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Health-Risking Informal Service: Price, Prevalence and Law Enforcement

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by

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Abstract

The health-risking informal service is transacted when the expected extra satisfaction rate exceeds the ratio of the expected extra cost to the formal service price. Its prevalence decreases with the costs of risk bearing for the providers and clients. Law-enforcement effort lowers (raises) the informal service equilibrium price when the ratio of the providers' and the clients' degrees of absolute risk aversion is greater (smaller) than the ratio of the law-enforcement elasticities of their cost bearing. Spending on law enforcement is efficient when the public cost of the expected chain-infection stemming from the informal service exceeds a threshold level. (*JEL* I19, K32)

Key words: Unsafe sex service, risk bearing, sexually transmitted diseases, public costs, law enforcement

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1. Introduction

This paper analyzes a market where, due to rewarding clients with a greater stimulation and providers with a higher price, a health-risking service is illegally traded alongside a safe and legal alternative. The service suppliers are numerous, registered and their operation is randomly inspected. Formally, the suppliers offer the legal service in a regulated fixed price. Informally, they solicit clients to have, or are induced by clients to provide, the more stimulating illegal service for an unreported, hence untaxed, additional payment. The apparently legal sex markets in several Australian and North American states and the quasi-legal sex markets in many other states and countries are notable venues for such formal and informal transactions. Despite the lack of economic studies on this issue, the analysis of the demand and supply of informal, unprotected sex service in an apparently legal market is of a special interest as the prevalence of this service contributes to the spread of HIV/AIDS.

In a legal sex industry, periodical tests for sexually transmitted diseases are mandatory for service providers. The right of clients to see the providers' medical certificates helps eliminating those who are positively tested at the beginning of the period. Despite these precautionary measures the market is porous. Periodical medical tests are not mandatory for clients and hence it is possible that several unaware, or selfish, clients carry sexually transmitted diseases. Providers engaged in informal service with these clients might contract those diseases and pass them on to other clients prior to their exclusion from the market by the subsequent medical test.

In view of the health risk and the risk associated with illegal activity, providers endowed with a strong risk aversion refrain from offering, and are not induced to supply, the informal service at the average market rate. However, familiarity and intimacy erode caution. Even a strongly risk-averse provider might be eventually induced by a persistent regular client to supply an informal service for a handsome extra payment. Likewise, a strongly risk averse client might be sooner or later allured by his regular service provider to experience the more stimulating service.

In addition to having different degrees of risk aversion, providers, and similarly clients, can be dissimilar in other respects. Providers may have different physical characteristics, interpersonal skills and, subsequently, appeal. Clients may have different sexual preferences and incomes. It is therefore sensible to divide the sex market into independent sub markets, each is relevant for a fairly homogeneous group of clients and their matching fairly homogeneous group of providers. This paper analyzes the effect of providers' and clients' costs of risk aversion on the price and prevalence of the informal service and on the usefulness of law-enforcement effort in such sub markets. Due to the intimate nature of the service, data are not available and hence the analysis deals with theoretical aspects *per se*.

The analysis assumes, for tractability, that all intended providers are simultaneously tested for sexually transmitted diseases on prescheduled dates. The market is, therefore, spanned over a period defined by two successive tests. The actual service providers are those tested negative at the beginning of the period. They are rational, risk averse, identical, and operating independently. The clients are rational, risk averse and anonymous. Some of the clients, though identical in all other respects to the rest, are already carrying sexually transmitted diseases but are not aware of this fact. The service providers do not know which clients carry these diseases. Since clients do not necessarily use the service of one provider exclusively, each provider regards all the clients, new and regulars, as having equal probability of carrying sexually transmitted diseases. In addition to the toll of health hazard, the overall costs of risk bearing for the providers and the clients rise with the law-enforcement effort.

The analysis of the price and prevalence of informal transactions in a sub market of sex between two successive tests is structured as follows. The supply of the informal service is derived in Section 2. Section 3 analyzes the demand for the informal service. Section 4 summarizes the effects of the providers' and clients' costs of risk bearing on the informal service equilibrium price and prevalence and draws the implications of these effects for the impact of law-enforcement effort on the price of the informal service. Section 5 derives the public-cost minimizing law-enforcement spending. Section 6 briefly concludes.

2. The Supply of the Informal Service

The analysis of the supply of the informal service is based on the following assumptions. The number of formal and informal service providers, *n*, during the period is large. The service providers are identical, full-time, independent, risk averse, rational operators. The durations of formal and informal services are fixed and identical and hence the sum of the number of formal service (N_F^S) and the number of informal services (N_{IF}^S) offered by any service provider during the period is fixed and known - N .

The cost associated with each formal service is constant, c_F. The cost associated with any informal service, c_{IF} , is a normally distributed random variable with mean m_c and with variance, s_c^2, containing two components. The first component, $s_{H_S}^2$, reflects the provider's risk of contracting a disease during an informal service and its implications for her future health, employment and income. The second component,

$s_s^2(g)$, represents the provider's risk stemming from random inspection of the nature of her service. It is an increasing function of the spending on law-enforcement effort, *g*, during the period ($\partial s_s^2(g)/\partial g > 0$). The larger the spending on law-enforcement effort the higher the likelihood of inspection and adverse effects on the service provider's license and flow of future incomes. As these components are positive and additive,

$$s_c^2 = s_{H_S}^2 + s_s^2(g) > 0 . \qquad (1)$$

The price of the formal service (P_F) is regulated and fixed. The price of the informal service (P_{IF}) is higher than the price of the formal service. From the perspective of the individual service provider, these prices are exogenously determined. As the operator is registered and as it is a common knowledge that N services are given during a period by a full-time operator, every service is reported to the tax office and a flat tax rate, $0 < t < 1$, is applied. The informal services are falsely reported as formal services.

Summing up, the service provider's net profit (p) from operation during the period is normally distributed with

$$E(p) = (1-t)(P_F - c_F)N + [(P_{IF} - P_F] - (m_c - c_F)]N_{IF}^S \qquad (2)$$

and

$$\text{var}(p) = [s_{H_S}^2 + s_s^2(g)]N_{IF}^{S\,2} . \qquad (3)$$

The service provider's utility from the net profit is negatively exponential with a unit upper-bound, reflecting a constant degree of absolute risk aversion $R_s > 0$:

$$u^s = 1 - \exp(-R_s p) . \qquad (4)$$

Maximizing the service provider's expected utility $(1 - E(\exp(-R_s p)))$ with respect to N_{IF}^S is equivalent to maximizing the power term in the moment-generating function of the provider's normally distributed profit evaluated at $-R_s$.[1] Namely,

$$\max_{N_{IF}^S}\{(1-t)(P_F - c_F)N + [(P_{IF} - P_F) - (m_c - c_F)]N_{IF}^S - 0.5R_s[s_{H_s}^2 + s_s^2(g)]N_{IF}^{S\,2}\}.$$

As $R_s[s_{H_s}^2 + s_s^2(g)] > 0$, the second-order condition for maximum is satisfied and an interior solution exists so long that the price differential $(P_{IF} - P_F)$ exceeds the expected cost differential $(m_c - c_F)$. The necessary condition for maximum implies that any provider's supply of informal service is:

$$N_{IF}^S = \frac{(P_{IF} - P_F) - (m_c - c_F)}{R_s[s_{H_s}^2 + s_s^2(g)]}. \tag{5}$$

Consequently, the aggregate supply of the formal service during the period ($\widehat{N}_{IF}^s$) is given by

$$\widehat{N}_{IF}^s = \frac{n[(P_{IF} - P_F) - (m_c - c_F)]}{R_s[s_{H_s}^2 + s_s^2(g)]}. \tag{6}$$

3. The Demand for the informal service

The analysis of the demand for the informal service is conducted within the following framework. The number of clients, *m*, of the formal and informal services is large. Some of the clients carry sexually transmitted diseases. They are not aware of this fact. They are anonymous and identical to the rest of the clients in any other respect. The clients are risk averse, rational and aware of the risks associated with informal services. Although the analysis does not explicitly deal with the scheduling

[1] See Freund (1956) for the development of this procedure, Hammond (1974) for a discussion of its generality, and Yassour et al. (1981), Collender and Zilberman (1985), Levy et al. (1989) and Levy (1992) for various applications.

of the formal and informal services over the period, the following argument is made. Due to health-risk, clients might have been inclined to demand informal services at the early part of a period defined by two consecutive tests - close to the day on which the providers have passed the medical test. However, such an inclination would have triggered a concerted law-enforcement effort in the early part of the period, thereby generating a high risk of disruption and reputation loss for clients and offsetting the health-risk moderating effect of a concentrated early demand for informal services. For this reason, and due to the habitual nature of sexual activity, the demand for the informal service is likely to be evenly spread over the period.

For each client,

N_F^D is the number of formal services sought during the period,

N_{IF}^D is the number of informal services sought during the period,

r_F is the return on the risk-free formal service,

r_{IF} is a normally distributed random return on the risky informal service with mean m_{IF} and variance s_{IF}^2, and

y is a predetermined income accruing during the period.

Since the informal service involves more exciting characteristics than the formal service, $m_{IF} > r_F$.[2] Taking the scalar $a > 0$ to represent the rate of the expected satisfaction differential between the informal and formal services,

$$m_{IF} = (1 + a)r_F \,. \tag{7}$$

The variance of the return on the informal service represents the risk stemming from an unsafe service for the client's health, $s_{H_d}^2 > 0$. It also reflects the client's concerns about the inconvenience and loss of reputation accompanying a possible inspection of

[2] A similar assumption is made by Levy (2002) with respect to the satisfaction from unprotected sex relative to condom-protected sex.

the nature of the service, $s_d^2(g) > 0$ - an increasing ($\partial s_d^2(g)/\partial g > 0$) function of the spending on law-enforcement effort (g) during the period.[3] Namely,

$$s_{IF}^2 = s_{H_d}^2 + s_d^2(g) > 0\,. \tag{8}$$

Since sexual activity is strongly habitual, and for tractability, a fixed fraction $e > 0$ of the client's income is spent on formal and informal services and the client's periodical utility from formal and informal services is independent from the client's utility from spending on other goods, $\hat{u}^d((1-e)y)$. It is portrayed by a negative exponential function displaying a unit upper-bound and a constant degree of absolute risk aversion $R_d > 0$. Namely, the client's periodical utility is given by

$$u^d = \hat{u}^d((1-e)y) + \{1 - \exp\{-R_d[r_F N_F^D + r_{IF} N_{IF}^D]\}\}\,. \tag{9}$$

As a fixed fraction of the client's income is spent on formal and informal services and as the prices of the formal and informal services are exogenously given, the client's number of formal services during the period can be expressed as

$$N_F^D = \frac{e}{P_F} y - \frac{P_{IF}}{P_F} N_{IF}^D\,. \tag{10}$$

Consequently, the client's budget-constrained periodical utility is

$$u^d = \hat{u}^d((1-e)y) + 1 - \exp\{-R_d[r_F(\frac{ey}{P_F} - \frac{P_{IF}}{P_F} N_{IF}^D) + r_{IF} N_{IF}^D]\}\,. \tag{11}$$

The client chooses the combination of numbers of formal services and informal services that maximizes his expected utility. Recalling the definition of the moment generating function and that r_{IF} is normally distributed, maximizing $E(u^d(\cdot))$ with respect to N_{IF}^D is equivalent to maximizing

[3] This assumption is consistent with the argument made by Behrens, Caulkins, Tragler and Feichtinger (1997) and by Tragler, Caulkins and Feichtinger (2001) that law-enforcement effort increases the costs of risk bearing for illicit-drug users.

$$v = r_F (\frac{ey}{P_F} - \frac{P_{IF}}{P_F} N_{IF}^D) + (1+a) r_F N_{IF}^D - 0.5 R_d [s_{H_d}^2 + s_d^2(g)] N_{IF}^{D\,2} . \quad (12)$$

As $R_d [s_{H_d}^2 + s_d^2(g)] > 0$ the second-order condition for maximum is satisfied, and so long that the relative expected return on the formal service $(1+a)$ exceeds the relative price (P_{IF} / P_F), there exists an interior solution. The necessary condition for maximum implies that each client's demand for illegal service during the period is

$$N_{IF}^D = \frac{(1+a-\frac{P_{IF}}{P_F}) r_F}{R_d [s_{H_d}^2 + s_d^2(g)]} . \quad (13)$$

Consequently, the aggregate demand for the informal service ($\hat{N}_{IF}^d$) during the period is

$$\hat{N}_{IF}^d = \frac{m(1+a-\frac{P_{IF}}{P_F}) r_F}{R_d [s_{H_d}^2 + s_d^2(g)]} . \quad (14)$$

4. Risk Bearing and the Informal Service Price and Prevalence

In view of the aggregate supply and demand equations (6) and (14), the informal service equilibrium price is

$$P_{IF}^* = \frac{m(1+a) r_F R_s [s_{H_s}^2 + s_s^2(g)] + n(P_F + m_c - c_F) R_d [s_{H_d}^2 + s_d^2(g)]}{m \frac{r_F}{P_F} R_s [s_{H_s}^2 + s_s^2(g)] + n R_d [s_{H_d}^2 + s_d^2(g)]} \quad (15)$$

and the prevalence of the informal service (i.e., the equilibrium number of informal services transacted during the period) is

$$\hat{N}_{IF}^* = \frac{a - [(m_c - c_F)/P_F]}{(1/nP_F) R_s [s_{H_s}^2 + s_s^2(g)] + (1/mr_F) R_d [s_{H_d}^2 + s_d^2(g)]} . \quad (16)$$

(See detailed derivation in the Appendix.)

COROLLARY 1: The rate of the expected extra satisfaction from the informal service ($a = m_{IF} / r_F - 1$) must be greater than the ratio of the informal service expected extra cost to the formal service price ($(m_c - c_F)/P_F$) for the informal service to be traded.

This corollary is straightforwardly obtained from equation (16). The underlying rationale is as follows. From equation (14), the demand for the informal service is positive only when $a > \dfrac{P^*_{IF} - P_F}{P_F}$. From equation (6), the supply of the informal service is positive only when $P^*_{IF} - P_F > m_c - c_F$. Consequently, if a positive quantity of the informal service is traded then $a > \dfrac{P^*_{IF} - P_F}{P_F} > \dfrac{m_c - c}{P_F}$.

COROLLARY 2: The prevalence of the informal service decreases with the costs of risk bearing for the providers and the clients. (Straightforward from equation (16).)

PROPOSITION 1: The equilibrium price of the informal service rises with the service providers' costs of risk bearing ($R_s[s^2_{H_s} + s^2_s(g)]$) and decreases with the clients' costs of risk bearing ($R_d[s^2_{H_d} + s^2_d(g)]$). (See proof in the Appendix.)

Recalling that $\partial s^2_s(g)/\partial g > 0$ and $\partial s^2_d(g)/\partial g > 0$, the costs of risk bearing for the informal service providers and their clients rise with the spending on law-enforcement effort. In view of Proposition 1, the incremental costs of risk bearing for the service providers and clients lower the prevalence of the informal service. As suggested by the following proposition, the direction of the net effect of the spending on law enforcement effort on the equilibrium price of the informal service depends on the ratio of the providers' and the clients' degrees of absolute risk aversion and on the ratio of the providers' and the clients' elasticities of cost bearing, which are defined as:

$$x_s \equiv \frac{\partial R_s[s_{H_s}^2 + s_s^2(g)]}{\partial g} \frac{g}{R_s[s_{H_s}^2 + s_s^2(g)]} = \frac{(\partial s_s^2(g)/\partial g)g}{[s_{H_s}^2 + s_s^2(g)]} \tag{19}$$

and

$$x_d \equiv \frac{\partial R_d[s_{H_d}^2 + s_d^2(g)]}{\partial g} \frac{g}{R_s[s_{H_s}^2 + s_s^2(g)]} = \frac{(\partial s_d^2(g)/\partial g)g}{[s_{H_d}^2 + s_d^2(g)]}. \tag{20}$$

PROPOSITION 2: If $\frac{x_s}{x_d} > \frac{R_s}{R_d}$, then the equilibrium price of the informal service rises with law-enforcement effort. If $\frac{x_s}{x_d} < \frac{R_s}{R_d}$, then the equilibrium price of informal service falls with law-enforcement effort. (See proof in the Appendix.)

5. Public-Cost Minimizing Law-Enforcement

As the probability and intensity of epidemics such as HIV/AIDS rise with the prevalence of unsafe sex, spending on law-enforcement effort that suppresses the prevalence of the informal service in the apparently legal market of sex may be desirable from the perspectives of the general public. The total public costs of the prevalence and control of the informal service during the period are

$$C = bN_{IF}^* + g$$

where, b is the public cost of the average chain of infections stemming from an informal service within the period, and where N_{IF}^{*} is given by equation (16) with, for tractability, linear specifications of $s_{s}^{2}(g)$ and $s_{d}^{2}(g)$:

$$\widehat{N}_{IF}^{*} = \frac{a-(m_c-c_F)/P_F}{(1/nP_F)R_s[s_{H_s}^{2}+s_{s}^{2}g]+(1/mr_F)R_d[s_{H_d}^{2}+s_{d}^{2}g]}. \tag{21}$$

As C is convex in *g*, the public-cost minimizing law-enforcement periodical spending satisfies

$$\frac{\partial C}{\partial g} = \frac{-b[a-(m_c-c_F)/P_F][(1/nP_F)R_s s_{s}^{2}+(1/mr_F)R_d s_{d}^{2}]}{\{(1/nP_F)R_s[s_{H_s}^{2}+s_{s}^{2}g]+(1/mr_F)R_d[s_{H_d}^{2}+s_{d}^{2}g]\}^2}+1=0 \tag{22}$$

which implies

$$g^{o2}+2\underbrace{\left(\frac{(1/nP_F)R_s s_{H_s}^{2}+(1/mr_F)R_d s_{H_d}^{2}}{(1/nP_F)R_s s_{s}^{2}+(1/mr_F)R_d s_{d}^{2}}\right)}_{g}g^{o}$$
$$-\underbrace{\left(\frac{b[a-(m_c-c_F)/P_F]}{[(1/nP_F)R_s s_{s}^{2}+(1/mr_F)R_d s_{d}^{2}]}-\frac{[(1/nP_F)R_s s_{H_s}^{2}+(1/mr_F)R_d s_{H_d}^{2}]^2}{[(1/nP_F)R_s s_{s}^{2}+(1/mr_F)R_d s_{d}^{2}]^2}\right)}_{d}=0 \quad .(23)$$

Subsequently, the public-cost minimizing law-enforcement spending during the period is

$$g^{o}=\sqrt{(g^{2}+d)}-g\ . \tag{24}$$

PROPOSITION 3: Spending on law enforcement is efficient (in the sense of minimizing the public costs of informal services) when

$$b > \frac{[(1/nP_F)R_s s_{H_s}^{2}+(1/mr_F)R_d s_{H_d}^{2}]^2}{[a-(m_c-c_F)/P_F][(1/nP_F)R_s s_{s}^{2}+(1/mr_F)R_d s_{d}^{2}]}.$$

(See proof in the Appendix.)

This proposition indicates that policing the sex market is desirable when the public costs of the expected chain-infection stemming from an informal service during the period exceed a threshold level. This level rises with the providers' and clients' costs of risk bearing stemming from the informal service's health hazard relative to their returns on the safe formal service. It is inversely related to the difference between the rate of the expected extra satisfaction for the client from the informal service and the ratio of the informal service expected extra cost to the formal service price. The threshold level is also inversely related to the provider and client's costs of risk bearing stemming from law enforcement relative to their returns on the safe formal service.

6. Concluding Remarks

The behavior of rational providers and clients in an apparently legal sex market, where a health-risking service is illegally traded alongside a safe and legal alternative, was analyzed. In this market, the suppliers are registered, randomly inspected and periodically tested for sexually transmitted diseases. They formally offer the legal service in a regulated fixed price. Informally, they solicit clients to have, or are swayed by clients to provide, the more exciting illegal service for an unreported, thus untaxed, extra payment. The analysis demonstrated the effects of the costs of the risk borne by expected-utility-maximizing providers and clients on the informal service equilibrium price and prevalence and their implications for the effects of law-enforcement effort.

The analysis led to the following conclusions. The rate of the expected extra satisfaction from the informal service must be greater than the ratio of the informal service expected extra cost to the formal service price for the informal service to be traded. The prevalence of the informal service decreases with the costs of risk bearing for the providers and the clients. The equilibrium price of the informal service rises with the service providers’ costs of risk bearing and falls with the clients’ costs of risk bearing. Consequently, spending on law enforcement effort lowers (raises) the equilibrium price of the informal service if the ratio of the providers’ and the clients’ degrees of absolute risk aversion is greater (smaller) than the ratio of the elasticities of their cost bearing with respect to law enforcement. Spending on law enforcement is efficient when the public cost of the expected chain-infection stemming from an informal service exceeds a critical level.

In the absence of instantaneous, inexpensive medical tests, engagement in unprotected informal sex service is comparable to playing Russian Roulette. The development and introduction of instantaneous, mandatory medical tests that are inexpensively and mutually applied by the service providers and their clients before engagement will eliminate the risk of contracting diseases for the participants in the sex market and for the general public and may eventually render the currently informal service formal.

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APPENDIX

The informal service equilibrium price and prevalence: From equation (6) and (14), the informal service market-clearing condition is

$$\frac{m(1+a-\frac{P_{IF}^*}{P_F})r_F}{R_d[s_{H_d}^2+s_d^2(g)]}=\frac{n[(P_{IF}^*-P_F)-(m_c-c_F)]}{R_s[s_{H_s}^2+s_s^2(g)]} \qquad \text{(A.1)}$$

which implies that the informal service equilibrium price is

$$P_{IF}^*=\frac{m(1+a)r_F R_s[s_{H_s}^2+s_s^2(g)]+n(P_F+m_c-c_F)R_d[s_{H_d}^2+s_d^2(g)]}{m\frac{r_F}{P_F}R_s[s_{H_s}^2+s_s^2(g)]+nR_d[s_{H_d}^2+s_d^2(g)]}\,. \qquad \text{(A.2)}$$

Equation 14 also implies

$$P_{IF}^*=(1+a)P_F-\frac{P_F R_d[s_{H_d}^2+s_d^2(g)]}{mr_F}\widehat{N}_{IF}^d\,. \qquad \text{(A.3)}$$

Equation (6) implies,

$$P_{IF}^*=P_F+m_c-c_F+\frac{R_s[s_{H_s}^2+s_s^2(g)]}{n}\widehat{N}_{IF}^s\,. \qquad \text{(A.4)}$$

In equilibrium,

$$(1+a)P_F-\frac{P_F R_d[s_{H_d}^2+s_d^2(g)]}{mr_F}\widehat{N}_{IF}^*=P_F+m_c-c_F+\frac{R_s[s_{H_s}^2+s_s^2(g)]}{n}\widehat{N}_{IF}^* \qquad \text{(A.5)}$$

implying

$$\widehat{N}_{IF}^*=\frac{a-(m_c-c_F)/P_F}{(1/nP_F)R_s[s_{H_s}^2+s_s^2(g)]+(1/mr_F)R_d[s_{H_d}^2+s_d^2(g)]}\,. \qquad \text{(A.6)}$$

PROOF OF PROPOSITION 1: By differentiating equation (15) with respect to the provider's cost of risk bearing,

$$\frac{\partial P_{IF}^*}{\partial R_s[s_{H_s}^2+s_s^2(g)]}=\frac{m(1+a)r_F\{m\frac{r_F}{P_F}R_s[s_{H_s}^2+s_s^2(g)]+nR_d[s_{H_d}^2+s_d^2(g)]\}}{\{m\frac{r_F}{P_F}R_s[s_{H_s}^2+s_s^2(g)]+nR_d[s_{H_d}^2+s_d^2(g)]\}^2}$$

$$-\frac{m\frac{r_F}{P_F}\{m(1+a)r_F R_s[s_{H_s}^2+s_s^2(g)]+n(P_F+m_c-c_F)R_d[s_{H_d}^2+s_d^2(g)]\}}{\{m\frac{r_F}{P_F}R_s[s_{H_s}^2+s_s^2(g)]+nR_d[s_{H_d}^2+s_d^2(g)]\}^2}$$

$$=\frac{[a-(m_c-c_F)/P_F]mnr_F R_d[s_{H_d}^2+s_d^2(g)]}{\{m\frac{r_F}{P_F}R_s[s_{H_s}^2+s_s^2(g)]+nR_d[s_{H_d}^2+s_d^2(g)]\}^2} \qquad \text{.(A.7)}$$

Recalling corollary 1,

$$\frac{\partial P_{IF}^*}{\partial R_s[s_{H_s}^2 + s_s^2(g)]} = \frac{[a - (m_c - c_F)/P_F]mnr_F R_d[s_{H_d}^2 + s_d^2(g)]}{\{m\frac{r_F}{P_F}R_s[s_{H_s}^2 + s_s^2(g)] + nR_d[s_{H_d}^2 + s_d^2(g)]\}^2} > 0 . \qquad (A.8)$$

By differentiating equation (15) with respect to the client's cost of risk bearing,

$$\frac{\partial P_{IF}^*}{\partial R_d[s_{H_d}^2 + s_d^2(g)]} = \frac{n(P_F + m_c - c_F)\{m\frac{r_F}{P_F}R_s[s_{H_s}^2 + s_s^2(g)] + nR_d[s_{H_d}^2 + s_d^2(g)]\}}{\{m\frac{r_F}{P_F}R_s[s_{H_s}^2 + s_s^2(g)] + nR_d[s_{H_d}^2 + s_d^2(g)]\}^2}$$

$$- \frac{n\{m(1+a)r_F R_s[s_{H_s}^2 + s_s^2(g)] + n(P_F + m_c - c_F)R_d[s_{H_d}^2 + s_d^2(g)]\}}{\{m\frac{r_F}{P_F}R_s[s_{H_s}^2 + s_s^2(g)] + nR_d[s_{H_d}^2 + s_d^2(g)]\}^2}$$

$$= \frac{[(P_F + m_c - c_F)/P_F - (1+a)]nmr_F R_s[s_{H_s}^2 + s_s^2(g)]}{\{m\frac{r_F}{P_F}R_s[s_{H_s}^2 + s_s^2(g)] + nR_d[s_{H_d}^2 + s_d^2(g)]\}^2}$$

$$= -\frac{[a - (m_c - c_F)/P_F]mnr_F R_s[s_{H_s}^2 + s_s^2(g)]}{\{m\frac{r_F}{P_F}R_s[s_{H_s}^2 + s_s^2(g)] + nR_d[s_{H_d}^2 + s_d^2(g)]\}^2}$$

.(A.9)

Recalling corollary 1,

$$\frac{\partial P_{IF}^*}{\partial R_d[s_{H_d}^2 + s_d^2(g)]} = -\frac{[a - (m_c - c_F)/P_F]mnr_F R_s[s_{H_s}^2 + s_s^2(g)]}{\{m\frac{r_F}{P_F}R_s[s_{H_s}^2 + s_s^2(g)] + nR_d[s_{H_d}^2 + s_d^2(g)]\}^2} < 0 . \qquad (A.10)$$

PROOF OF PROPOSITION 2:

$$\frac{\partial P_{IF}^*}{\partial g} = \frac{\partial P_{IF}^*}{\partial R_s[s_{H_s}^2 + s_s^2(g)]}\frac{\partial s_s^2(g)}{\partial g} + \frac{\partial P_{IF}^*}{\partial R_d[s_{H_d}^2 + s_d^2(g)]}\frac{\partial s_d^2(g)}{\partial g} \qquad (A.11)$$

and in recalling (A.8) and (A.10),

$$\frac{\partial P_{IF}^*}{\partial g} = \frac{mnr_F[a - (m_c - c_F)/P_F]\{R_d[s_{H_d}^2 + s_d^2(g)]\frac{\partial s_s^2(g)}{\partial g} - R_s[s_{H_s}^2 + s_s^2(g)]\frac{\partial s_d^2(g)}{\partial g}\}}{\{m\frac{r_F}{P_F}R_s[s_{H_s}^2 + s_s^2(g)] + nR_d[s_{H_d}^2 + s_d^2(g)]\}^2}$$

.(A.12)

As the denominator of this expression is positive, and as $a > (m_c - c_F)/P_F$ (by the corollary), $\frac{\partial P_{IF}^*}{\partial g} > 0$ when $R_d[s_{H_d}^2 + s_d^2(g)]\frac{\partial s_s^2(g)}{\partial g} > R_s[s_{H_s}^2 + s_s^2(g)]\frac{\partial s_d^2(g)}{\partial g}$ or $\frac{\partial P_{IF}^*}{\partial g} > 0$ when $R_d[s_{H_d}^2 + s_d^2(g)]\frac{\partial s_s^2(g)}{\partial g} < R_s[s_{H_s}^2 + s_s^2(g)]\frac{\partial s_d^2(g)}{\partial g}$.

By rearranging terms and recalling equations (19) and (20), these conditions can be equivalently expressed as displayed in the proposition.

PROOF OF PROPOSITION 3: Equation (24) implies that $g^o > 0$ when $d > 0$. This condition is satisfied when

$$b[a - (m_c - c_F)/P_F] > \frac{[(1/nP_F)R_s s^2_{H_s} + (1/mr_F)R_d s^2_{H_d}]^2}{(1/nP_F)R_s s^2_s + (1/mr_F)R_d s^2_d} \qquad \text{(A.13)}$$

which can be equivalently rendered, in recalling corollary 1, as

$$b > \frac{[(1/nP_F)R_s s^2_{H_s} + (1/mr_F)R_d s^2_{H_d}]^2}{[a - (m_c - c_F)/P_F][(1/nP_F)R_s s^2_s + (1/mr_F)R_d s^2_d]}. \qquad \text{(A.14)}$$