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with Varying Carrying Capacity and Concerns**

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# An Analysis of the World's Environment and Population with Varying Carrying Capacity and Concerns\*

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## Abstract

Due to the open-access nature of the environment we consider an *ad hoc* adjustment of people's footprints. People's environmental concerns are intensified (diminished) as the quality of the environment falls below (rises above) a threshold. Changes in the quality of the environment affect Earth's carrying capacity. We claim that without technological, social and international progress the interplay between the non-optimally changing environmental concerns and carrying capacity embarks the world's environment and human population on a clockwise oscillating course that leads to a unique interior steady state with population similar to the current one residing in a slightly more degraded environment. (*JEL* O13, Q20)

**Keywords:** Environment; Population; Dynamics; Carrying Capacity; Environmental Concerns

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## 1. Introduction

Since the beginning of the industrial revolution the world's population has grown from less than a billion to almost seven billion. Accompanied by changes in per capita income, life-expectancy, preferences, technology and production scale and composition, this population growth has intensified the pressure on the natural environment and its resources. In turn, the environmental degradation has raised concerns for the state of the planet and its future suitability for life. Whether the conflict between the exploitation of the environment and concerns for the environment will be resolved in an uninhabitable planet has been debated since the publication of Thomas Robert Malthus' first essay on the principle of population in 1798. We attempt to contribute to this debate by constructing, analyzing and empirically assessing a population-environment system in which Earth's carrying capacity decreases and people's concerns rise and moderate their footprint when the environment is degraded. We motivate our approach with a brief literature review focused on two seminal studies that reach different conclusions.

In *Limits to Growth* — a study of the consequences of a rapidly growing world population commissioned by the Club of Rome — Donella H. Meadows, Dennis L. Meadows, Jørgen Randers and William W. Behrens III (1972) come to the conclusion that output-growth would likely not be impeded by lack of resources before it was impeded by pollution. Missing in their simulation model of the world is a link between pollution and pollution prevention. The rationale for such a link is growing concerns. Indeed, analyses of the Health of the Planet Survey, the World Values Survey and the International Social Survey Program indicate that during the last twenty years concerns for the environment have not only intensified in rich countries, as advocated by the affluence hypothesis (Diekmann and Franzen, 1999; Franzen, 2003), but also in poor ones (Inglehart, 1995, 1997; Dunlap, Gallup and Gallup, 1993; Dunlap and Mertig, 1997). Supporting arguments and evidence of rising environmental concerns are also presented in studies of the Environmental Kuznets Curve (Shafik and Bandyopadhyay, 1992; Selden and Song, 1994; Grossman and Krueger, 1995; Andreoni and Levinson, 2001; Chavas, 2004).

In *A Question of Balance* William D. Nordhaus (2008) provides an integrated assessment model for global warming by elaborately incorporating cost-benefit aspects of abatement of greenhouse gas emissions into Frank P. Ramsey's (1928) model of optimal economic growth. Unlike Meadows *et al.* (1972), his DICE model

has a feedback loop between the atmospheric carbon dioxide and abatement activities. With optimal aggregate feedback and the modest abatement costs estimated in the Intergovernmental Panel on Climate Change's Assessment Reports, environmental catastrophe is not predicted. However, as admitted by William D. Nordhaus' (1992) use of expressions such as "*idealized competitive markets*" and "*major leap of faith*" (p. 7, second paragraph), optimal aggregate emission abatement is neither a market realization nor a likely outcome of international negotiations.

The Earth's atmosphere and much of the contents of the Earth's surface and crust do not have the property of exclusivity: they belong to everyone and no one. Lack of exclusivity encourages free riding in sharing the costs of abatement activities. The larger the costs of abatement activities the stronger is the inclination to free ride. Recalling Robert Mendelsohn's (2008) arguments, the full costs of abatement activities are not modest. Thus, the real system of the environment and human population does not have an optimal feedback.

In the following sections we conduct a theoretical and empirical investigation of the possible joint course of the environment and human population and its implications for survival within an analytically manageable *ad hoc* model of the environment and population with varying carrying capacity and environmental concerns. We treat the whole biosphere as an open-access resource and construct, in Section 2, a modified Lotka-Volterra model of the environment and population. A Lotka-Volterra model is also used by James A. Brander and Scott M. Taylor (1998) for explaining the growth and decline of an early civilization whose essential renewable natural resources had been subjected to open-access harvesting. Our version includes logistic regeneration of the environment and population and takes the environment as limiting the carrying capacity for people and concerns as moderating environmental degradation. Both the number of people and people's choice on how much care to take of the environment, their environmental footprint, also determine the change in the environment. Earth's carrying capacity declines as the environment deteriorates and the intensity of the feedback is associated with the human population's aggregate level of environmental concerns. We regard people as reacting to environmental degradation by decreasing their individual exploitation of the environment.

The model's phase-plane analysis, in Section 3 and Section 4, highlights the interplay between carrying capacity and environmental concerns in shaping the joint

course of the environment and population. The investigation of the model's dynamic properties, in Section 4, and their empirical assessment, in Section 5, suggest that extinction, or convergence to a much smaller population and a much lower environmental quality, are not internally generated scenarios for the human race.

## 2. Model

The model comprises the motion equations of the physical environment and human population. In view of the objective of our investigation these motion equations are taken to be deterministic — shocks (such as solar plasma bursts, volcanic eruptions, asteroid impact, nuclear accidents and epidemics) are ignored. While the size of Earth's physical environment is roughly fixed, the quality of Earth's environment (defined as the suitability of Earth's environment for human life) may vary over time. We denote Earth's quality adjusted physical environment at time  $t$  by  $E(t) \geq 0$  and the population of human beings by  $P(t) \geq 0$ .

The state of the environment is controlled by its natural regeneration,  $G_e(t)$ , and human exploitation. We assume that the physical environment is naturally regenerated as a logistic function of its current state. The regeneration function depends upon an intrinsic growth rate,  $g_e$ , and a maximal quality adjusted physical environment,  $E_{\max}$ ,

$$(1) \quad G_e(t) = g_e E(t) \left( 1 - \frac{E(t)}{E_{\max}} \right).$$

People's exploitation, their environmental footprint, depends both on the state of the environment and on the level of human population. The weaker the people's concerns for the physical environment, *ceteris paribus*, the larger their production and consumption footprints on the physical environment. People are quality responsive: as the environment deteriorates, awareness of, and, in turn, concerns for, the state of the environment are intensified. We model people's response to the state of the environment with a complacency threshold: a quality adjusted physical environment,  $E_{\text{comp}}$  ( $E_{\text{comp}} < E_{\max}$ ), above (below) which the individual footprint (*IFP*) on the environment is larger (smaller) than a positive scalar  $\beta$ . The higher the costs of abatement the larger is  $\beta$ . We refer to  $\beta$  as the footprint-complacency coefficient. This feedback is represented by the following *ad hoc* behavioral rule

$$(2) \quad IFP(t) = \beta \frac{E(t)}{E_{\text{comp}}}.$$

Since there are  $P$  people (identical, for tractability), each detracting  $IFP$  from the environmental stock, the change in the quality adjusted physical environment is

$$(3) \quad \dot{E}(t) = G_e(t) - IFP(t)P(t) = g_e E(t) \left( 1 - \frac{E(t)}{E_{\max}} \right) - \beta \frac{E(t)}{E_{\text{comp}}} P(t).$$

Next we turn to the equation describing population growth and its relation to the environment. Due to the fixed size of Earth's physical environment, a carrying capacity is incorporated into the formalization of the human population growth. Studies of wildlife population's survival and management typically employ growth functions embodying fixed, exogenously determined carrying capacity (Clark, 1976; Berck, 1979; Berck and Perloff, 1984; Horan and Bulte, 2004). Unlike wildlife, humans' impact on Earth's carrying capacity is significant. We assume that humans cannot live in a quality adjusted physical environment  $E_{\text{ext}}$  and lower. We refer to  $E_{\text{ext}}$  as the extinction threshold. We further assume that at any point in time the physical environment's capacity to carry humans,  $\hat{P}(t)$ , rises with the current deviation of the quality adjusted physical environment from the extinction threshold. For instance, higher environmental quality in the form of lower greenhouse-gas concentrations results in higher potential food production. The carrying capacity is also influenced by technology, healthcare, social interaction and international relations, which we model as an exogenous function of time. For instance, both peace and property rights contribute to capital formation, production and marketing. Consequently, we specify the physical environment's capacity to carry humans as

$$(4) \quad \hat{P}(t) = (\alpha + \gamma t)[E(t) - E_{\text{ext}}]$$

where  $\alpha > 0$  and  $\gamma \geq 0$  are scalars. The term  $(\alpha + \gamma t) > 0$  is the ratio of the maximum sustainable human population to the level of the environment above the extinction threshold. A continuous overall technological, healthcare, social and international progress is depicted by  $\gamma > 0$ , whereas stagnation is represented by  $\gamma = 0$ . Though not considered in this paper,  $\gamma < 0$  is possible. In particular, international relations might deteriorate to a destructive conflict that more than offsets the carrying-capacity gains from improvements in production and healthcare technologies. The multiplicative specification reflects that, even in the presence of a continuous combined progress, the carrying capacity of Earth might decline as the physical environment deteriorates and vanishes when the extinction threshold is reached. By

incorporating this specification of the carrying capacity into a logistic growth function,  $g_p P(t)[1 - P(t)/\hat{P}(t)]$ , the motion-equation of the human population is

$$(5) \quad \dot{P}(t) = g_p P(t) \left( 1 - \frac{P(t)}{(\alpha + \gamma t)[E(t) - E_{ext}]} \right)$$

where  $g_p$  is a positive scalar indicating the human population's intrinsic growth rate.

The motion equations (3) and (5) constitute a model of the environment and population. A continuous combined process of technological, healthcare, social and international relation improvements ( $\gamma > 0$ ) renders this differential equation-system non-autonomous and hence precludes interior steady states. We ask whether such a multi-facet progress also prevents a corner steady state – uninhabitable planet. We claim that coupled with diminishing complacency it does. We support this claim by demonstrating, in the following sections, that even in the absence of future progress and as long as there is no regression (namely,  $\gamma = 0$ ) the quality adjusted physical environment does not converge to  $E_{ext}$  and the human population is not driven to extinction.

### 3. Unique, Interior Steady State

Recalling equations (3) and (5) and assuming that  $\gamma = 0$ , the isocline  $\dot{E} = 0$  is given by  $E = E_{\max} - [(\beta E_{\max}) / (g_e E_{comp})]P$  and the isocline  $\dot{P} = 0$  by  $E = E_{ext} + (1/\alpha)P$ . Since the intercept of the negatively sloped isocline  $\dot{E} = 0$  is larger than the intercept of the positively sloped isocline  $\dot{P} = 0$  these linear isoclines intersect one another once, and their intersection point is in the positive orthant of the  $P - E$  plane. Namely, in the absence of further technological, healthcare, social and international progress, or regression, the environment-population system has a unique, interior steady state. The steady-state quality adjusted physical environment is

$$(6) \quad E^* = E_{ext} + \frac{1}{\alpha} \left( \frac{E_{\max} - E_{ext}}{\frac{1}{\alpha} + \frac{\beta / E_{comp}}{g_e / E_{\max}}} \right)$$

and the steady-state human population is

$$(7) \quad P^* = \frac{E_{\max} - E_{\text{ext}}}{\frac{1}{\alpha} + \frac{\beta / E_{\text{comp}}}{g_e / E_{\max}}}$$

Equations (6) and (7) suggest that as long as a lack of progress is not accompanied by absolute complacency ( $E_{\text{comp}} = 0$ ) the steady-state quality adjusted physical environment is higher than the extinction threshold ( $E_{\text{ext}}$ ) and, consequently, the stationary human population is not nil. The higher the population's complacency threshold ( $E_{\text{comp}}$ ), the more distant the steady-state quality of the physical environment is from the extinction threshold and, due to a greater carrying capacity, the larger is the stationary population of human beings. These equations also suggest that the stationary population and the steady-state quality adjusted physical environment increase with the environment's intrinsic recovery rate ( $g_e$ ) and the maximal quality adjusted physical environment ( $E_{\max}$ ), and decrease with the footprint-complacency coefficient ( $\beta$ ). The steady-state population also decreases with the extinction threshold ( $E_{\text{ext}}$ ). The steady-state population further decreases with the stock of the quality adjusted extra (beyond  $E_{\text{ext}}$ ) environmental resources required for sustaining a human being under perpetual stagnation ( $1/\alpha$ ). As the subsequent positive effect of the population decline on the stationary quality of the environment can be dominated by the larger per capita requirement of environmental stock,  $\partial(E^* - E_{\text{ext}})/\partial(1/\alpha) = \{1 - 1/[1/\alpha + \beta E_{\max} / g_e E_{\text{comp}}]\}P^*$  is not necessarily positive.

#### 4. Local and possibly global convergence and no extinction

We argue that changing carrying capacity and environmental concerns are likely to engender a cyclical environment-population course that convergence to the steady state. The underlying rationale is as follows. With the quality of the environment being initially high, excess carrying capacity is large and concerns for the environment are low. Hence, population grows rapidly and so also does its aggregate footprint. As the environment deteriorates the excess carrying capacity diminishes and, in turn, population growth decelerates. At the same time, concerns for the environment rise. Negative population growth and rising concerns moderate the aggregate footprint and, subsequently, the environment starts improving. As the

environment gradually improves, carrying capacity is slightly increased. Population growth is resumed and is accompanied for a while by moderated concerns. Then, with a bit larger aggregate footprint the environment slightly deteriorates, population growth diminishes and concerns rise, and so on, with gradual convergence to steady state.

A formal identification of the joint course of the environment and human population in the neighborhood of the steady state requires an evaluation of the Jacobian of the motion-equations (3) and (5) with  $\gamma = 0$  in the steady state indicated by (6) and (7),<sup>1</sup>

$$(8) \quad J = \begin{bmatrix} \partial \dot{E}^* / \partial E & \partial \dot{E}^* / \partial P \\ \partial \dot{P}^* / \partial E & \partial \dot{P}^* / \partial P \end{bmatrix} = \begin{bmatrix} -\frac{g_e E^*}{E_{\max}} & -\frac{\beta E^*}{E_{\text{comp}}} \\ \alpha g_p & -g_p \end{bmatrix}.$$

The characteristic roots of this Jacobian are

$$(9) \quad \lambda_{1,2} = \frac{1}{2} \left\{ -\left[ g_p + \frac{g_e E^*}{E_{\max}} \right] \pm \sqrt{\left[ g_p + \frac{g_e E^*}{E_{\max}} \right]^2 - 4 g_p E^* \left[ \frac{g_e}{E_{\max}} + \frac{\alpha \beta}{E_{\text{comp}}} \right]} \right\}.$$

The real part of both eigenvalues is negative because the trace of J is negative and the discriminant is smaller than the trace squared. The discriminant can be either sign, so the roots can be either two negative real roots or a complex conjugate pair with a negative real part. Therefore, the population and the environment converge either directly or in an inward spiral to the steady state.

We can also show global properties with a phase-plane diagram, Figure 1. Since  $\partial \dot{E} / \partial P = -\beta E / E_{\text{comp}} < 0$ , the vertical arrows in the phases above (below) the isocline  $\dot{E} = 0$  point downward (upward). As  $\partial \dot{P} / \partial E = g_p \{ P / [\alpha(E - E_{\text{ext}})] \}^2 > 0$ , the horizontal arrows point rightward (leftward) in the phases above (below) the isocline  $\dot{P} = 0$ . The phase plane and one additional argument show the global properties of the environment-population system.

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<sup>1</sup>  $\dot{E}_E = g_e - 2g_e E^* / E_{\max} - \beta P^* / E_{\text{comp}}$ . Note that  $\dot{E} = 0$  implies  $\beta P^* / E_{\text{comp}} = g_e [1 - E^* / E_{\max}]$ , which by substitution into  $\dot{E}_E$  in turn implies  $\dot{E}_E = g_e - 2g_e E^* / E_{\max} - g_e + g_e E^* / E_{\max} = -g_e E^* / E_{\max}$ . Recalling that  $E^* = E_{\text{ext}} + (1/\alpha)P^*$ ,  $\partial \dot{P} / \partial E = \alpha g_p P^{*2} / [\alpha(E^* - E_{\text{ext}})]^2 = \alpha g_p$  and  $\partial \dot{P} / \partial P = g_p - 2g_p P^* / [\alpha(E^* - E_{\text{ext}})] = -g_p$ .

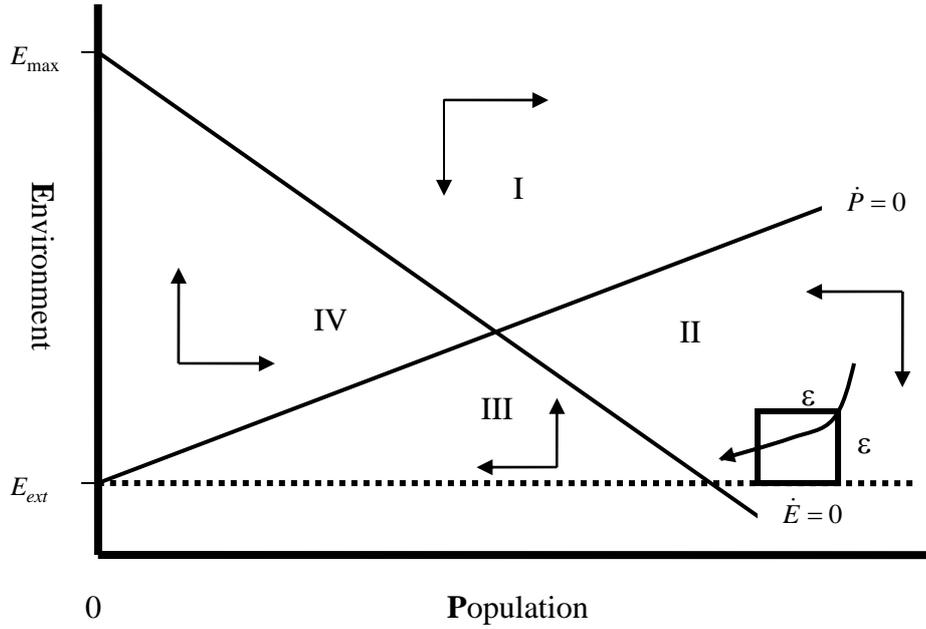


FIGURE 1. PHASE PLANE WITH NO EXTINCTION

The phase-plane diagram includes a dotted line at  $E = E_{\text{ext}}$ . We show that if  $E$  begins above  $E_{\text{ext}}$  it never reaches  $E_{\text{ext}}$ . From the diagram we see that  $E$  potentially reaches  $E_{\text{ext}}$  only in phase II. We have drawn a square of size  $\varepsilon$  along the dotted line and cornered on a possible population-environment path in that phase. For  $E$  to reach  $E_{\text{ext}}$  it must hit the bottom of such a square rather than exit through the left side of the square. We consider  $\varepsilon = E(t) - E_{\text{ext}}$ . So in equation (5) we can, by the choice of  $\varepsilon$ , make  $\dot{P}$  arbitrarily negative with  $\lim_{\varepsilon \rightarrow 0} \dot{P}/P = -\infty$ . Equation (3) for  $\dot{E}$  is bounded from below when  $\varepsilon$  approaches zero, which presumes that  $E$  approaches  $E_{\text{ext}}$ . The limiting value is

$$(10) \quad \dot{E} = g_e E_{\text{ext}} \left( 1 - \frac{E_{\text{ext}}}{E_{\text{max}}} \right) - \beta \frac{E_{\text{ext}}}{E_{\text{comp}}} P$$

and it is bounded from below for every  $t$ . By choosing  $\varepsilon$  sufficiently small,  $\dot{P} < \dot{E} < 0$  everywhere within the square. Hence the path moves faster to the left than downward and covers the distance  $\varepsilon$  to the left before it can cover that distance downwards. Therefore, the path exits the square to the left without hitting the bottom. This rules out  $E$  falling to the level of  $E_{\text{ext}}$ . Since population extinction can only happen in phase II and on, or below,  $E_{\text{ext}}$  (see the arrows in the phase diagram), population extinction cannot occur in our model.

Next we use the phase diagram to show a global sufficient condition for the population and environment path to converge to the steady state. Looking back at Figure 1, the direction of the path in every phase has one arrow that points inwards toward the equilibrium and another that points away. For instance, in phase I, the  $E$  is above  $E^*$  but is moving downwards, while  $P$  will be carried beyond  $P^*$  in that phase. We bound the true path by a rectangular path that omits the convergent direction. So in phase I, we consider a path that only increases  $P$ ; in phase II it only decreases  $E$ , and so on. The true path is closer to the equilibrium than this rectangular path. The bounding path is a cobweb in the sense of the Cobweb Theorem of Mordecai Ezekiel (1938). From the Cobweb Theorem we know when the slope of supply exceeds that of demand in absolute value, oscillations are damped. In Figure 1,  $\dot{P} = 0$  plays the role of supply and  $\dot{E} = 0$  plays the role of demand. Hence, the bounding path converges whenever the slope of  $\dot{P} = 0$  is greater in absolute value than that of  $\dot{E} = 0$ . Since the true path is more inclined toward the steady state than the bounding path, the true path also converges. This property also prevails in the case where the slopes are equal. In this case, the true path must be closer to the equilibrium at each corner of the cobweb. For instance, in phase I the bounding path is straight across, whereas the true path is across and down. So the true path moves toward the center at each corner of the cobweb and must also converge. Comparing the slopes of  $\dot{P} = 0$  and  $\dot{E} = 0$  we find that the sufficient, but not necessary, condition for global convergence is

$$(11) \quad \beta \frac{E_{\max}}{E_{\text{comp}}} \leq g_e \alpha .$$

Namely, if the maximal individual footprint ( $\beta E_{\max} / E_{\text{comp}}$ ) does not exceed the maximal marginal growth of the carrying capacity ( $g_e \alpha$ ), the joint course of the population and the environment with  $\gamma = 0$  converges to the steady state from any initial point ( $P_0 > 0, E_0 > E_{\text{ext}}$ ). Figure 2 shows a convergent case.

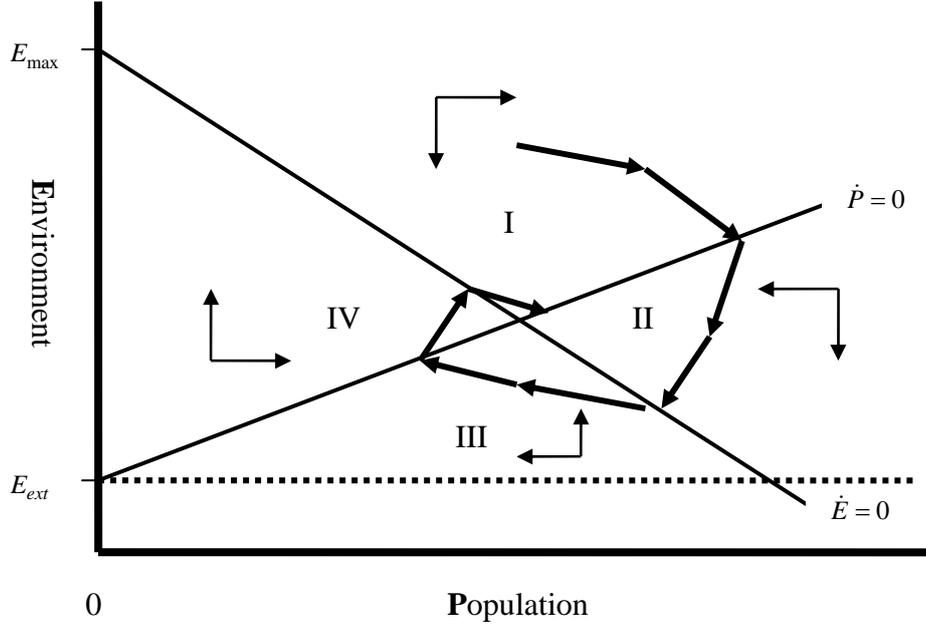


FIGURE 2. PHASE PLANE WITH CONVERGENCE

### 5. Empirical Assessment of the Model and the Non-Extinction Claim

For quantifying the steady state and examining its aforementioned local and global properties of convergence an estimation of the model's parameters is attempted. By rearranging the terms in equations (3) and (5), taking time to be discrete and adding zero-mean and finite-variance random disturbances  $\varepsilon_t$  and  $\nu_t$ , the following regression-equations of the rates of change in the quality-adjusted physical environment and human population are obtained

$$(12) \quad e_t \equiv \frac{E_t - E_{t-1}}{E_{t-1}} = g_e - \left( \frac{g_e}{E_{\max}} \right) E_t - \left( \frac{\beta}{E_{\text{comp}}} \right) P_t + \varepsilon_t$$

$$(13) \quad p_t \equiv \frac{P_t - P_{t-1}}{P_{t-1}} = g_p \left( 1 - \frac{P_t}{(\alpha + \gamma t)[E_t - E_{\text{ext}}]} \right) + \nu_t.$$

The estimation of these equations requires time-series observations on the world's population and state of environment. Due to the prominence of the ocean-warming and the (associated) climate-change problems and data availability our construction of the index of the state of the global environment is based on the principal greenhouse gas stock. Approximately eighty percent of the total warming potential of the major greenhouse gases is due to Carbon Dioxide. Complete time-series on the other greenhouse gases are not available. Our estimation uses the data on carbon-dioxide

concentration (CDC) recorded by the United States' National Oceanic and Atmospheric Administration (NOAA) since 1958 – the average annual mole fraction of carbon-dioxide in one million molecules of dried air at 3,400 meters above sea level on Mount Mauna Loa (4,169 meters), Hawaii. Time series on other important environmental variables such as forest cover, flow of rivers and changes in soil quality are only partially available and for much shorter periods. We consider  $CDC_{1958}$  as a benchmark and use  $[CDC_t / CDC_{1958}]^{-1}$  as an indicator of  $E_t$ . A time-series data on the world's population for the same period, 1959-2009, is extracted from the World Development Indicators (WDI, The World Bank Group, 2007) and the International Data Base Information Gateway – U.S. Census Bureau.

The position of the NOAA Mauna Loa Observatory well above local human-generated influences and faraway from major urban and industrial centers is suitable for monitoring key atmospheric constituents that are capable of forcing climate change. However, climate change is a long-run effect of carbon-dioxide emissions. Originated by human and natural activities and highly concentrated at the boundary layer (bottom) of the troposphere of densely populated regions, carbon-dioxide emissions also pose immediate health hazards. For this reason, we construct a measure of the entire stock (Q) of atmospheric carbon dioxide and alternatively use it for assessing the quality of the environment in the estimation of the model's parameters. As described in Appendix A, our assessment of the stock of carbon dioxide in the entire atmosphere is based on measurements of the global emissions of carbon dioxide provided by the WDI for the period 1960-2005.

We consider  $Q_{1959}$  as a benchmark and use  $[Q_t / Q_{1959}]^{-1}$  as an alternative indicator of  $E_t$ . The third column of Table A1 in Appendix A displays the computed values of this index. For comparison, the fourth column presents the values of  $[CDC_t / CDC_{1958}]^{-1}$  for the same period. The computed indices lie in the (0,1) interval and continually decrease. Between 1959 and 2005 the global carbon-dioxide emission based index  $[Q_t / Q_{1959}]^{-1}$  has decreased by 35.3 percent, whereas the Mauna Loa's recording based index  $[CDC_t / CDC_{1958}]^{-1}$  by 26.8 percent. The faster decreasing  $[Q_t / Q_{1959}]^{-1}$  provides a stronger representation of the tripling of the world's annual carbon-dioxide emissions during this short period and their much

longer average resident time in the atmosphere. As can be seen from Figure 3, the scatter diagram with  $[Q_t / Q_{1959}]^{-1}$ , indicated by squared dots, is more convex and provides a closer resemblance of the predicted population and environment joint course in phase I, Figure 2, than the scatter diagram with  $[CDC_t / CDC_{1958}]^{-1}$ , indicated by the round dots.

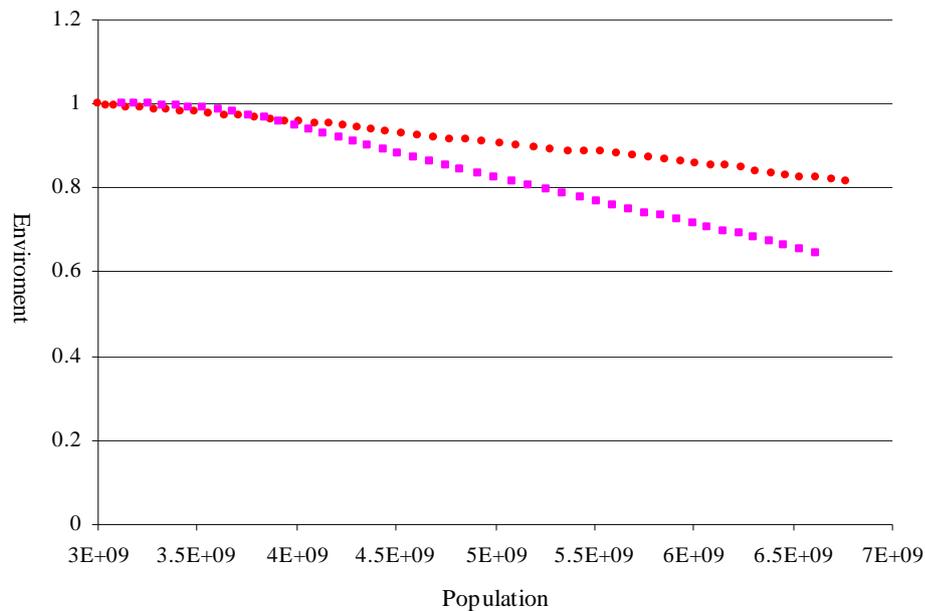


FIGURE 3. POPULATION AND THE ENVIRONMENT

We report the estimation results with each of the said environmental indices. We acknowledge that due to the exclusion of other important environmental factors such as forest cover, flow of rivers and changes in soil quality and due to the fact that our data encompass only a short episode in phase I of the history of Earth and the human population, our estimation cannot be definitive. Using Lee and Strazicich's (2003) unit-root test, our growth rate time-series of  $e$  and  $p$  are found to be  $I(0)$ . The least squares estimates of the parameters of equation (12) are obtained with Newey-West (1987, 1994) heteroskedasticity and autocorrelation consistent (HAC) adjustment. The estimates of the parameters of equation (13) are obtained with non-linear least squares. The estimates of the parameters and their t-statistics obtained with E

approximated by  $[CDC_t / CDC_{1958}]^{-1}$  are presented in Table 1. The estimates of the parameters and their t-statistics obtained with E approximated by  $[Q_t / Q_{1959}]^{-1}$  are presented in Table 2.

Table 1. Estimated parameters with  $[CDC_t / CDC_{1958}]^{-1}$

Ordinary least squares estimation results of the environment motion equation (12)				Non-linear least squares estimation results of the population motion equation (13)			
Parameter	$g_e$	$\frac{g_e}{E_{\max}}$	$\frac{\beta}{E_{\text{comp}}}$	$g_p$	$\alpha$	$\gamma$	$E_{\text{ext}}$
Estimate	-0.10955	0.094951	3.85E-12	0.045684	9,886,779,151	358,216,206	0.505425
t-statistic	-1.63380	1.631777	1.340454	20.97377	158.69221	11.61252	20.96628
Adjusted R-squared 0.321 F-statistic 12.56342 Probability (F-statistic) 0.000043 Estimation with Newey-West HAC adjustment All parameters are not significant at 10% level				Centered R-squared 0.7870 R-bar-squared 0.7728 Convergence obtained in 3 iterations All parameters significant at the 1% level			

Table 2. Estimated parameters with  $[Q_t / Q_{1959}]^{-1}$

Ordinary least squares estimation results of the environment motion equation (12)				Non-linear least squares estimation results of the population motion equation (13)			
Parameter	$g_e$	$\frac{g_e}{E_{\max}}$	$\frac{\beta}{E_{\text{comp}}}$	$g_p$	$\alpha$	$\gamma$	$E_{\text{ext}}$
Estimate	0.264574	0.188557	2.37E-11	0.027960	14,109,821,229	154,806,111	0.129553
t-statistic	6.198403	5.971179	7.093538	43.15132	197.95749	3.19970	2.00838
Adjusted R-squared 0.834 F-statistic 111.14 Probability (F-statistic) 0.000 Estimation with Newey-West HAC adjustment All parameters significant at the 1% level				Centered R-squared 0.951 R-bar-squared 0.948 Convergence obtained in 6 iterations All parameters significant at the 1% level			

The estimates of the environment motion equation (12) obtained with  $[CDC_t / CDC_{1958}]^{-1}$  are not statistically significant, whereas those obtained with  $[Q_t / Q_{1959}]^{-1}$  are statistically significant and have the expected signs. The model fits the cumulated emission based environmental index time-series better than it fits the strictly CDC based one. In the cumulated emissions case, the estimation results of

$\beta / E_{comp}$  do not reject the hypothesis of an *ad hoc* environmental-concern mechanism with a complacency threshold. In both cases, the estimation results of the population motion equation (13) have the expected signs and are statistically significant. In particular, the estimation results of  $\alpha$ ,  $\gamma$  and  $E_{ext}$  do not reject the changing carrying-capacity hypothesis. The statistically significant, positive estimate of  $\gamma$  suggests an overall progress.

The environmental index  $[CDC_t / CDC_{1958}]^{-1}$  is based on carbon-dioxide emissions that are diluted by the volume of the atmosphere above the habitable and uninhabitable surfaces of Earth. The alternative index,  $[Q_t / Q_{1959}]^{-1}$ , better represents the state of the troposphere's boundary layer in the habitable areas. For this reason, and as the estimation results of both the environment and population motion equations obtained with  $[Q_t / Q_{1959}]^{-1}$  are statistically significant, we use the estimates reported in Table 2 for assessing the steady state and its stability. The substitution of these estimates into equations (6) and (7) implies that the steady-state figures of the environment and human population are  $E^* = 0.588758$  and  $P^* = 6,479,304,144$ . These steady-state figures do not take into account progress.<sup>2</sup> Recalling equation (8), the Jacobian of the linearized environment-population system evaluated with these steady-state figures is

$$(14) \quad J = \begin{bmatrix} -0.111014491 & -1.395E-11 \\ 394510601.56 & -0.02796 \end{bmatrix}.$$

In turn,  $trJ = -0.13897449$  and  $\Delta \equiv (trJ)^2 - 4 \det J = -0.01511565$ . As both the trace and discriminant are negative, the characteristic roots of the estimated Jacobian are complex conjugate pair with a negative real part. That is, the population and the

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<sup>2</sup> Michael P. Todaro and Stephen C. Smith (2006) suggest a convergence of the world's population to about 10 billion.

environment converge to a steady state from any initial point in the vicinity of steady state. However, the substitution of the estimates into inequality (11) reveals that the sufficient condition for global converge to the steady state is not satisfied.

## 6. Conclusion

Due to the open-access nature of the environment we use *ad hoc* relationships in the construction of an analytically manageable model of the environment and population with varying carrying capacity and concerns. Our phase-plane analysis and estimation results suggest that in the absence of further progress, or regression, the proposed model has a unique, interior, steady state with a population similar to the present size and with a slightly lower environmental quality. Since the present population size and quality of the environment are in the vicinity of the locally stable steady state, we interpret our empirical findings as supporting the conceptually generated converging population and environment path. Namely, with non-optimally changing carrying capacity and environmental concerns and in the absence of future progress, or regression, the global environment and the human race are on a damped cyclical course of decline and revival that leads to an interior steady state with familiar population size and environmental quality. Consequently, we conclude that extinction, or convergence to a much smaller population and a much lower environmental quality, are not likely courses for a species that, in addition to displaying increasing environmental concerns, generates improvements in technology, healthcare provision, social order and international affairs. These findings do not support Donella H. Meadows *et al.*'s (1972) pessimistic outlook. Despite the *ad hoc* nature of our model, our findings are more in line with William D. Nordhaus' (2008) optimization based conclusions.

## APPENDIX A

### THE CARBON-DIOXIDE-EMISSION BASED INDEX $[Q_t / Q_{1959}]^{-1}$

The World Development Indicators (WDI, The World Bank Group, 2007) include forty-six observations on the world's annual emissions of carbon dioxide ( $q$ ) between 1960 and 2005. In assessing the undocumented global stock of atmospheric carbon dioxide ( $Q$ ) in each of the aforementioned years, the following law of motion is assumed

$$(A1) \quad Q_t - (1 - \delta)Q_{t-1} = q_t$$

for every  $t = 1, 2, 3, \dots, 46$  with  $\delta > 0$  denoting a time-invariant annual rate of natural depletion of atmospheric carbon dioxide. By induction,

$$(A2) \quad Q_t = (1 - \delta)^t Q_0 + \sum_{j=0}^{t-1} (1 - \delta)^j q_{t-j}$$

where,  $Q_0$  is the 1959 global stock of atmospheric carbon dioxide. Taking this initial stock as a benchmark we propose

$$(A3) \quad [Q_t / Q_0]^{-1} = \frac{Q_0}{(1 - \delta)^t Q_0 + \sum_{j=1}^t (1 - \delta)^j q_{t-j}}$$

as an indicator of the state of the global environment. As can be seen from the second column in Table 1, the aggregate emissions of carbon dioxide strongly increased since 1962. It can be expected that carbon-dioxide emissions dominated the natural annual depletion and rendered  $0 < \hat{E} < 1$  over the period 1962 to 2005.

The 1959 global stock of atmospheric carbon dioxide is unknown. Recalling equation (A1),  $Q_2 - Q_1 = -\delta(1 - \delta)Q_0 - \delta q_1 + q_2$ . Let  $Q_2 - Q_1 = \theta q_2$ , then  $Q_0 = [(1 - \theta)q_2 - \delta q_1] / [\delta(1 - \delta)]$ , where  $\theta < 1$  is an unknown scalar. The substitution of this expression into (A3) renders the proposed index as

$$(A4) \quad [Q_t / Q_0]^{-1} = \frac{(1 - \theta)q_2 - \delta q_1}{\delta(1 - \delta)} \frac{1}{(1 - \delta)^{t-1} [(1 - \theta)q_2 / \delta - q_1] + \sum_{j=1}^t (1 - \delta)^j q_{t-j}}$$

where 1960 is year 1 and 1961 is year 2. The computation of this index of the state of the environment with annual carbon-dioxide emission figures depends on the values of  $\delta$  and  $\theta$ .

Our choice of  $\delta$  is based on Forster *et al.* (2007) who argue that, typically, about 0.5 of a carbon-dioxide pulse to the atmosphere is removed within 30 years, a further 0.3 is removed within a few centuries, and the rest remains for many thousands of years. Solving equation (A1) with hypothetically no further emissions ( $q_t = 0$ ),  $Q_t = c(1 - \delta)^t$ ,  $c > 0$ . Substituting the approximated thirty-year period resident time for 50 percent of the stock,  $(1 - \delta_{highest})^{30} = 0.5$  (where 0.5 is the share of the remaining stock). Consequently,  $\delta_{highest} \approx 0.023$  for the depleted half of the stock. The annual depletion rate of the 30 percent of the stock that is removed within a few centuries is smaller. As the number of centuries is unknown, this rate is arbitrarily set to be slightly less than half of  $\delta_{highest}$ : namely,  $\delta_{medium} = 0.010$ . The annual depletion rate of the remaining 20 percent that stay in the atmosphere for many thousands of years is negligible:  $\delta_{lowest} = 0$ . Consequently, the weighted average annual depletion rate of the atmospheric carbon dioxide is

$$(A5) \quad \delta = 0.5\delta_{highest} + 0.3\delta_{medium} + 0.2\delta_{lowest} = 0.0145.$$

Our choice of  $\theta$  takes into account that  $\theta q_2 = Q_2 - Q_1 = q_2 - \delta Q_1$  or, equivalently, that  $\theta = 1 - \delta(Q_1 / q_2)$ . With  $\delta = 0.0145$ ,  $\lim \theta = 0$  when  $(Q_1 / q_2) \rightarrow 68.965$ . If the 1960 stock was 69 times the emissions in 1961, then  $\theta$  is zero. In view of the low annual depletion rate and the thousands of years of deforestation and emission of carbon dioxide into the atmosphere, and in view of the small increase (0.659%) in the aggregate carbon-dioxide emissions in 1961,  $Q_1 / q_2$  (the ratio of the 1960 stock to the 1961 emissions of carbon dioxide) should be sufficiently high for  $\theta$  to be negligible.

The third column of Table A1 displays the computed values of the atmospheric carbon-dioxide stock and the values of  $[Q_t / Q_{1959}]^{-1}$  obtained with  $\delta = 0.0145$  and  $\theta = 0$ .

Table A1: World carbon-dioxide stock, environment indices and population

Year	Carbon-dioxide Atmospheric Stock ( $Q_t$ in kilotons)	$[Q_t / Q_{1959}]^{-1}$	$[CDC_t / CDC_{1958}]^{-1}$	World's Population (P)
1960	655476965.5	1.000096	0.997065	3121477101
1961	655476965.5	1.000096	0.994774	3186981140
1962	655865349.5	0.999504	0.992244	3252585073
1963	656790373.9	0.998096	0.990564	3319792342
1964	658288225.5	0.995825	0.988611	3390053636
1965	660266326.3	0.992842	0.987314	3459775928
1966	662801984.5	0.989044	0.983197	3530674067
1967	665689259.8	0.984754	0.980817	3605947470
1968	669183197.5	0.979612	0.978145	3681596861
1969	673403241.1	0.973473	0.973384	3760216598
1970	678573358.1	0.966056	0.970216	3837305018
1971	684236428.4	0.958061	0.968313	3913581553
1972	690432936.2	0.949462	0.964972	3990363847
1973	697404298.6	0.939972	0.958445	4065380485
1974	704307552.3	0.930758	0.957022	4138250636
1975	711004452.8	0.921992	0.954392	4210942357
1976	718586200.2	0.912264	0.951604	4284497599
1977	726592956.3	0.902211	0.946671	4359247244
1978	734762078.5	0.892180	0.942071	4434466506
1979	743845996.3	0.881285	0.938239	4511281324
1980	752596677.4	0.871038	0.932975	4590035431
1981	760619577.6	0.861850	0.929052	4668825873
1982	768375921.7	0.853150	0.926030	4747053320
1983	775957510.8	0.844815	0.921654	4826996384
1984	784088686.9	0.836054	0.917453	4910665750
1985	792666217.0	0.827007	0.91358	4996354042
1986	801727596.8	0.817659	0.910107	5083156555
1987	811181538.7	0.808130	0.905439	5170308311
1988	821271502.3	0.798202	0.899075	5259139944
1989	831673161.6	0.788218	0.895381	5344209637
1990	842198796.7	0.778367	0.892196	5426373277
1991	852894242.2	0.768607	0.888883	5509691674
1992	863049883.7	0.759562	0.886912	5591798510
1993	873087580.3	0.750830	0.885222	5674980207
1994	883412082.4	0.742055	0.881051	5755872246
1995	894052207.2	0.733224	0.876213	5836558560
1996	904996050.2	0.724357	0.872006	5916098534
1997	916176919.5	0.715517	0.869343	5995874180
1998	927041778.2	0.707131	0.862156	6073279636
1999	937683144.4	0.699106	0.858315	6150949153
2000	948800418.8	0.690915	0.855387	6227969448
2001	960412348.7	0.682561	0.851537	6305155602
2002	972028113.7	0.674405	0.846745	6381200063
2003	984702890.0	0.665724	0.840864	6457749643
2004	998608186.1	0.656454	0.836989	6534293221
2005	1013385407.0	0.646881	0.832052	6610256630

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