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Emission-Photosynthesis Imbalance and Climate Change: Forest Land under Intensified Uncertainty and Expected Utility Maximization

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This paper introduces photosynthesis into the motion-equation of the atmospheric stock of carbon-dioxide as a counterpart endogenous factor to emissions of this principal greenhouse gas from lands occupied by humans. By doing so, the paper links the stock of atmospheric carbon-dioxide, hence climate-change and uncertainty, to the allocation of usable land to humans and forest. The public planners can control this allocation and, consequently, the atmospheric stock of carbon-dioxide, climate-change and future usable land by setting land-rates in accordance with use. The analysis considers two types of land-use for expected utility maximizing humans and derives the effects of the land-rates and photosynthesis' efficiency on the size of the forest.

JEL Classification: Q52, Q54

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1. Introduction

The current trend of global warming is mainly a reflection of a chronic imbalance between the emission and recycling of carbon-dioxide. Since the beginning of agricultural settlements humans have contributed to this imbalance by clearing lands from the natural recyclers of this principal greenhouse gas and by burning fossil fuels. During the last 11,000 years the world forest-cover has been reduced from about 50 percent of the Earth's land to about 30 percent. About three quarters of this decline occurred in the last 200 years — a period dominated by coal, gas and oil fueled mechanization of production and transportation, which facilitated an unprecedented growth and spread of output and population. This process has intensified after the Second World War with the industrialization of developing countries. Evidently, the background atmospheric concentration of carbon-dioxide recorded since 1958 at Mount Mauna Loa, Hawaii, displays an oscillating trajectory, with peaks (troughs) at the end (beginning) of the plants' active season, along an upwardly inclined convex contour, the Keeling Curve. When contrasted with deep historical data on carbon-dioxide concentration in bubbles trapped in Antarctic ice-cores, the Keeling Curve reflects unprecedented levels and rates of accumulation of carbon-dioxide. Despite the globally growing concerns reported by Dunlap et al. (1993), Inglehart (1995, 1997), Diekmann and Franzen (1999) and Franzen (2003), the humans' aggregate carbon-dioxide emissions increased by forty-five percents and the world forest cover decreased at an average rate of about 8 million hectares per annum between 1990 and 2010.

While the equilibrium climate sensitivity to atmospheric greenhouse gasses is expected to be relatively small — about three degrees Celsius — the tails of the probability density functions of the surface-temperature change and damage are argued by Weitzman (2009, 2011) to be fat due to compounded uncertainty. In which case, wait-and-see is a high-risk strategy. Yet, remedial actions are impeded by the nature of the atmosphere. The atmosphere is an indivisible open-access natural resource. In the absence of property rights and clear identity of the sources of the stock of carbon-dioxide in any particular location, formation of markets for the aforesaid externalities created by this stock — global warming and climate change — is impossible. Economic analyses of the imbalance in the atmospheric carbon-dioxide cycle and remedial policies have typically focused on human emissions of this gas and other greenhouse gasses. Following Weitzman's (1974) seminal approach, the relative efficiency of quantity and price based instruments for controlling the atmospheric stock of greenhouse gasses have been analyzed by Pizer (2002), Hoel and Karp (2002), Newell and Pizer, (2003), Fischer and Newell (2008) and many other researchers. The implementation of these instruments involves large costs of monitoring emissions. It also requires international cooperation for sharing the burden and attaining globally significant outcomes (Levy, 2011).

Unlike the atmosphere, land — the main platform of humans' and plants' activities — is divisible, controlled by states, and classified by type of use. Hence, controlling the use of land by land-rates (rental rates in the case of state ownership and tax rates in the case of private ownership) can serve as an alternative method for controlling humans' carbon-dioxide emissions as well as plants' photosynthesis of this gas. The large monitoring costs of carbon emissions from this divisible platform can be avoided by setting the land-rates according to use.

This paper considers both humans' emissions and plants' photosynthesis as endogenous determinants of the atmospheric stock of carbon-dioxide and, consequently, global warming and climate change. The paper analyzes theoretically the division of usable land between the human emitters and the natural recyclers of this principal greenhouse gas, who are exposed to their imbalanced-interaction-driven climate change. While the habitat of the natural recyclers, forest, is confined to their initial endowment of land, humans can expand their control over the usable land by renting at a flat rate that varies with the type of use. Land is taken to be essential input in humans' production and, for tractability, two types of use of land by humans are considered. The analysis regards humans as sophisticated rational beings. They take into account the effect of the usable-land allocation on the accumulation of carbon-dioxide in the atmosphere and the effects of the consequent climate change on the size of the usable land, on the mean and variance of production, and on the mean and variance of the atmospheric carbon-dioxide photosynthesized by plants. These effects are described in section 2. In addition to income, humans' expected utility is affected by their concerns about the accumulation of carbon-dioxide in the atmosphere. As presented in section 3, the quantity of usable land rented is chosen to maximize their expected utility. The public planner controls the outcomes by setting the land-rates according to use and technology. The effects of the land-rates and photosynthesis' efficiency on the size of the forest are derived in section 4.

2. Model specifications

Two uncoordinated groups of human land-users are considered: rural dwellers (henceforth farmers) and urban dwellers (henceforth manufacturers). Each of these groups is assumed to be homogeneous with regard to preferences, ability and technology. With S_t denoting the atmospheric carbon-dioxide stock in period t, EF_t the farmers' carbon-dioxide emissions in period t, EM_t the manufacturers carbon-dioxide emissions in period t, EW_t the carbon-dioxide emissions by wildlife in period t, Q_t the quantity of atmospheric carbon-dioxide recycled by the forest in period t, and $0 < \delta < 1$ the rate of decay of atmospheric carbon-dioxide, and with the amount of carbon-dioxide photosynthesized on farm-land by domesticated plants assumed to be negligible, the change in atmospheric carbon-dioxide stock in period t is expressed as:

$$S_t - S_{t-1} = EF_t + EM_t + EW_t - Q_t - \delta S_{t-1} . \tag{1}$$

Because of increasing spread and frequency of climate-based disasters (e.g., floods, blizzards, frosts, hurricanes, cyclones, droughts and bushfires) the size of the Earth's usable land decreases from a maximum $\bar{L}$ with the divergence of the current climate from the ideal climate. For simplicity, the ideal climate is taken to be the same for farmers, manufacturers and forests. The divergence from the ideal climate is driven by the deviation of the current atmospheric stock of carbon-dioxide from the level associated with the ideal climate, $\bar{S}$. With a computationally convenient quadratic representation of this deviation and with $\mu > 0$ indicating the usable-land-loss coefficient, the total usable land at the beginning of period t, is

$$L_{t-1} = \frac{\bar{L}}{1 + \mu(S_{t-1} - \bar{S})^2} . \tag{2}$$

Some of the usable land is held by N_f identical (for simplicity) farmers, each renting an equal amount of land (ℓ_f) at a flat land-rate of r_h dollars per acre, and by N_m identical (for simplicity) manufacturers, each renting an equal amount of land (ℓ_m) at a flat land-rate of r_m dollars per acre. The rest of the usable land (L_n) is covered by undomesticated plants; namely, forest:

$$L_{nt} = L_{t-1} - N_f \ell_{ft} - N_m \ell_{mt} \,. \tag{3}$$

With the quantity of carbon-dioxide emitted by each acre used by farmers and manufacturers assumed to be $\alpha_f > 0$ and $\alpha_m > 0$, respectively, the aggregate emissions of carbon-dioxide by humans at t are

$$EF_t + EM_t = N_f \alpha_f \ell_{ft} + N_m \alpha_m \ell_{mt} \,. \tag{4}$$

Farming, manufacturing and photosynthesis are not performed in fully controlled environments and all lands are assumed to be exposed to the same climate-based random disturbances. A deviation from $\overline{S}$ leads to climate deterioration and, thereby, decline in productivity. With β_f indicating the average productivity of the land used by farmers under ideal climate and $\theta_f > 0$ the sensitivity of per acre farm-output mean to a divergence from the ideal climate, each farmer's output in period t (y_{ft}) is:

$$y_{ft} = \left[\frac{\beta_f}{1 + \theta_f (S_{t-1} - \overline{S})^2} + \varepsilon_t \right] \ell_{ft} \tag{5}$$

where ε_t denotes the climate-based random disturbance. This disturbance is assumed to be normally distributed with zero mean and a variance, σ_t^2, that increases with the divergence from the climate-wise ideal atmospheric carbon-dioxide stock:

$$\sigma_t^2 = \overline{\sigma}^2 [1 + \gamma (S_{t-1} - \overline{S})^2] \,. \tag{6}$$

The positive scalar $\overline{\sigma}^2$ indicates the random disturbance's variance under ideal climate, and $\gamma > 0$ the sensitivity of the per acre farm-output variance to a divergence from the ideal climate.

The activities of the other two land-occupants considered in this model are affected by the common random disturbance, ε_t, in different intensities. As manufacturing is performed in a better controlled environment than farming, the effect of the common random disturbance on manufacturing production is lower. By letting the relative effect on manufacturing be indicated by $0 < \omega < 1$, the random disturbance in manufacturing is indicated by $\omega \varepsilon_t \sim \mathscr{N}(0, \omega^2 \sigma_t^2)$. With β_m indicating the mean output per acre held by manufacturers under ideal climate and $\theta_m > 0$ the sensitivity of per acre manufacturing output mean to a divergence from the ideal climate, each manufacturer's output in period t (y_{mt}) is:

$$y_{mt} = \left[\frac{\beta_m}{1+\theta_m (S_{t-1} - \bar{S})^2} + \omega\varepsilon_t \right] \ell_{mt} \,. \tag{7}$$

By letting the relative effect of the common disturbance on undomesticated plants' activity be indicated by $\psi > 0$, the random disturbance in photosynthesis can be indicated by $\psi\varepsilon_t \sim \mathscr{N}(0, \psi^2\sigma_t^2)$. With $\varphi > 0$ indicating the per acre photosynthesis mean under ideal climate and $\eta > 0$ the sensitivity of per acre photosynthesis-mean to a divergence from the ideal climate, the quantity of atmospheric carbon-dioxide photosynthesized by the forest in period t is:

$$Q_t = \left[\frac{\varphi}{1+\eta(S_{t-1} - \bar{S})^2} + \psi\varepsilon_t \right] (L_{t-1} - N_f \ell_f - N_m \ell_m) \;. \tag{8}$$

The substitution of (4) and (8) into (1) implies

$$S_t - S_{t-1} = N_f \alpha_f \ell_f + N_m \alpha_m \ell_m + EZ_t - \left[\frac{\varphi}{1+\eta(S_{t-1} - \bar{S})^2} + \psi\varepsilon_t \right] (L_{t-1} - N_f \ell_f + N_m \ell_m) - \delta S_{t-1}$$

.(9)

As it is not the focus of the paper, and for simplicity, the effect of climate change on all other forms of wildlife is ignored and EW_t is taken to be exogenous.

3. Uncoordinated land distribution with expected utility maximizing humans

Farmers' and manufacturers' utilities at period t (u_{ft} and u_{mt}, respectively) increase with their disposable income and decrease with their concerns about a further rise at t in the atmospheric carbon-dioxide stock, which is already above the optimal stock $\bar{S}$. With the price of the farm's product indicated by p_f, and with $\nu_f > 0$ indicating farmers' constant (for simplicity) marginal willingness to pay for avoiding a rise in the atmospheric stock of carbon-dioxide, each farmer's decision problem is portrayed as choosing the parcel of land that maximizes his expected utility from his willingness-to-pay deducted personal disposable income, $x_{ft} = p_f y_{ft} - r_f \ell_{ft} - \nu_f (S_t - S_{t-1})$. By recalling (5) and (8) and rearranging terms,

$$\begin{aligned} x_{ft} = {} & \left[\frac{p_t \beta_f}{1+\theta_f (S_{t-1} - \bar{S})^2} - r_f \right] \ell_{ft} \\ & - \nu_f \{ N_f \alpha_f \ell_{ft} + N_m \alpha_m \ell_{mt} + EW_t - \left[\frac{\varphi}{1+\eta(S_{t-1} - \bar{S})^2} \right] (L_{t-1} - N_f \ell_{ft} - N_m \ell_{mt}) - \delta S_{t-1} \} \\ & + [p_f \ell_{ft} - (L_{t-1} - N_f \ell_{ft} - N_m \ell_{mt}) \psi \nu_f] \varepsilon_t . \end{aligned} \tag{10}$$

It is normally distributed with mean

$$\mu_{x_{ft}} = \left[\frac{p_f \beta_f}{1+\theta_f (S_{t-1} - \overline{S})^2} - r_f \right] \ell_{ft}$$
$$- \nu_f \{ N_f \alpha_f \ell_{ft} + N_m \alpha_m \ell_{mt} + EW_t - \left[\frac{\varphi}{1+\eta(S_{t-1} - \overline{S})^2} \right] (L_{t-1} - N_f \ell_{ft} - N_m \ell_{mt}) - \delta S_{t-1} \} \tag{11}$$

and variance

$$\sigma^2_{x_{ft}} = [p_f \ell_{ft} - (L_{t-1} - N_f \ell_{ft} - N_m \ell_{mt}) \psi \nu_f]^2 \overline{\sigma}^2 [1+\gamma(S_{t-1} - \overline{S})^2]\,. \tag{12}$$

Following Freund's (1956) convenient formulation of expected utility from a normally distributed income, let $u_{ft} = 1 - \exp(-R_f x_{ft})$ and $R_f > 0$ represent the farmers' degree of absolute risk aversion. Then, maximizing Eu_{ft} is equivalent to maximizing $\mu_{x_{ft}} - 0.5 R_f \sigma^2_{x_{ft}}$, where the second term can be interpreted as the farmer's costs of risk bearing. Recalling (11) and (12), each farmer's decision on land-holding is reached by solving the following control problem:

$$\max_{\ell_{ft}} \left\{ \begin{array}{l} \left[\dfrac{p_f \beta_f}{1+\theta_f (S_{t-1} - \overline{S})^2} - r_f \right] \ell_{ft} \\ -\nu_f \left[N_f \alpha_f \ell_{ft} + N_m \alpha_m \ell_{mt} + EW_t - \dfrac{\varphi}{1+\eta(S_{t-1} - \overline{S})^2} (L_{t-1} - N_f \ell_{ft} - N_m \ell_{mt}) - \delta S_{t-1} \right] \\ -0.5 R_f \{ [p_f \ell_{ft} - (L_{t-1} - N_f \ell_{ft} - N_m \ell_{mt}) \psi \nu_f]^2 \overline{\sigma}^2 [1+\gamma(S_{t-1} - \overline{S})^2] \} \end{array} \right\}$$

Since $-R_f \overline{\sigma}^2 [1+\gamma(S_{t-1} - \overline{S})^2][(p_f + N_f \psi \nu_f)^2 < 0$, the second-order condition for maximum is satisfied. From the first-order condition and equation (2), the expected utility maximizing land held by each farmer is

$$\ell^*_{ft} = \left[\frac{\psi \nu_f}{p_f + N_f \psi \nu_f} \right] \left[\frac{\overline{L}}{1+\mu(S_{t-1} - \overline{S})^2} - N_m \ell_{mt} \right]$$
$$+ \frac{\dfrac{p_f \beta_f}{1+\theta_f (S_{t-1} - \overline{S})^2} - r_f - \nu_f N_f \left[\alpha_f + \dfrac{\varphi}{1+\eta(S_{t-1} - \overline{S})^2} \right]}{R_f \overline{\sigma}^2 [1+\gamma(S_{t-1} - \overline{S})^2](p_f + N_f \psi \nu_f)^2}. \tag{13}$$

By symmetry, with similar assumptions about the manufacturers' utility and objective and with p_m denoting the manufactured good price, the expected utility maximizing land held by each manufacturer is

$$\ell^*_{mt} = \left[\frac{\psi\nu_m}{p_m\omega + N_m\psi\nu_m}\right]\left[\frac{\bar{L}}{1+\mu(S_{t-1}-\bar{S})^2} - N_f\,\ell_{ft}\right] + \frac{\dfrac{p_m\beta_m}{1+\theta_m(S_{t-1}-\bar{S})^2} - r_m - \nu_m N_m\left[\alpha_m + \dfrac{\varphi}{1+\eta(S_{t-1}-\bar{S})^2}\right]}{R_m\bar{\sigma}^2[1+\gamma(S_{t-1}-\bar{S})^2](p_m\omega + N_m\psi\nu_m)^2}. \tag{14}$$

With accurate expectations about the counterpart's demand for usable land being assumed, equations (13) and (14) imply that the expected utility maximizing land per farmer and manufacturer are:

$$\begin{aligned}
\ell^*_{ft} = {} & \left\{\frac{\left[\dfrac{\psi\nu_f}{p_f + N_f\psi\nu_f}\right]\left[1 - \dfrac{N_m\psi\nu_m}{p_m\omega + N_m\psi\nu_m}\right]}{1 - \left[\dfrac{\psi\nu_f N_f}{p_f + N_f\psi\nu_f}\right]\left[\dfrac{\psi\nu_m N_m}{p_m\omega + N_m\psi\nu_m}\right]}\right\}\frac{\bar{L}}{1+\mu(S_{t-1}-\bar{S})^2} \\
& + \left\{\frac{1}{1 - \left[\dfrac{\psi\nu_f N_f}{p_f + N_f\psi\nu_f}\right]\left[\dfrac{\psi\nu_m N_m}{p_m\omega + N_m\psi\nu_m}\right]}\right\}\frac{\dfrac{p_f\beta_f}{1+\theta_f(S_{t-1}-\bar{S})^2} - r_f - \nu_f N_f\left[\alpha_f + \dfrac{\varphi}{1+\eta(S_{t-1}-\bar{S})^2}\right]}{R_f\bar{\sigma}^2[1+\gamma(S_{t-1}-\bar{S})^2](p_f + N_f\psi\nu_f)^2} \\
& - \left\{\frac{\left[\dfrac{\psi\nu_f}{p_t + N_f\psi\nu_f}\right]N_m}{1 - \left[\dfrac{\psi\nu_f N_f}{p_f + N_f\psi\nu_f}\right]\left[\dfrac{\psi\nu_m N_m}{p_m\omega + N_m\psi\nu_m}\right]}\right\}\frac{\dfrac{p_m\beta_m}{1+\theta_m(S_{t-1}-\bar{S})^2} - r_m - \nu_m N_m\left[\alpha_m + \dfrac{\varphi}{1+\eta(S_{t-1}-\bar{S})^2}\right]}{R_m\bar{\sigma}^2[1+\gamma(S_{t-1}-\bar{S})^2](p_m\omega + N_m\psi\nu_m)^2}
\end{aligned} \quad .(15)$$

and

$$\begin{aligned}
\ell^*_{mt} = {} & \left\{\frac{\left[\dfrac{\psi\nu_m}{p_m\omega + N_m\psi\nu_m}\right]\left[1 - \dfrac{N_f\psi\nu_f}{p_f + N_f\psi\nu_f}\right]}{1 - \left[\dfrac{N_m\psi\nu_m}{p_m\omega + N_m\psi\nu_m}\right]\left[\dfrac{N_f\psi\nu_f}{p_f + N_f\psi\nu_f}\right]}\right\}\frac{\bar{L}}{1+\mu(S_{t-1}-\bar{S})^2} \\
& + \left\{\frac{1}{1 - \left[\dfrac{N_m\psi\nu_m}{p_m\omega + N_m\psi\nu_m}\right]\left[\dfrac{N_f\psi\nu_f}{p_f + N_f\psi\nu_f}\right]}\right\}\frac{\dfrac{p_m\beta_m}{1+\theta_m(S_{t-1}-\bar{S})^2} - r_m - \nu_m N_m\left[\alpha_m + \dfrac{\varphi}{1+\eta(S_{t-1}-\bar{S})^2}\right]}{R_m\bar{\sigma}^2[1+\gamma(S_{t-1}-\bar{S})^2](p_m\omega + N_m\psi\nu_m)^2} \\
& - \left\{\frac{\left[\dfrac{N_f\psi\nu_m}{p_m\omega + N_m\psi\nu_m}\right]}{1 - \left[\dfrac{N_m\psi\nu_m}{p_m\omega + N_m\psi\nu_m}\right]\left[\dfrac{N_f\psi\nu_f}{p_f + N_f\psi\nu_f}\right]}\right\}\frac{\dfrac{p_f\beta_f}{1+\theta_f(S_{t-1}-\bar{S})^2} - r_f - \nu_f N_f\left[\alpha_f + \dfrac{\varphi}{1+\eta(S_{t-1}-\bar{S})^2}\right]}{R_f\bar{\sigma}^2[1+\gamma(S_{t-1}-\bar{S})^2](p_f + N_f\psi\nu_f)^2}
\end{aligned} \quad .(16)$$

The expressions in the large parentheses in the first terms on the right-hand side of equations (15) and (16) indicate positive marginal effects of usable land on the expected utility maximizing land holding of individual farmers and manufacturers. The expressions in the large parentheses in the second terms on the right-hand side of these equations reveal positive effects of the ratio of the self marginal net expected benefit to marginal risk on the expected utility maximizing land holding of individual farmers and manufacturers. The expressions in large parentheses in the third terms on the right-hand side of the said equations indicate negative effects of the ratio of the counterpart's agent marginal net expected benefit to marginal risk on the expected utility maximizing land holding of individual farmers and manufacturers. Consequently, the effects of the model parameters $\overline{L}$, μ, α_f, α_m, β_f, β_m, R_f, R_m, and $\overline{\sigma}^2$ are clear. Both the farmer's and manufacturer's expected utility maximizing land-parcels increase with the size of the usable land under ideal climate, with self output per acre, with the counterpart's emissions per acre, with the counterpart's production sensitivity to climate change, and with the counterpart's degree of absolute risk aversion. Their expected utility maximizing land-parcels decrease with the coefficient of the land-loss engendered by climate change, with self emissions per acre, with the sensitivity of self production to climate change, with the random disturbance's variance under ideal climate, with self degree of absolute risk aversion, and with the counterpart's output per acre.

The directions of the effects of photosynthesis' efficiency — the mean recycled carbon-dioxide by an acre of forest — on the farmer's and manufacturer's demands for land are not clear. The following propositions are obtained by differentiating (15) and (16) with respect to this factor.

Proposition 1 (photosynthesis' efficiency effect on farm-land)

$$\left\{\frac{\partial \ell^*_{ft}}{\partial\left(\dfrac{\varphi}{1+\eta(S_{t-1}-\overline{S})^2}\right)}\right\} \begin{matrix}>\\=\\<\end{matrix} 0 \quad \text{if}$$

$$\left\{\frac{\dfrac{\psi\nu_f\nu_m N_m{}^2}{p_t+N_f\psi\nu_f}}{R_m\overline{\sigma}^2[1+\gamma(S_{t-1}-\overline{S})^2](p_m\omega+N_m\psi\nu_m)^2}\right\} \begin{matrix}>\\=\\<\end{matrix} \left\{\frac{\nu_f N_f}{R_f\overline{\sigma}^2[1+\gamma(S_{t-1}-\overline{S})^2](p_f+N_f\psi\nu_f)^2}\right\}.$$

Proposition 2. (photosynthesis' efficiency effect on manufacturer-land)

$$\left\{\frac{\partial \ell^*_{mt}}{\partial\left(\dfrac{\varphi}{1+\eta(S_{t-1}-\bar{S})^2}\right)}\right\} \begin{matrix} > \\ = \\ < \end{matrix} 0 \quad \text{if}$$

$$\left\{\frac{\dfrac{\psi\nu_m\nu_f N_f{}^2}{p_m\omega+N_m\psi\nu_m}}{R_f\bar{\sigma}^2[1+\gamma(S_{t-1}-\bar{S})^2](p_f+N_f\psi\nu_f)^2}\right\} \begin{matrix} > \\ = \\ < \end{matrix} \left\{\frac{\nu_m N_m}{R_m\bar{\sigma}^2[1+\gamma(S_{t-1}-\bar{S})^2](p_m\omega+N_m\psi\nu_m)^2}\right\}.$$

The usable land reserved by the expected utility maximizing farmers and manufacturers for the forests is:

$$L^*_{nt} = L_{t-1} - N_f\ell^*_{ft} - N_m\ell^*_{mt} \ . \tag{17}$$

By differentiation and collecting terms, the following proposition indicates, as can be intuitively expected, a positive effect of photosynthesis' efficiency on the land left by humans for forest. It further displays the factors comprising this positive effect and suggests that the effect of photosynthesis' efficiency on the forest-land decreases with the divergence from the ideal climate, with the sensitivity of the per acre farm-output variance to a divergence from the ideal climate, with the random disturbance's variance under ideal climate, and with the farmers' and manufacturers' degrees of absolute risk aversion.

Proposition 3 (the effect of photosynthesis' efficiency on forest-land)

$$\frac{\partial L^*_{nt}}{\partial\left(\dfrac{\varphi}{1+\eta(S_{t-1}-\bar{S})^2}\right)}$$

$$=\left\{\frac{\dfrac{\nu_m N_m{}^2\left[1-\dfrac{N_f\psi\nu_f}{p_f+N_f\psi\nu_f}\right]}{R_m\bar{\sigma}^2[1+\gamma(S_{t-1}-\bar{S})^2](p_m\omega+N_m\psi\nu_m)^2}+\dfrac{\nu_f N_f{}^2\left[1-\dfrac{\psi\nu_m N_m}{p_m\omega+N_m\psi\nu_m}\right]}{R_f\bar{\sigma}^2[1+\gamma(S_{t-1}-\bar{S})^2](p_f+N_f\psi\nu_f)^2}}{1-\left[\dfrac{N_m\psi\nu_m}{p_m\omega+N_m\psi\nu_m}\right]\left[\dfrac{N_f\psi\nu_f}{p_f+N_f\psi\nu_f}\right]}\right\} > 0.$$

4. Policy implications for forests

The public planner's control instruments of the aggregate size of the forest in a world with uncoordinated expected utility maximizing human activities are the land-rates for farming and manufacturing. The objective, hence setting, of these use-based land-rates is different from that of *ad valorem* rates, which fully, or partially, disregard negative external effects generated on sites. There may be various criteria for selecting these use-based land-rates. Similar to macroeconomic stabilization policy, a possible practical criterion is minimizing the variance of the atmospheric carbon-dioxide stock while restricting the mean to be equal to a target level, $\hat{S}_t$. Using equation (9) and (6) to compute the atmospheric carbon-dioxide stock's mean and variance, the public planner's decision problem under this criterion is formally expressed as

$$\min_{(r_{ft},r_{mt})}\{[\psi(L_{t-1}-N_f\ell^*_{ft}+N_m\ell^*_{mt})]^2[1+\gamma(S_{t-1}-\bar{S})^2]\bar{\sigma}^2\} \tag{18}$$

subject to

$$\begin{aligned}&\left[\alpha_f+\frac{\varphi}{1+\eta(S_{t-1}-\bar{S})^2}\right]N_f\ell^*_f+\left[\alpha_m+\frac{\varphi}{1+\eta(S_{t-1}-\bar{S})^2}\right]N_m\ell^*_m\\&+EZ_t-\left[\frac{\varphi}{1+\eta(S_{t-1}-\bar{S})^2}\right]L_{t-1}+(1-\delta)S_{t-1}=\hat{S}_t\end{aligned} \tag{19}$$

where ℓ^*_f is given by (15) and ℓ^*_m by (16). This problem can be numerically solved with calibrated parameters of an expanded version of the model to a larger, more realistic, number of land usages.

Regardless of the land-rates' selection criteria, the combined effect of the policy instruments on the forest-land can be obtained by total differentiation of equation (17). With inter-sectoral mobility being taken into account:

$$\begin{aligned}dL^*_{nt}=&-\left[\frac{\partial N_f}{\partial r_f}\ell^*_{ft}+\frac{\partial\ell^*_{ft}}{\partial r_f}N_f+\frac{\partial N_m}{\partial r_f}\ell^*_{mt}+\frac{\partial\ell^*_{mt}}{\partial r_f}N_m\right]dr_f\\&-\left[\frac{\partial N_f}{\partial r_m}\ell^*_{ft}+\frac{\partial\ell^*_{ft}}{\partial r_m}N_f+\frac{\partial N_m}{\partial r_m}\ell^*_{mt}+\frac{\partial\ell^*_{mt}}{\partial r_m}N_m\right]dr_m\end{aligned} \quad .(20)$$

It is sensible to assume that $\partial N_f/\partial r_f<0$, $\partial N_m/\partial r_f>0$, $\partial N_f/\partial r_m>0$ and $\partial N_m/\partial r_m<0$, but moderated by costs of migration and adjustment. Since the total number of humans is fixed, $\partial N_f/\partial r_f=-\partial N_m/\partial r_f$ and $\partial N_m/\partial r_m=-\partial N_f/\partial r_m$. In view of this property and by recalling (15) and (16) and collecting terms,

$$dL^{*}_{nt}=\left\{\frac{\dfrac{N_f\left\{1-\left[\dfrac{N_m\psi\nu_m}{p_m\omega+N_m\psi\nu_m}\right]\right\}}{R_f\overline{\sigma}^2[1+\gamma(S_{t-1}-\overline{S})^2](p_f+N_f\psi\nu_f)^2}}{\left\{1-\left[\dfrac{N_f\psi\nu_f}{p_f+N_f\psi\nu_f}\right]\left[\dfrac{N_m\psi\nu_m}{p_m\omega+N_m\psi\nu_m}\right]\right\}}-\frac{\partial N_f}{\partial r_f}(\ell^{*}_{ft}-\ell^{*}_{mt})\right\}dr_f$$
$$+\left\{\frac{\dfrac{N_m\left\{1-\left[\dfrac{N_f\psi\nu_f}{p_f+N_f\psi\nu_f}\right]\right\}}{R_m\overline{\sigma}^2[1+\gamma(S_{t-1}-\overline{S})^2](p_m\omega+N_m\psi\nu_m)}}{\left\{1-\left[\dfrac{N_f\psi\nu_f}{p_f+N_f\psi\nu_f}\right]\left[\dfrac{N_m\psi\nu_m}{p_m\omega+N_m\psi\nu_m}\right]\right\}}-\frac{\partial N_f}{\partial r_m}(\ell^{*}_{ft}-\ell^{*}_{mt})\right\}dr_m \quad .(21)$$

The following propositions about the effects of each of the two policy instruments separately are obtained from (21) straightforwardly.

Proposition 4 (farmer's land-rate effect on forest)**:** If

$$\left\{\frac{\dfrac{N_f\left\{1-\left[\dfrac{N_m\psi\nu_m}{p_m\omega+N_m\psi\nu_m}\right]\right\}}{R_f\overline{\sigma}^2[1+\gamma(S_{t-1}-\overline{S})^2](p_f+N_f\psi\nu_f)^2}}{\left\{1-\left[\dfrac{N_f\psi\nu_f}{p_f+N_f\psi\nu_f}\right]\left[\dfrac{N_m\psi\nu_m}{p_m\omega+N_m\psi\nu_m}\right]\right\}}\right\}\begin{matrix}>\\=\\<\end{matrix}\left\{\frac{\partial N_f}{\partial r_f}(\ell^{*}_{ft}-\ell^{*}_{mt})\right\}$$

then the size of the forest-land increases, does not change, decreases with the land-rate for farmers.

Proposition 5 (manufacturer's land-rate effect on forest)**:** If

$$\left\{\frac{\dfrac{N_m\left\{1-\left[\dfrac{N_f\psi\nu_f}{p_f+N_f\psi\nu_f}\right]\right\}}{R_m\overline{\sigma}^2[1+\gamma(S_{t-1}-\overline{S})^2](p_m\omega+N_m\psi\nu_m)}}{\left\{1-\left[\dfrac{N_f\psi\nu_f}{p_f+N_f\psi\nu_f}\right]\left[\dfrac{N_m\psi\nu_m}{p_m\omega+N_m\psi\nu_m}\right]\right\}}\right\}\begin{matrix}>\\=\\<\end{matrix}\left\{\frac{\partial N_f}{\partial r_m}(\ell^{*}_{ft}-\ell^{*}_{mt})\right\}$$

then the size of the forest-land increases, does not change, decreases with the land-rate for manufacturers.

5. Conclusion

The atmosphere is the recipient of both the beneficial and harmful gaseous emissions of the inhabitants of Earth. In addition to emissions from parcels of land used by humans, photosynthesis is incorporated into the motion-equation of the atmospheric stock of carbon-dioxide as an endogenous variable. By doing so, the stock of atmospheric carbon-dioxide is linked to the division of usable land between humans and plants. This division and, consequently, the atmospheric stock of carbon-dioxide, climate change and future usable land can be controlled by setting land-rates in accordance with current use. As can be seen from propositions 4 and 5, the farmer-manufacturer expected utility maximizing land-parcel differential $(\ell^*_{\mathrm{ft}} - \ell^*_{\mathrm{mt}})$ is crucial for evaluating the effects of the land-rates on the size of the world's forest. As can be further seen from equations (15) and (16), the sign of the said land-parcel differential is not clear and may vary over time with as the stock of atmospheric carbon-dioxide evolves. Presently, the average land-parcel per farmer is greater than the manufacturer's average parcel. But one may argue that with climate-change induced food-supply disruptions and shortages the level of this inequality and even its direction can change. The terms on the left-hand side of the expression in propositions 4 and 5 are positive and $\partial \mathrm{N_f} / \partial \mathrm{r_f} < 0$ and $\partial \mathrm{N_f} / \partial \mathrm{r_m} > 0$. Hence, while the expected utility maximizing parcel of land per farmer is still larger than the manufacturer's parcel (i.e., $\ell^*_{\mathrm{mt}} - \ell^*_{\mathrm{ft}} < 0$), the higher the land-rates, the larger is the forest. These positive effects of the land-rates on the forest-land are moderated by the agents' marginal costs of risk bearing.

References

Diekmann, Andreas, and Axel Franzen (1999), "The Wealth of Nations and Environmental Concern", *Environment and Behavior* 31: 540– 49.

Dunlap, Riley E., George H. Gallup, and Alec M. Gallup (1993), "Of Global Concern: Results of the Health of the Planet Survey", *Environment* 35(7– 15): 33–39.

Fischer, Carolyn and Richard G. Newell (2008), "Environmental and technology policies for climate mitigation", *Journal of Environmental Economics and Management* 55 (2), pp. 142–162.

Franzen, Axel (2003), "Environmental Attitudes in International Comparison: An Analysis of the ISSP Surveys 1993 and 2000", *Social Science Quarterly* 84(2): 297-308.

Freund, R.J. (1956), "An introduction of risk into a risk programming model", *Econometrica* 24, 253-63.

Hoel, Michael and Larry Karp (2002), "Taxes versus quotas for a stock pollutant", *Resource and Energy Economics* 24 (4), pp. 367–384.

Inglehart, Ronald (1995), "Public Support for the Environmental Protection: Objective Problems and Subjective Values in 43 Societies", *PS: Political Science & Politics* 28: 57– 72.

Inglehart, Ronald (1997), *Modernization and Postmodernization: Cultural, Economic, and Political Change in 43 Societies*. Princeton, N.J.: Princeton University Press.

Levy, Amnon (2011), "Abstinence with Reputation Loss, Understating Expectations and Guilt and the Effectiveness of Emission Tax", University of Wollongong Economics Working Paper Series 2011, 11-01.

Newell, Richard G. and William A. Pizer (2003), "Regulating stock externalities under uncertainty", *Journal of Environmental Economics and Management* 45 (2), pp. 416–432.

Pizer, William A. (2002), "Combining price and quantity controls to mitigate global climate change", *Journal of Public Economics* 85 (3), pp. 409–434.

Weitzman, Martin L. (1974) "Prices vs. quantities", *Review of Economic Studies* 41 (4), 477–491.

Weitzman, Martin L. (2009), "on the Modeling and Interpreting the economics of Catastrophic climate Change", *The Review of Economics and Statistics* XCI(1), February,1-19.

Weitzman, Martin L. (2011), “Fat-Tailed Uncertainty in the Economics of Catastrophic Climate Change, February 23, REEF Symposium on Fat Tails.