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Optimal Control of Broadcasting Spectrum with a Variety-Reception Tradeoff and Consumers' Income Sensitivity

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Abstract

A socially desirable number of royalties-paying users of a state-owned broadcasting spectrum is derived within an optimal control framework where the adjustment of the number of users to above-normal profits is adversely affected by sunk costs. The optimal control takes into account the tradeoff between the benefits from higher variety and royalties' revenues and the costs of the intensified interferences associated with entry. It also considers the positive information-dissemination effect and the negative effort-diversion effect of broadcasts on aggregate income. The broadcasting industry's optimal steady state is identified and its stability is analyzed.

Keywords: Economics; Optimal Control; Spectrum; OTA Broadcasts; Variety; Interferences; Royalties

JEL Classification : C61, C62, D61, K23, L52

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1. INTRODUCTION

The consumers of over-the-air (OTA) broadcasts value both variety and reception. With the current technology and constrained broadcasting spectrum, an improvement in any of these factors comes at the expense of the other. An expansion of the number of OTA broadcasters increases programs' variety and the heightened competition improves programs' contents, but the increased spectral congestion intensifies interferences. This tradeoff is the focal theme of our analysis of the optimal control of a spectrum allocated to OTA broadcasts. Our focus on this theme is briefly motivated by historical and contemporary evidence from the OTA broadcasting industries in technologically advanced countries.

Due to the public good nature of OTA broadcasts and their educational, cultural and political impacts and due to scarce bandwidth and high sunk costs, broadcasts and entry rules have been tightly regulated in all major OECD countries (cf., Webbink 1973). Until the late 1970s the television broadcasting industries in OECD countries comprised only a handful of licensed and highly protected public and commercial firms. Since the 1980s alternative transmission techniques, such as satellite and cable, have created a more favorable environment for entry into the television broadcasting industry. Yet veteran incumbent OTA broadcasters' focus on nation-wide audience and their market shares have remained very high (cf., Motta and Polo, 1997; Caves, 2006). The radio broadcasting industries have been less concentrated and more localized, but also entry to these industries has remained highly regulated due to sunk costs and tight spectrum constraint. In many European metropolitan areas, OTA radio broadcasting includes 20 to 80 FM stations and similar numbers of AM stations. Metropolitan areas in Italy offer the largest FM variety. With only 50 kHz separation between stations, they have the most heavily congested FM broadcasting spectrum in Europe.

Congestion generates interferences. The intensified tradeoff between variety and reception has been recently stressed by the Electronic Communication Committee (ECC) within the European Conference of Postal and Telecommunications Administrations (CEPT, 2010).[1] The recent adoption of digital transmission

[1] "The available [FM] spectrum (20.5 MHz) constitutes a limited resource that is used intensively in Europe. In many countries the introduction of new FM services is difficult and may lead to an unacceptable degradation of existing services." (CEPT, 2010, Section 1, P. 5) … "The FM spectrum is in many areas overcrowded and may be reaching saturation if the high quality of reception and existing coverages must be retained. This results in FM services increasingly being interference-limited by

technologies has expanded the scope for program variety in both the television and radio broadcasting industries. Digital technology is spectral efficient — it can increase the number of channels six-fold (cf., Adda and Ottaviani, 2005). The adoption of digital technology by American OTA broadcasters in 2007 and the subsequent turning off of analogue signals in 2008 have freed a significant UHF space. However, the spectral gains have not relaxed the US television broadcasting industry's spectrum constraint. The spectral dividend of the digital switchover was mainly auctioned off to large telecommunications carriers in order to accommodate the deployment of 4G mobile-phone networks. Similar diversion of the digital switchover's spectral dividend is expected in the rest of the OECD countries. The situation is more complex for radio broadcasting as only few countries have successfully adopted and rolled out digital platforms for radio transmissions, and even fewer have clear digital switchover plans for analogue radio broadcasting. Still, buffer zones between broadcasters' bands have to be reduced in order to accommodate new entrants to the OTA broadcasting industry.

Despite the digital switchover, expansion of the OTA broadcasting industry can be expected to intensify the tradeoff between variety and reception in the pursuit of the overall quality of programs. The variety-reception tradeoff is likely to be most prominent under a deregulatory scheme that allows free entry and exit. In view of the recent broadcasting spectrum deregulatory trends (cf., De Vany, 1998; Hazlett, 2008) and the variety-reception tradeoff, our theoretical analysis explores the optimal steady-state number of OTA broadcasters and its stability when entry and exit are motivated by above-normal profits and moderated by sunk costs. We treat the broadcasting spectrum as a state-owned, time-invariant, scarce natural resource. As in the case of any other state-owned natural resource, governments are entitled to royalties on its use. Hence, in addition to the direct benefits from the service provided by the broadcasting industry, there are indirect benefits—the public services financed by the states' royalties on this natural resource.

We construct a conceptual framework where the state's royalties are allowed to vary over time so as to maximize the sum of the discounted direct and indirect benefits stemming from the use of the broadcasting spectrum. The number of broadcasters is allowed to adjust to the above-normal profit from broadcasting at a

design or otherwise and these higher interference levels may have to be accepted to allow the introduction of many more additional services". (CEPT, 2010, Section 2, P. 5) rate moderated by sunk costs. The broadcasting industry's above-normal profit is assumed to rise with the quality of the broadcasting industry's service. On the one hand, entry increases variety, heightens competition and, in turn, raises the quality of the industry's service. On the other hand, entry increases spectral congestion and the intensified interferences lower the quality of the industry's service.

The possible sensitivity of the consumers' incomes to the quality of the industry's service is another central theme in our analysis. In constructing the optimal control model, we take into account two opposing effects of the quality of the service on the consumers' incomes. On the one hand, the information disseminated by the broadcasts enhances knowledge, forms standards of performance and generates transactions. On the other hand, broadcasts divert time from work and other modes of investment in human and social capitals. The higher the overall quality of the industry's broadcasts (variety and reception) is, the stronger these opposing effects. The net effect of the quality of the industry's service on the consumers' aggregate income is not clear.

We derive the steady state of the royalties-based optimally controlled industry, present its comparative statics and identify the conditions that allow for a stable path to the optimal steady state along which the number of broadcasters gradually converges to the optimal steady state. In contrast to the observed consolidation and return to concentration in the aftermath of deregulatory reforms in the United States, Italy, Germany and Japan (cf., Noam, 1992; Motta and Polo, 1997; Hazlet, 2005), our analysis of royalties-based optimal control of the OTA broadcasting industry reveals a possible convergence to a steady state with a larger number of broadcasters.

To set the stage and motivate the royalties-based optimal control of the OTA broadcasting industry, Section 2 presents the industry's basic dynamics and Section 3 computes and illustrates the industry's steady state under *ad hoc* fixed royalties and adjustment to above-normal profits. Section 4 formalizes the multifaceted effects of broadcasts on the consumers' utility: a quality enhanced positive direct effect, a negative indirect effect of diverting budget from other, private, goods, the aforementioned opposing effects of the quality of the broadcasting industry's service on the consumers' aggregate income and budget, and the positive effect of the spectrum royalties on the provision of public goods. Section 4 assembles these utility aspects and the OTA broadcasting industry's dynamics described in Section 2 into the construction of the social planner's optimal control problem. Section 5 derives the optimal steady-state number of the OTA broadcasters and its comparative statics. Section 6 analyzes the possibility of convergence to the industry's optimal steady state.

2. DYNAMICS OF THE INDUSTRY

Let n(t) denote the number of suppliers (broadcasters) of OTA transmitted programs (broadcasts) at time t. At every instance t each broadcaster uses a single channel and delivers a single program. Let the broadcasters be technologically and location-wise identical and paying royalties, g(t), to the government for using a band at t. Also let the width of each band (channel) be technologically determined and fixed, ω, and the bands evenly spread along a fixed homogeneous spectrum space set aside for the broadcasting industry, $\hat{S}$. Then the buffer zones between bands evenly diminish as the number of broadcasters increases and broadcasts are equally receivable by any consumer. For tractability, let us further assume that the consumers are located at an identical, physically unobstructed, distance from the broadcasters (e.g., a flat circular area with broadcasters at its centre and residents in its circumference). Then all broadcasts are equally receivable by all. In our setting, the programs' consumers are also users of broadcast time. Namely, they advertise their services during programs.

Broadcasters enter (exit) the industry as long as the above-normal profit (ANP) from broadcasting is positive (negative). That is,

$$\dot{n}(t) = \phi ANP(t), \tag{1}$$

where $\phi > 0$ reflects the speed of adjustment (ease of entry and exit). Sunk costs deter entry and exit. The larger the broadcasters' sunk costs are, the lower the speed of adjustment of the number of broadcasters to the above-normal profit from broadcasting. The sunk costs are assumed to be time-invariant and hence ϕ is taken to be a scalar. With $\hat{S}$ denoting the spectrum available to the broadcasting industry, $0 \leq n(t) \leq \hat{S}/\omega$.

From the perspective of the consumers, the overall quality of the aerially transmitted programs, Q(t), rises with variety, depth and reception. While the variety and depth of programs rise with the number of channels and their competition, interferences intensify as the buffer zone between the channels diminishes. In other words, reception is positively related to the size of the unused spectrum (S), which is given by:

$$S(t) = \hat{S} - \omega n(t)\,. \qquad (2)$$

Consequently,

$$Q(t) = q(n(t), S(t)) = q(n(t), (\hat{S} - \omega n(t)))\,, \qquad (3)$$

where, $q(0, \hat{S}) = 0$. The direct, variety, effect of the number of channels on quality, is positive but not increasing: $q_n > 0$ and $q_{nn} \le 0$. The indirect effect of the number of channels on quality, through deteriorating reception, is negative: $-\omega q_s$, where $q_s > 0$ and (for simplicity) unchanged, $q_{ss} = 0$. Up to a critical number of channels, $\tilde{n}$ ($\tilde{n} < \hat{S}/\omega$), the positive variety effect dominates the negative interference effect: $Q_n = (q_n - \omega q_s) \overset{>}{\underset{<}{=}} 0$ for $n \overset{<}{\underset{>}{=}} \tilde{n}$.

The overall demand for broadcasts increases with quality. Consequently, the broadcasting industry's aggregate revenue from advertisements and subscription fees at any t is R(Q(t)) with $R(0) = 0$, $R_Q > 0$ and, for tractability, $R_{QQ} = 0$. Assuming that the consumers do not have favourite channels, the industry's aggregate revenue is equally distributed. The instantaneous operational cost of each channel is, for simplicity, time-invariant, c, and so also is the (foregone) normal profit attainable in other industries, π.

In sum, the change in the number of broadcasters (channels) is given by:

$$\dot{n}(t) = \phi\{[R(q(n(t), (\hat{S} - \omega n(t))))/n(t)] - [c + \pi + g(t)]\}\,. \qquad (4)$$

The royalties charged on bands reduce the above-normal profit and, subsequently, the number of broadcasters. In turn, the variety and depth of programs is reduced, but the reception of each broadcast is improved. If the former (latter) effect dominates the latter (former), the industry's overall revenue decreases (increases), the number of broadcasters is diminished (increased), and so forth. As long as the broadcasting industry is not in the optimal steady state, time-invariant royalties are not optimal. In the following sections we firstly demonstrate the role of fixed *ad hoc* royalties in the said process and then the determination of the optimal royalties.

3. STEADY STATE OF THE INDUSTRY UNDER FIXED AD HOC ROYALTIES

Recalling our assumptions, $R(0) = 0$ and the slope of the industry's revenue curve is

$$\frac{dR}{dn} = R_Q (q_n - \omega q_s) \overset{>}{\underset{<}{=}} 0 \text{ for } n \overset{<}{\underset{>}{=}} \tilde{n} \qquad (5)$$

as depicted in Figure 1 by the inverted parabola. With fixed royalties, g_0, the industry's cost function is linear in *n*, $C(t) = (c + \pi + g_0)n(t)$. The interior steady state of the industry is in the intersection between the industry's revenue curve and cost line, which, as displayed by the arrows along the horizontal axis, is asymptotically stable. The larger the speed of adjustment is, the faster the convergence of the number of OTA broadcasters to the steady state number, n^{ss}. In the extreme case of $\phi \to \infty$, adjustment to steady state is immediate. Namely, $n(t) = n^{ss}$ at any t as the immediate adjustment of the broadcasters' number exhausts the broadcasting industry's above normal profit:

$$ANP(t) = R(q(n^{ss}, (\hat{S} - \omega n^{ss}))) / n^{ss} - (c + \pi + g_0) = 0 \quad \forall t. \tag{6}$$

High sunk costs and strict regulations diminish the speed of adjustment. In the polar extreme case of no-adjustment ($\phi = 0$), the number of OTA broadcasters is time invariant, n_{NA}, and the above normal profit is a scalar, Δ, which can be positive, negative or zero. In this case,

$$R(q(n_{NA}, (\hat{S} - \omega n_{NA}))) - (c + \pi + g_0 + \Delta)n_{NA} = 0. \tag{7}$$

If, by fluke, $\Delta = 0$, the number of OTA broadcasters under no-adjustment is identical to the aforementioned steady-state number of OTA broadcasters.

To illustrate the difference in the number of OTA broadcasters under the said two polar cases, we consider some computationally convenient specifications. In particular, we specify the overall quality of the service of the OTA broadcasting industry for the consumers to be given by

$$Q(t) = n(t)[\hat{S} - \omega n(t)] \tag{8}$$

where, as indicated by Eq. (2), $\hat{S} - \omega n$ is the size of the unused spectrum set aside for broadcasting. This parabolic specification of Q reflects properties that can be resulted from the opposite effects of n on variety and reception: $Q_n = \hat{S} - 2\omega n \overset{>}{\underset{<}{=}} 0$ as $n \overset{<}{\underset{>}{=}} \hat{S}/2\omega$ and $Q_{nn} = -2\omega < 0$. Namely, the marginal effect of the number of OTA broadcasters on the overall quality of the service of the OTA broadcasting industry is initially positive but diminishing, and when the number of OTA broadcasters is larger than $\hat{S}/2\omega$ the positive effect of n on variety is dominated by the negative effect of congestion on reception.

We assume that the marginal revenue (i.e., increment in revenues from subscription fees and commercial advertisements) from the quality of the broadcasting industry service is constant (p) and, for simplicity, also time-invariant. That is, p can be interpreted as the consumers' price of broadcasts' quality. In which case, the OTA broadcasting industry revenue function is

$$R(t) = pQ(t)\,. \tag{9}$$

By substituting the right-hand side of Eq. (8) for Q into this specification of R, the industry's revenue function is

$$R(t) = p\hat{S}n(t) - p\omega n(t)^2 \tag{10}$$

and the industry's above-normal profit is

$$ANP(t) = (p\hat{S} - p\omega n(t)) - (c + \pi + g)\,. \tag{11}$$

By setting ANP to zero, the steady-state number of OTA broadcasters is

$$n^{ss} = \frac{1}{\omega}\left[\hat{S} - \frac{c + \pi + g}{p}\right]. \tag{12}$$

This steady-state number of OTA broadcasters is smaller than the number that can be accommodated, $\hat{S}/\omega$. The smaller the broadcasting service's mark-up $[p/(c + \pi + g)]$ is, the larger the difference between the number of broadcasters that can be accommodated and the steady state number. By lowering the royalties per band, the government increases the mark-up for OTA broadcasters and, in turn, their steady-state number.

By substituting (10) into (7), the number of OTA broadcasters under no-adjustment (n_{NA}) is

$$n_{NA} = \frac{1}{\omega}\left[\hat{S} - \frac{c + \pi + g + \Delta}{p}\right]. \tag{13}$$

The number of OTA broadcasters under no-adjustment with positive (negative) above normal profit is smaller (larger) than the steady-state number of OTA broadcasters. As displayed by Figure 1, these numbers are given by the intersection of the dashed lines $(c + \pi + g + \Delta)n$ with the industry total revenue curve. As the rigidities (high sunk costs and regulations on entry and exit) causing no-adjustment are moderated, convergence to steady state begins. The greater the speed of adjustment is, the faster the convergence of the number of the OTA broadcasters from n_{NA} to n^{ss}.

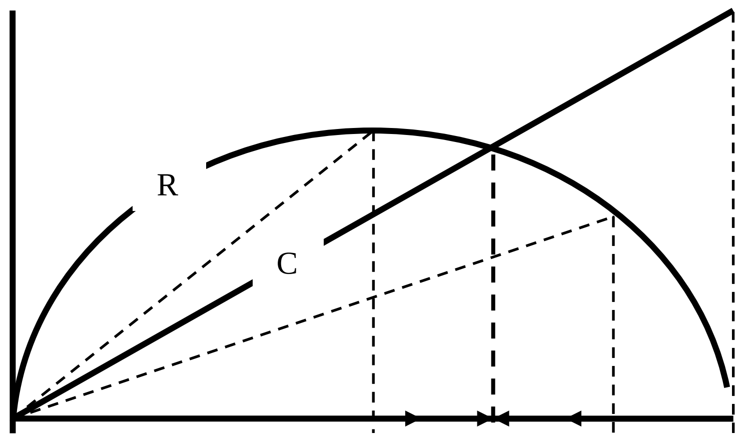

Figure 1. Number of broadcasters under fixed royalties

4. OPTIMAL CONTROL OF THE INDUSTRY

Let us now analyze a socially optimal determination of royalties and adjustment of broadcasters' number. In the proposed setting, technology is fixed, the government allows entry and exist, yet it indirectly controls the number of broadcasters by choosing the trajectory of royalties {g} per band that maximizes the consumers' lifetime utility. The royalties received by the government from the broadcasters at t, $n(t)g(t)$, are immediately directed to finance public services.

Consumers are infinitely lived and have an aggregate income, Y(t), of which R(q(t)) is spent on access to, and advertisements in, the OTA broadcasts and the remainder, Y(t)-R(q(t)), on private goods. Broadcasts have two opposing effects on aggregate income. On the one hand, they disseminate information that enhances knowledge, forms standards of performance and generates transactions. On the other hand, they divert time from work and active investment in human and social capitals. These opposing effects are intensified by the quality of the broadcasts. With $\varphi_1 > 0$ indicating the information dissemination effect, $\varphi_2 > 0$ the production-effort diversion effect of broadcasts, and $\hat{Y} > 0$ the aggregate income attainable when the said effects offset one another (or nil), the consumers' aggregate income is:

$$Y(t) = \hat{Y} + (\varphi_1 - \varphi_2)Q(t) . \quad (14)$$

With (14) and the explicit specifications (8) and (9) of the broadcasting industry's quality of service and revenue and with $\alpha > 0$ indicating the consumers' direct marginal instantaneous utility from the quality of the broadcasts and $\beta > 0$ their marginal instantaneous utility from the private goods, the consumers' instantaneous utilities from the quality of the broadcasts (u_1) and from consuming the private good (u_2) are:

$$u_1(t) = \alpha[\hat{S}n(t) - \omega n(t)^2] \tag{15}$$

and

$$u_2(t) = \beta\{\hat{Y} + (\varphi_1 - \varphi_2 - p)[\hat{S}n(t) - \omega n(t)^2]\}\,. \tag{16}$$

In addition, the consumers' derive instantaneous utility (u_3) from the investment of the spectrum's royalties, n(t)g(t), in the provision of public goods. In contrast to the constant marginal utilities assumed in (15) and (16), diminishing marginal utility from the spectrum's royalties spent on the provision of the public goods, $u_3'' < 0$, cannot be compromised for analytical simplicity since *g* is the public planner's control variable. A second order polynomial that ensures a diminishing positive marginal utility from the public goods at the steady-state financing level n*g* and reflects the consumers' dislike of deviations from that optimal stationary contribution of the spectrum's royalties to the provision of public goods is considered:

$$u_3(t) = \gamma_1[n(t)g(t)] - \gamma_2[n(t)g(t) - n^*g^*]^2 \tag{17}$$

where $\gamma_1 > \gamma_2 > 0$.[2]

With the time index omitted for compactness, the consumers' overall instantaneous utility is:[3]

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[2] Other specifications (e.g., $u_3 = \gamma\sqrt{ng},\ \gamma > 0$) were considered. The specification indicated in (17) facilitates the computation of the steady state.

[3] As there is no strong interaction between public goods' consumption and private goods' consumption, the assumed separability of the utilities derived from these consumptions is sensible. In justifying the assumed separability between the utilities generated from the use of the broadcasting service and from the rest of the private good consumption, we stress that these service and private good consumption are aggregates and that the sign of the cross derivatives of a utility function defined on these aggregates is not clear. Some components of the broadcasting service are substitute to some components of the consumer's aggregate consumption of the rest of the private goods, but complementing others. For example, watching OTA broadcast sport competitions complements the use of some household's facilities and utilities and the consumption of home-made food's ingredients, but substitutes attendance of sport competitions, use of transportation and stadium related services and consumption of fast food. For this reason, and for tractability, the cross derivative is assumed to be nil.

$$u = u_1 + u_2 + u_3$$
$$= \alpha(\hat{S}n - \omega n^2) + \beta[\hat{Y} + (\varphi_1 - \varphi_2 - p)(\hat{S}n - \omega n^2)] + \gamma_1[ng] - \gamma_2[ng - n^* g^*]^2. \quad (18)$$

The public planner's decision-problem is postulated to be choosing the trajectory of royalties that maximizes $\int_0^\infty e^{-\rho t} u(t)dt$ subject to:

$$\dot{n} = \phi[pn(\hat{S} - \omega n)/n - (c + g + \pi)]. \quad (21)$$

The current value Hamiltonian associated with this problem is

$$H = \beta\hat{Y} + [\alpha + \beta(\varphi_1 - \varphi_2 - p)](\hat{S}n - \omega n^2) + \gamma_1[ng] - \gamma_2[ng - n^* g^*]^2$$
$$+ \lambda\phi[p(\hat{S} - \omega n) - c - g - \pi]. \quad (22)$$

The co-state variable, λ, reflects the public planner's current shadow value of broadcasts' variety and competition. The Hamiltonian is concave in the control variable. As long as $2[\alpha + \beta(\varphi_1 - \varphi_2 - p)]\omega + 2\gamma_2 g^2 > 0$, it is also concave in the state variable. In which case, the following Pontryagin's maximum-principle conditions are, by the Mangasarian (1966) theorem, sufficient:

$$\dot{\lambda} = -H_n + \rho\lambda = -[\alpha + \beta(\varphi_1 - \varphi_2 - p)](\hat{S} - 2\omega n)$$
$$-[\gamma_1 - 2\gamma_2(ng - n^* g^*)]g + (\phi p\omega + \rho)\lambda \quad (23)$$

$$H_g = [\gamma_1 - 2\gamma_2(ng - n^* g^*)]n - \lambda\phi = 0 \quad (24)$$

$$\dot{n} = \phi[p(\hat{S} - \omega n) - c - g - \pi] \quad (25)$$

$$\lim_{t\to\infty} e^{-\rho t} H(t) = 0. \quad (26)$$

By differentiating (24) with respect to time, substituting (23) for $\dot{\lambda}$ and (24) for λ and rearranging terms, the Euler equation for the optimal change in royalties on bands is:

$$\dot{g} = \frac{\phi[\alpha + \beta(\varphi_1 - \varphi_2 - p)](\hat{S} - 2\omega n) + [\gamma_1 - 2\gamma_2(ng - n^* g^*)][\phi g - (\phi p\omega + \rho)n]}{2\gamma_2 n^2}$$
$$+ \frac{[\gamma_1 - 2\gamma_2(2ng - n^* g^*)]}{2\gamma_2 n^2}\dot{n} \quad .(27)$$

The change in the optimal royalties is moderated by the public planner's rate of time preference and the broadcasters' marginal return on the quality of programs. The change in the optimal royalties rises with the consumers' marginal direct utility from the broadcasts and with the net effect of the broadcasts on the consumers' aggregate income ($\varphi_1 - \varphi_2$), proportionally to the product of the consumers' marginal utility from the private goods and the available broadcasting spectrum.

5. INDUSTRY'S OPTIMAL STEADY STATE

From the necessary conditions (23), (24) and (25) we obtain that in steady state:

$$-[\alpha+\beta(\varphi_1-\varphi_2-p)](\hat{S}-2\omega n^*)-\gamma_1 g^*+(\phi p\omega+\rho)\lambda^*=0 \quad (27)$$

$$\lambda^*=(\gamma_1/\phi)n^* \quad (28)$$

$$g^*=p(\hat{S}-\omega n^*)-c-\pi . \quad (29)$$

By substituting (28) and (29) into (27) the steady-state number of broadcasters is:

$$n^*=\frac{[\alpha+\beta(\varphi_1-\varphi_2-p)+\gamma_1 p]\hat{S}}{2[\alpha+\beta(\varphi_1-\varphi_2-p)]\omega+\gamma_1[2p\omega+\rho/\phi]} . \quad (30)$$

Corollary 1. The steady-state number of broadcasters rises with the consumers' marginal direct utility from broadcasts.

Proof: $\dfrac{\partial n^*}{\partial\alpha}=\dfrac{\gamma_1(\rho/\phi)\hat{S}}{\{2[\alpha+\beta(\varphi_1-\varphi_2-p)]\omega+\gamma_1[2p\omega+\rho/\phi]\}^2}>0$.

Corollary 2. If the broadcasts' information-dissemination effect on the consumers' aggregate income (φ_1) dominates (is dominated by) the sum of the production-effort-diversion effect (φ_2) and the consumers' price of broadcasts' quality (p), then the steady-state number of broadcasters increases (decreases) with the consumers' marginal utility from the other private goods.

Proof: $\dfrac{\partial n^*}{\partial\beta}=\dfrac{\gamma_1(\rho/\phi)(\varphi_1-\varphi_2-p)\hat{S}}{\{2[\alpha+\beta(\varphi_1-\varphi_2-p)]\omega+\gamma_1[2p\omega+\rho/\phi]\}^2}\overset{>}{\underset{<}{=}}0$ as $(\varphi_1-\varphi_2-p)\overset{>}{\underset{<}{=}}0$.

Corollary 3. If the broadcasts' information-dissemination effect on the consumers' aggregate income dominates (is dominated by) the sum of the production-effort-diversion effect and the consumers' price of broadcasts' quality, then the steady-state number of broadcasters decreases (increases) with the consumers' marginal utility from the public goods financed by the spectrum royalties:

Proof: $\dfrac{\partial n^*}{\partial\gamma_1}=\dfrac{-(\rho/\phi)[\alpha+\beta(\varphi_1-\varphi_2-p)]\hat{S}}{\{2[\alpha+\beta(\varphi_1-\varphi_2-p)]\omega+\gamma_1[2p\omega+\rho/\phi]\}^2}\overset{<}{\underset{>}{=}}0$ as $(\varphi_1-\varphi_2-p)\overset{>}{\underset{<}{=}}0$.

Corollary 4. The steady-state number of broadcasters rises with the broadcasts' information-dissemination effect on the consumers' aggregate income.

Proof: $\dfrac{\partial n^*}{\partial\varphi_1}=\dfrac{\beta\gamma_1(\rho/\phi)\hat{S}}{\{2[\alpha+\beta(\varphi_1-\varphi_2-p)]\omega+\gamma_1[2p\omega+\rho/\phi]\}^2}>0$.

Corollary 5. The steady-state number of broadcasters decreases with the broadcasts' production-effort-diversion effect on the consumers' aggregate income.

Proof: $\frac{\partial n^*}{\partial \varphi_2} = \frac{-\beta\gamma_1(\rho/\phi)\hat{S}}{\{2[\alpha+\beta(\varphi_1-\varphi_2-p)]\omega+\gamma_1[2p\omega+\rho/\phi]\}^2} < 0$.

Corollary 6. If the public planner's rate of time preference is larger (smaller) than $2\omega\phi[\varphi_2+(1-\gamma_1)p-\alpha-\beta\varphi_1]/\beta$, then the steady-state number of broadcasters decreases (increases) with the consumers' price of broadcasts' quality.

Proof: $\frac{\partial n^*}{\partial p} = \frac{-\gamma_1\{\beta(\rho/\phi)+2\omega[\alpha+\beta(\varphi_1-\varphi_2-p)+\gamma_1 p]\}\hat{S}}{\{2[\alpha+\beta(\varphi_1-\varphi_2-p)]\omega+\gamma_1[2p\omega+\rho/\phi]\}^2} \overset{<}{\underset{>}{=}} 0$

as $\beta(\rho/\phi)+2\omega[\alpha+\beta\varphi_1+\gamma_1 p] \overset{>}{\underset{<}{=}} 2\omega(\varphi_2+p)$, which by rearrangement can be displayed as $\rho \overset{>}{\underset{<}{=}} 2\omega\phi[\varphi_2+(1-\gamma_1)p-\alpha-\beta\varphi_1]/\beta$.

Corollary 7. The steady-state number of broadcasters decreases with the band's width:

$$\frac{\partial n^*}{\partial \omega} = \frac{-2[\alpha+\beta(\varphi_1-\varphi_2-p)+\gamma_1 p]^2\hat{S}}{\{2[\alpha+\beta(\varphi_1-\varphi_2-p)]\omega+\gamma_1[2p\omega+\rho/\phi]\}^2} < 0$$

Corollary 8. If the consumers' price of broadcasts' quality is larger (smaller) than $[\beta(\varphi_2+p-\varphi_1)-\alpha]/\gamma_1$ (the consumers' net loss of utility from private consumption due to a marginal improvement in the broadcasts' quality, deflated by the consumers' marginal utility from the spectrum's royalties), then the steady-state number of broadcasters decreases (increases) with the public planner's rate of time preferences.

Proof: $\frac{\partial n^*}{\partial \rho} = \frac{-(\gamma_1/\phi)[\alpha+\beta(\varphi_1-\varphi_2-p)+\gamma_1 p]\hat{S}}{\{2[\alpha+\beta(\varphi_1-\varphi_2-p)]\omega+\gamma_1[2p\omega+\rho/\phi]\}^2} \overset{<}{\underset{>}{=}} 0$

as $\alpha+\beta\varphi_1+\gamma_1 p \overset{>}{\underset{<}{=}} \beta(\varphi_2+p)$, which by rearrangement can be displayed as $p \overset{>}{\underset{<}{=}} [\beta(\varphi_2+p-\varphi_1)-\alpha]/\gamma_1$. The

Corollary 9. If the marginal revenue from the broadcasts' quality is larger (smaller) than $[\beta(\varphi_2+p-\varphi_1)-\alpha]/\gamma_1$, then the steady-state number of broadcasters increases (decreases) with the speed of adjustment to the broadcast industry's above normal profit.

Proof: $\frac{\partial n^*}{\partial \phi} = \frac{\gamma_1 \rho[\alpha + \beta(\varphi_1 - \varphi_2 - p) + \gamma_1 p]\hat{S}}{\{2\phi[\alpha + \beta(\varphi_1 - \varphi_2 - p)]\omega + \gamma_1[2p\omega + \rho/\phi]\}^2} \gtreqless 0$

as $\alpha + \beta\varphi_1 + \gamma_1 p \gtreqless \beta(\varphi_2 + p)$, which by rearrangement can be displayed as

$p \gtreqless [\beta(\varphi_2 + p - \varphi_1) - \alpha]/\gamma_1$.

Recalling that entry and exist are deterred by sunk costs, the steady-state number of broadcasters decreases (increases) with the broadcasters' sunk costs if the consumers' price of broadcasts' quality is larger (smaller) than the consumers' net loss of utility from private consumption due to a marginal improvement in the broadcasts' quality, deflated by the consumers' marginal utility from the public goods financed by the spectrum's royalties.

Corollary 10. The steady-state number of broadcasters decreases with the size of the spectrum allocated to broadcasting, but otherwise decreases, if the difference between the information-dissemination effect and the production-effort-diversion effect on the consumers' aggregate income satisfies the following inequality:

$$p - \frac{1}{\beta}\max\{(\alpha + \gamma_1 p), \left(\frac{\alpha}{\omega} + \gamma_1 p + \frac{0.5\rho}{\phi\omega}\right)\} < (\varphi_1 - \varphi_2) < p - \frac{1}{\beta}\min\{(\alpha + \gamma_1 p), \left(\frac{\alpha}{\omega} + \gamma_1 p + \frac{0.5\rho}{\phi\omega}\right)\}.$$

Proof: $\frac{\partial n^*}{\partial \hat{S}} = \frac{[\alpha + \beta(\varphi_1 - \varphi_2 - p) + \gamma_1 p]}{2[\alpha + \beta(\varphi_1 - \varphi_2 - p)]\omega + \gamma_1[2p\omega + \rho/\phi]}$. The numerator and denominator have the same sign only when the said inequality is satisfied.

From equation (29), the directions of the effects of the model parameters on the steady-state royalties on bands (g*) are opposite to those on the steady-state number of broadcasters.

6. STABILITY OF THE OPTIMAL STEADY STATE

For assessing the stability of the steady state (n*,g*) we evaluate the Jacobian of the differential equation system (25) and (27) in steady state

$$\begin{bmatrix} \dot{n}_n^* & \dot{n}_g^* \\ \dot{g}_n^* & \dot{g}_g^* \end{bmatrix} = \begin{bmatrix} -\phi p\omega & -\phi \\ \frac{2\omega\phi[\gamma_2 g^*(2pn^* - g^*/\omega) - \alpha - \beta(\varphi_1 - \varphi_2 - p) - \gamma_1 p] + (2\gamma_2 g^* n^* - \gamma_1)\rho}{\gamma_2 n^{*3}} & \{\phi[p\omega - g^*/n^*] + \rho\} \end{bmatrix} \quad (31)$$

and its eigenvalues

$$\mu_{1,2} = 0.5[\rho - \phi(g^*/n^*)] \pm 0.5\sqrt{[\rho - \phi(g^*/n^*)]^2 - 4\phi\left(\dot{g}_n^* - \phi p\omega[(p\omega - g^*/n^*) + \rho]\right)} . \quad (32)$$

An inspection of equation (32) suggests that if the public planner's rate of time preference and the broadcasters' sunk costs are sufficiently high, the Jacobian's trace in steady state, $[\rho - \phi(g^*/n^*)]$, might be positive. In which case, the steady state is not asymptotically stable. This argument is based on dominant direct effects of the public planner's rate of time preference and the broadcasters' sunk costs on $[\rho - \phi(g^*/n^*)]$. Their indirect effects on $[\rho - \phi(g^*/n^*)]$ through g^*/n^* are not clear. From equation (29), $g^*/n^* = [(p\hat{S} - c - \pi)/n^*] - p\omega$. Recalling corollaries 8 and 9, $[(p\hat{S} - c - \pi)/n^*] - p\omega$ increases (decreases) with the public planner's rate of time preference, but decreases (increases) with the broadcasters' sunk costs, if the marginal revenue from the broadcasts' quality is larger (smaller) than $[\beta(\varphi_2 + p - \varphi_1) - \alpha]/\gamma_1$. Yet even with $\rho - \phi(g^*/n^*) > 0$, there can be convergence to steady state if $\Omega \equiv \dot{g}_n^* - \phi p\omega[(p\omega - g^*/n^*) + \rho] < 0$. In this case, $\mu_1 > 0$ and $\mu_2 < 0$ and the steady state is a saddle point. By appropriate setting of the royalties, the public planner can let the broadcasting industry gradually approach the steady state along the initially nearest arm of the single stable manifold. This case is illustrated by Figure 2. While it is clear that the slope of the isoclines $\dot{n} = 0$ is negative $(-p\omega)$, the slope of the isoclines $\dot{g} = 0$ is not clear:

$$\frac{dg}{dn}(\dot{g} = 0) = -\frac{2\omega\phi[\gamma_2 g^*(2pn^* - g^*/\omega) - \alpha - \beta(\varphi_1 - \varphi_2 - p) - \gamma_1 p] + (2\gamma_2 g^* n^* - \gamma_1)\rho}{\gamma_2 n^{*3}\{\phi[p\omega - g^*/n^*] + \rho\}} . \quad (33)$$

As can be seen from this expression, with ρ being large the positive slope of the isoclines $\dot{n} = 0$ portrayed in Figure 2 is supported by a large marginal utility from the public good in steady state: $\gamma_1 > 2\gamma_2 g^* n^*$. A positive slope is also supported by a large direct marginal utility from the quality of the broadcast (α) and an information-dissemination effect that dominates the effort-diversion effect of broadcasts on the consumers' aggregate income ($\varphi_1 > \varphi_2$). Starting from a highly regulated industry with a small number of broadcasters (n_0), the left upward sloped converging arm is relevant for the public planner. Along this arm the number of broadcasters increases gradually despite the rising royalties.

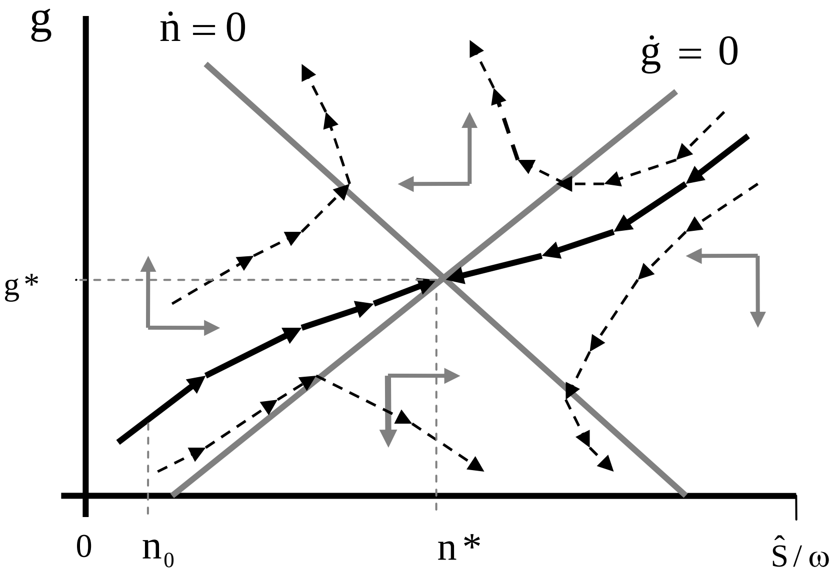

Figure 2. An unstable steady state — a saddle point

In contrast, if the public planner's rate of time preference and the broadcasters' sunk costs are sufficiently low so that $\rho - \phi(g^*/n^*) < 0$, and if $\Omega \equiv \dot{g}_n^* - \phi p\omega[(p\omega - g^*/n^*) + \rho] > 0$, then the steady state (n*,g*) is asymptotically stable. If, in addition, $[\rho - \phi(g^*/n^*)]^2 < 04\phi\Omega$, the steady state is note a node and the industry's optimal trajectory to the steady state is characterised by damped oscillations of the number of broadcasters and royalties, as displayed in Figure 3.

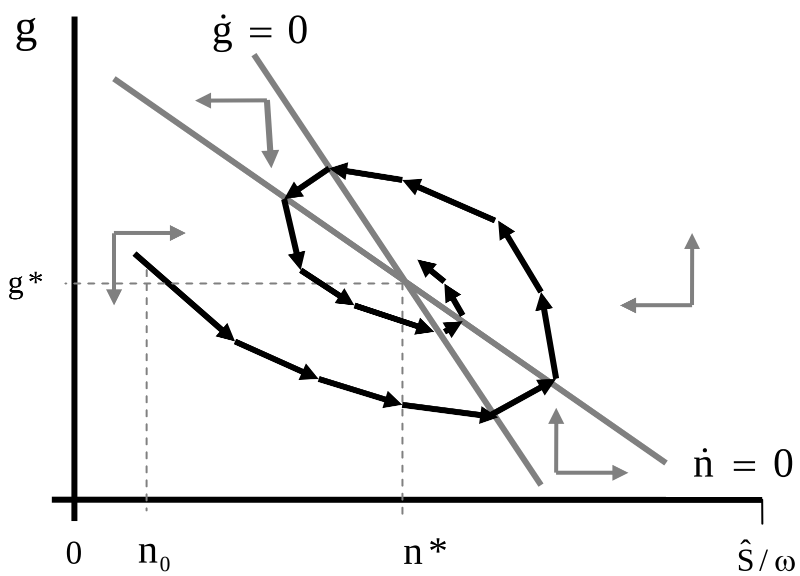

Figure 3. A stable steady state approached along a spiral

7. CONCLUSION

The advent of digital transmission technologies has done little to relieve constraints on the amount of spectrum allocated to the broadcasting industry. The perennial tradeoff between variety and reception still prevails. This tradeoff is likely to be most prominent under a deregulatory scheme. Spectrum is a state-owned, time-invariant, scarce natural resource. As in the case of any other state-owned natural resource, governments are entitled to charge royalties on its use and can direct these revenues to finance public services. Therefore, in addition to the direct benefits from the service provided by the broadcasting industry, the indirect benefits to consumers from the public services financed by the royalties on this natural resource were taken into account in the determination of the socially optimal allocation of bands to broadcasters. For setting the state's royalties on spectrum, we proposed an optimal control model that takes into account the aforesaid aspects, entry and exit of broadcasters in accordance with above-normal profit and at a rate moderated by sunk costs, and a possible positive and negative effects of broadcasts on consumers' income. We derived the socially optimal steady-state of the broadcasting industry and its comparative statics and analyzed the possibility of convergence to this steady state. In contrast to the observed consolidation and return to concentration in the aftermath of deregulatory reforms in the United States, Italy, Germany and Japan, our analysis reveals that optimal control of the broadcasting industry with variable royalties on bands can gradually lead the industry to a steady state with a larger number of broadcasters. Extensions of the analysis may consider the alternative usages of the spectrum and determine the optimal portion of the spectrum allocated to OTA broadcasts and the optimal band-width.

REFERENCES

Adda, J., and Ottaviani, M. (2005), "The Transition to Digital Television", *Economic Policy*, 20(41), 160-209.

Caves, R. E. (2006), *Switching Channels: Organization and Change in TV Broadcasting*, Cambridge and London: Harvard University Press.

CEPT. (2010), "Future Possibilities for the Digitalisation of Band II (87.5 - 108 MHz)", Electronic Communications Committee (ECC) of the European Conference of the Postal and Telecommunications Administrations (CEPT), St. Petersburg, May 2010.

De Vany, A. (1998), "Implementing a Market-based Spectrum policy", *Journal of Law and Economics*, 41(2), 627-46.

Hazlet, T. W. (2005), "Cable Television", in S. Majumdar, V. Ingo, and M. Cave (eds.) *Handbook of Telecommunications Economics*, Amsterdam: Elsevier, North Holland.

Hazlett, T. W. (2008), "Optimal Abolition of FCC Spectrum Allocation", *Journal of Economic Perspectives*, 22(1), 103-128.

Mangasarian, O.L. (1966), “Sufficient Conditions for the Optimal Control of Nonlinear Systems”, *SIAM Journal Control* 4, 1966: 139-152.

Motta, M., and Polo, M. (1997), "Concentration and Public Policies in the Broadcasting Industry: The Future of Television", *Economic Policy*, 12(25), 295-334.

Noam, E. M. (1992), *Television in Europe*, New York: Oxford University Press.

Webbink, D. W. (1973), "Regulation, Profits and Entry in the Television Broadcasting Industry", *Journal of Industrial Economics*, 21(2), 167-76.