpha+\beta(\varphi_1-\varphi_2-p)]$ and $(\hat{S}-2\omega n^*)$ have different (identical) signs. Formally,

$$\frac{dn^*}{d\gamma}=\left[\frac{\frac{[(\rho/\phi)+p\omega][\alpha+\beta(\varphi_1-\varphi_2-p)](\hat{S}-2\omega n^*)}{[u_n(n^*)]^2}}{dV/dn^*}\right] \overset{>}{\underset{<}{=}} 0 \tag{42}$$

as $[\alpha+\beta(\varphi_1-\varphi_2-p)](\hat{S}-2\omega n^*) \overset{<}{\underset{>}{=}} 0$.

7. Conclusion

The advent of digital transmission technologies has done little to relieve the broadcasting industry's spectrum constraint. The perennial trade-off between variety and reception prevails and is likely to be most prominent under a deregulatory scheme. Spectrum is a state-owned, time-invariant, scarce natural resource. As in the case of any other state-owned natural resource, governments are entitled to charge royalties on its use and can direct these revenues to finance public services. Therefore, in addition to the direct benefits from the service provided by the broadcasting industry, the indirect benefits to consumers from the public services financed by the royalties on this natural resource were taken into account in the determination of the optimal allocation of bands to broadcasters. For setting the state's royalties on spectrum, we proposed an optimal control model that takes into account the aforesaid aspects, entry and exit of broadcasters in accordance with above-normal profit and at a rate moderated by sunk costs, and possible positive and negative effects of broadcasts on consumers' income. We derived the consumers' inter-temporal utility maximizing steady-state of the broadcasting industry and its comparative statics and analyzed the possibility of convergence to this steady state. In contrast to the observed consolidation and return to concentration in the aftermath of past deregulatory reforms in OECD countries, our analysis reveals that the optimal control of the broadcasting industry with variable royalties on bands may gradually lead the industry to a steady state with a larger number of broadcasters.

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