



**University of Wollongong
Economics Working Paper Series
2003**

<http://www.uow.edu.au/commerce/econ/wplist.html>

**Environmental Hazard and Residential Value, Location
and Dispersion**

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WP 03-16

December 2003

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Abstract

This paper provides microeconomic foundations to the relationship between the values of residential properties and the environmental quality of their location. It constructs an environmental-quality-adjusted lifetime-utility function by combining satisfaction from consumption over the lifespan with risk to life from living in an environmentally hazardous location. It employs this utility function to analyse willingness to pay for environmental quality, choice of location and residential dispersion and its relationship with income distribution.

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1. Introduction

The choice of a residential property and its location is one of the major and least reversible decisions in human life. The high user-cost of residential properties and the environmental quality of their locations have strong long-term implications for the consumption, lifestyle and health of the decision-makers and their dependents. The hypothesis that certain facilities and land use constitute environmental hazard and hence deter demand for adjacent residential properties, has been extensively tested. Applying hedonic pricing methods and other techniques to cross-section and time-series data, a large number of empirical studies, including the seminal articles by Harrison and Rubinfeld (1978) and Freeman (1979) and the more recent papers by Michaels and Smith (1990), Kohlhase (1991), Kiel and McClain (1995a, 1995b) and McCluskey and Rausser (2003), have lent support to a positive correlation between residential-property prices and distance from sources of environmental hazards and, consequently, to the assertion that the values of residential properties reflect people's concerns with the environmental quality of their location. These concerns arise mainly from worries about health risk and stigma (McClelland, Schulze and Hurd, 1990, Wandersman and Hallman, 1993; Lober and Green, 1994).

The objective of this paper is to provide analytical foundations for the effect of sources of environmental hazards on the values, location and dispersion of residential properties. The analysis begins with the assertion that life is uncertain and the probability of survival depends upon the environmental quality of the place of residence. From residents' perspective, environmental quality is eroded by any perceived type of hazard posed by nearby facilities and land use. Residents' lifetime utility from consumption increases with the probability of survival associated with the

environmental quality of their neighbourhood and the period of exposure to hazardous facilities and land use. Following a construction of an environmental-quality adjusted utility function, the paper analyses residents' willingness to pay for environmental quality, their choice of location and their aggregate level of residential dispersion and its relationship with income distribution. The analysis emphasises the roles of consumption-elasticity of utility and sensitivities of personal health and income and market rent to environmental hazard in the determination of residents' willingness to pay for environmental quality and in the determination of residential location and dispersion.

The analysis is organised as follows. Section 2 constructs a resident's environmental-quality-adjusted lifetime utility function by combining the resident's satisfaction from consumption over her lifespan with the risk to her life from residing in a hazardous environment. Section 3 employs this lifetime utility function to analyse the resident's willingness to pay for environmental quality. Section 4 analyses the resident's choice of location. Section 5 extends the analysis of the choice of location to the case where the resident's productivity is affected by the environmental quality of her neighbourhood. Section 6 analyses the residential dispersion of a heterogeneous population and its relationship with income distribution. Section 7 summarises the main results and indicates the possible relationship between income disparity, political power and the location and persistence of hazardous facilities and land use.

2. Resident's environmental-quality-adjusted utility

It is possible that on major and non-easily reversible¹ choices, such as place of residence, decisions are made in accordance with expected-lifetime-utility maximisation. Ms. Jones is an analytically competent lifetime-utility maximiser. Her rate of time preference, r_j , is time invariant and positive, but not very large, revealing that she cares about her future utilities from consumption. Being farsighted and feeling young, Ms. Jones' planning horizon is very long – infinite, for tractability.² However, Ms. Jones is aware of the uncertainty about her existence and of the effect of an environmental hazard near her place of residence on her probability of survival.

Ms. Jones' instantaneous income is y_{jt} , instantaneous residential rent (or user cost) is R_t , instantaneous spending on consumption is $y_{jt} - R_t$,³ and instantaneous utility from consumption is $u_j(y_{jt} - R_t)$, displaying $u'_j > 0$ and $u''_j < 0$. Being an expected-lifetime-utility maximiser, Ms. Jones multiplies her accumulated utility between the starting point of her planning horizon 0 to her possible time of death t ,

$\int_0^t e^{-r_j t} u_j(y_{jt} - R_t) dt$, by her probability of dying at time t , f_{jt} . The sum of all the

products of f_{jt} and $\int_0^t e^{-r_j t} u_j(y_{jt} - R_t) dt$ associated with any possible time of death

$0 \leq t \leq \infty$ is her expected lifetime-utility:

$$V_j = \int_0^{\infty} f_{jt} \int_0^t e^{-r_j t} u_j(y_{jt} - R_t) dt dt. \quad (1)$$

¹ Due to large financial, psychological and social costs.

² The assumption of infinite planning horizon can be further justified by considering Ms. Jones to be a representative of a household and/or a family.

³ There is no saving, or there is no distinction between saving and consumption, for simplicity.

Integrating by parts, Ms. Jones realises that her expected lifetime-satisfaction can be rendered as:

$$V_j = \int_0^{\infty} [1 - F_{jt}] e^{-rt} u_j(y_{jt} - R_t) dt \quad (2)$$

where, F_{jt} denotes the cumulative density function associated with f_{jt} and indicates her probability of dying by t and hence $1 - F_{jt}$ displays her probability of living beyond t (see Appendix A, Kamien and Schwartz, 1991; Levy, 2000, 2002a, 2002b). Ms. Jones interprets this expected lifetime utility as the sum of her discounted instantaneous utility from consumption accruing during her planning horizon and weighted by her probability of prevailing.

Spending most of her time at her home and neighbourhood, Ms. Jones believes that her probability of dying by t depends upon the quality of her neighbourhood's environment during the period $(0,t)$ and upon the sensitivity of her health to this quality. She takes her neighbourhood's environmental quality to be eroded by any perceived type of hazard posed by nearby facilities and undesired land use, which she expects to persist indefinitely. She therefore believes that the environmental quality of any location affected by these facilities and land use is time-invariant. She also believes that the rent and her income in any location are time-invariants. Hence, she intends to stay in one location after making her choice. In other words, Ms. Jones expects to be exposed to the (perceived) initial environmental quality, \mathbf{q} , of her chosen neighbourhood until the end of her life. She takes \mathbf{q} to belong to the unit interval $(0,1)$, where 1 represents the least hazardous environment and 0 the most hazardous environment.⁴

⁴ Consistently with the aforementioned empirical studies, \mathbf{q} may rise with the distance from the source(s) of the environmental hazard.

Ms. Jones considers the sensitivity of her health to the quality of her residential environment to be time-invariant and denotes it by a non-negative scalar \mathbf{a}_j ($0 \leq \mathbf{a}_j \leq 1$). Ms. Jones assesses that her probability of dying by t declines with \mathbf{q} and \mathbf{a}_j , but rises with the period of exposure $(0,t)$ to the hazardous facilities and land use. Namely, $F_{jt} = F_j(\mathbf{q}, \mathbf{a}_j, t)$ with $\partial F_j / \partial \mathbf{q} < 0$, $\partial F_j / \partial \mathbf{a} < 0$ and $\partial F_j / \partial t > 0$. Ms. Jones uses the following convenient explicit form to approximate her probability of dying by t :

$$F_{jt} = 1 - e^{-\mathbf{m}_j t} \mathbf{q}^{\mathbf{a}_j}. \quad (3)$$

She interprets \mathbf{a}_j as the elasticity of her health with respect to the environmental quality of her neighbourhood and $e^{-\mathbf{m}_j t}$ as the effect of the period of exposure to hazardous facilities and land use on her probability of survival. Since $\mathbf{m}_j = -[d(1 - F_j)/dt]/[1 - F_j]$, she takes \mathbf{m}_j to be a positive scalar indicating the rate of decline of her survival probability due to continued exposure to hazardous facilities and land use.

By substituting Eq. (3) into Eq. (2) and her expectation of time-invariant income and rent, Ms. Jones rewrites her expected lifetime utility as

$$V_j = \int_0^{\infty} e^{-(r_j + \mathbf{m}_j)t} \mathbf{q}^{\mathbf{a}_j} u_j(y_j - R) dt, \quad (4)$$

which she now interprets as the lifetime sum of her instantaneous utilities from consumption adjusted to the environmental quality of her location and discounted by her time preference and the effect of continued exposure to hazardous facilities and land use. By integrating the right-hand side of Eq. (4), Ms. Jones finds that, living the rest of her life in a location endowed with environmental quality \mathbf{q} , her expected

lifetime utility is equal to the ratio of her environmental-quality-adjusted instantaneous utility from consumption and her *full* discounting rate:

$$V_j = \mathbf{q}^{\mathbf{a}_j} u_j(y_j - R)/(r_j + \mathbf{m}_j). \quad (5)$$

The interpretation of this environmental-quality-adjusted-lifetime-utility function can be broadened to include the effect of stigma. When worries about the public image of her neighbourhood adversely affect Ms. Jones' health, \mathbf{a}_j can be redefined as the sum of her health sensitivity to environmental quality and her concern about neighbourhood's public image.

3. Residents' willingness to pay for environmental quality

Ms. Jones perceives location A to be less affected than location B by the hazardous facilities and land use. That is, $\mathbf{q}_A > \mathbf{q}_B$. Recalling Eq. (5) and assuming that her income is not affected by the environmental quality of her neighbourhood,⁵ Ms. Jones is indifferent between location A and location B so long that her full rents⁶, R_A^j and R_B^j , on structurally identical residential properties in these locations satisfy:

$$\mathbf{q}_A^{\mathbf{a}_j} u_j(y_j - R_A^j)/(r_j + \mathbf{m}_j) = \mathbf{q}_B^{\mathbf{a}_j} u_j(y_j - R_B^j)/(r_j + \mathbf{m}_j). \quad (6)$$

In other words, Ms. Jones is indifferent between the two locations if equality between the ratio of instantaneous utility from consumption and the inverse of the ratio of their environmental qualities' impact on her well-being exists:

⁵ This assumption is relaxed in section 5.

⁶ Including her travel costs to places of business and social activities.

$$\frac{u_j(y_j - R_A^j)}{u_j(y_j - R_B^j)} = (\mathbf{q}_B / \mathbf{q}_A)^{\mathbf{a}_j} . \quad (7)$$

Corollary 1: One is willing to pay a higher full rent for a residential property in an environmentally less hazardous location if one's health is sensitive to the environmental quality of one's place of residence.⁷ (See proof in Appendix B.)

When \mathbf{a}_j is taken to be the sum of person j 's health sensitivity to environmental quality and person j 's concern about her neighbourhood's public image, this corollary encompasses sensitivity to stigma.

Ms. Jones explores further her willingness to pay for the environmental-quality difference between location A and location B by using the analytically convenient isoelastic function of instantaneous utility from consumption

$$u_{ji} = (y_j - R_i^j)^{\mathbf{b}_j} , \quad 0 < \mathbf{b}_j < 1, \quad (8)$$

for any location $i = A, B$. By substituting this explicit form into Eq. (7), Ms. Jones finds that her willingness to pay extra rent for the environmental-quality difference between location A and location B is:

$$R_A^j - R_B^j = [1 - (\mathbf{q}_A / \mathbf{q}_B)^{-\mathbf{a}_j / \mathbf{b}_j}] (y_j - R_B^j) . \quad (9)$$

Ms. Jones assesses the life expectancy of residential properties to be very long and, for tractability, takes it to be infinite. Perceiving herself an ordinary member of the society, she takes the market capitalization rate of an ordinary residential property to

⁷ This corollary is consistent with Smith and Desvousges' (1986) finding that respondents would be willing to pay between \$2472 and \$3199 more for residential properties located a mile further from a hazardous waste landfill.

be equal to her full discounting rate ($r_j + m_j$). By summing her discounted willingness to pay extra rent over an infinite period, Ms. Jones realises that the difference between her highest bid on an ordinary residential property in location A (P_A^j) and her highest bid on an identical property in location B (P_B^j) should be:

$$P_A^j - P_B^j = [1 - (q_A / q_B)^{-a_j / b_j}] (y_j - R_B^j) / (r_j + m_j). \quad (10)$$

Corollary 2: The positive effect of the A-B environmental quality ratio on one's A-B highest-bid difference on structurally identical residential properties is intensified by one's income and health sensitivity, but is moderated by one's consumption elasticity of utility, rate of time preference and rate of decline of survival probability stemming from continued exposure to hazardous facilities and land use. (See proof in Appendix B.)

4. Optimal location of residence

Suppose that information about environmental qualities of all locations is complete and perfectly transmitted to Ms. Jones. There is a continuum of environmental qualities, and locations are ranked continuously by their environmental quality q within the unit interval (0,1). Consistent with the previous section's findings about her willingness to pay rent on identical residential properties in locations endowed with different environmental qualities, Ms. Jones observes that the market rents of ordinary residential properties rise with the environmental quality of

their location and can be approximated by a differentiable function $R(\mathbf{q})$ with $R'(\mathbf{q}) > 0$.⁸ By substituting this rent function into Eq. (5), Ms. Jones obtains that

$$\mathbf{q}_j^* = \arg \max \mathbf{q}^{a_j} u_j(y_j - R(\mathbf{q})) / (\mathbf{r}_j + \mathbf{m}_j) = \frac{\mathbf{a}_j u_j(y_j - R(\mathbf{q}_j^*))}{R'(\mathbf{q}_j^*) u'_j(y_j - R(\mathbf{q}_j^*))} \quad (11)$$

if $\frac{y_j - R(\mathbf{q}_j^*)}{R'(\mathbf{q}_j^*)} > \frac{1 - \mathbf{b}}{1 + \mathbf{a}} \mathbf{q}_j^*$ (see Appendix C). Ms. Jones realises that \mathbf{q}_j^* is independent of her full discounting rate due to her assumption, which she considers to be the most sensible one, of time-invariant relative environmental qualities of the various locations.

For tractability, Ms. Jones takes the market rent of an ordinary residential property to be linearly rising in its location's environmental quality from the lowest rent of R_0 in the environmentally most hazardous location ($\mathbf{q} = 0$) to the highest rent R_1 in the environmentally least hazardous location ($\mathbf{q} = 1$)

$$R = R_0 + (R_1 - R_0)\mathbf{q} \quad (12)$$

and also interprets $R_1 - R_0$ as the market rent-gradient. Subsequently, Ms. Jones obtains that her optimal residential location is where the quality of the environment is

$$\mathbf{q}_j^* = \left(\frac{\mathbf{a}_j}{\mathbf{a}_j + \mathbf{b}_j} \right) \left[\frac{y_j - R_0}{R_1 - R_0} \right] \quad (13)$$

⁸ This assumption is compatible with the findings of Michaels and Smith (1990) Kohlhase (1991), Kiel and McClain (1995a, 1995b) and McCluskey and Rausser (2003) when \mathbf{q} is taken to be the distance from hazardous facilities and land use.

if $\frac{y_j - R_0}{R_1 - R_0} > [1 + \frac{1 - \mathbf{b}_j}{1 + \mathbf{a}_j}] \mathbf{q}_j^*$ (see Appendix C).

Corollary 3: If $\frac{y_j - R_0}{R_1 - R_0} > [1 + \frac{1 - \mathbf{b}_j}{1 + \mathbf{a}_j}] \mathbf{q}_j^*$, the optimal environmental quality of one's place of residence is proportional to the ratio of the difference between one's income and rent in the environmentally most hazardous location to the rent-gradient. The proportion-coefficient rises with one's health sensitivity to environmental quality and declines with one's elasticity of utility from consumption. (See proof in Appendix C.)

Corollary 4: If $y_j \geq R_1 + (\mathbf{b}_j / \mathbf{a}_j)(R_1 - R_0)$, one's optimal place of residence is in the environmentally least hazardous location. (See proof in Appendix C.)

In other words, the environmentally least hazardous location is chosen for residence when one's income exceeds the rent in that location by at least the product of the rent-gradient and the ratio of one's consumption elasticity of utility to one's health sensitivity to environmental quality. Of course, if $y_j < R_0$, one cannot even afford residence in the environmentally most hazardous location.

5. Optimal location when environmental quality affects productivity

It is possible that income is affected by location. If productivity is improved by health and health is improved by environment quality (i.e., $\mathbf{a}_j > 0$), income rises with the environmental quality of the place of residence. Ms. Jones extends her

analysis to this case by considering a differentiable income function $y_j(\mathbf{q})$ with $y'_j(\mathbf{q}) > 0$. She obtains

$$\mathbf{q}_j^{**} = \arg \max_{\mathbf{q}} \mathbf{q}^{a_j} u_j(c_j(\mathbf{q})) / (\mathbf{r}_j + \mathbf{m}_j) = \frac{\mathbf{a}_j u_j(c_j(\mathbf{q}_j^{**}))}{c'(\mathbf{q}_j^{**}) u'_j(c_j(\mathbf{q}_j^{**}))} \quad (14)$$

so long that

$$(1 + \mathbf{a}) u'_j(c_j(\mathbf{q}_j^{**})) c'_j(\mathbf{q}_j^{**}) + \mathbf{q}_j^{**} [u''_j(c_j(\mathbf{q}_j^{**})) c'_j(\mathbf{q}_j^{**})^2 + u'_j(c_j(\mathbf{q}_j^{**})) (c''_j(\mathbf{q}_j^{**}))] < 0,$$

where $c_j(\mathbf{q}) \equiv y_j(\mathbf{q}) - R_j(\mathbf{q})$ (see Appendix C).

Ms. Jones assumes that her income rises linearly in her location's environmental quality from the lowest level of y_{0j} in the environmentally most hazardous location to the highest level y_{1j} in the environmentally least hazardous location

$$y_j = y_{0j} + (y_{1j} - y_{0j}) \mathbf{q} \quad (15)$$

and interprets $y_{1j} - y_{0j}$ as her personal income-gradient. By substituting Eq. (12) and Eq. (15) into Eq. (14), Ms. Jones obtains that her optimal location is where the quality of the environment is

$$\mathbf{q}_j^{**} = \left(\frac{\mathbf{a}_j}{\mathbf{a}_j + \mathbf{b}_j} \right) \left[\frac{y_{0j} - R_0}{(R_1 - R_0) - (y_{1j} - y_{0j})} \right] \quad (16)$$

if, and only if, $\frac{(y_{0j} - R_0)}{(R_1 - R_0) - (y_{1j} - y_{0j})} > [1 + \frac{1 - \mathbf{b}_j}{1 + \mathbf{a}_j}] \mathbf{q}_j^{**}$ (see Appendix C).

Corollary 5 (interior solution): The optimal place for a person, who is sensitive to environmental quality and facing a positive (negative) income-rent differential in the environmentally most hazardous location, is in an environment

$$0 < \left(\frac{\mathbf{a}_j}{\mathbf{a}_j + \mathbf{b}_j} \right) \left[\frac{y_{0j} - R_0}{(R_1 - R_0) - (y_{1j} - y_{0j})} \right] < 1 \quad \text{if, and only if, the effect of}$$

environmental quality on this person's income-rent differential is negative (positive).

(See proof in Appendix C.)

Corollary 6: If $\frac{(y_{0j} - R_0)}{(R_1 - R_0) - (y_{1j} - y_{0j})} > [1 + \frac{1 - \mathbf{b}_j}{1 + \mathbf{a}_j} \mathbf{q}_j^{**}]$, the chosen environmental

quality of the place of residence increases with the personal health sensitivity to the environmental quality, with the personal income-gradient and with the personal income-market-rent differential in the environmentally most hazardous location, but decreases with the personal utility's consumption elasticity and with the market rent-gradient. (See proof in Appendix C.)

Corollary 7 (corner solution): The optimal place of residence for a person whose health is sensitive to environmental quality and who is facing a non-negative (zero) income-rent differential in the most hazardous environment and a positive (negative) difference between his income-gradient and the market rent-gradient is in the least (most) hazardous location. (See proof in Appendix C.)

This corollary says that when people's income in the environmentally most hazardous location is at least as large as their rent and when the environmental-quality effect on their income is at least as large as the environmental-quality effect on their rent, they maximise their consumption and minimise their risk of dying by residing in the environmentally least hazardous location. However, when their income in the

environmentally most hazardous location is equal to the rent and the environmental-quality effect on the market rent exceeds the environmental quality effect on their income, people reside in the environmentally most hazardous location since they cannot afford a safer one.

The condition for choosing the environmentally least hazardous location is more generally articulated by the following proposition.

Corollary 8: If $y_{1j} \geq R_1 + (\mathbf{b}_j / \mathbf{a}_j)[(R_1 - R_0) - (y_{1j} - y_{0j})]$, one's optimal place of residence is in the environmentally least hazardous location. (See proof in Appendix C.)

This corollary suggests that the environmentally least hazardous location is chosen for residence when the individual's anticipated income in that location exceeds the rent in that location by at least the product of the difference between the market rent-gradient and her personal income-gradient and the ratio of her consumption elasticity of utility to her health sensitivity to environmental quality. It further implies that the minimum anticipated income (y_{1j}^{\min}) for residing in the environmentally least hazardous location is given by

$$y_{1j}^{\min} = R_1 + \frac{(\mathbf{b}_j / \mathbf{a}_j)(y_{0j} - R_0)}{1 + (\mathbf{b}_j / \mathbf{a}_j)} \quad (17)$$

and, consequently, $\frac{\partial y_{1j}^{\min}}{\partial (\mathbf{b}_j / \mathbf{a}_j)} = \frac{y_{0j} - R_0}{[1 + (\mathbf{b}_j / \mathbf{a}_j)]^2} \begin{matrix} > \\ < \end{matrix} 0$ as $y_{0j} \begin{matrix} > \\ < \end{matrix} R_0$.

6. Residential dispersion and income distribution

The previous sections' analyses of the choice of location suggest that in the absence of asymmetric information about environmental qualities and rents and in the presence of open access, the dispersion of expected-lifetime-utility-maximising people across locations endowed with different environmental qualities is due to personal differences in health sensitivity to environmental quality, in utility's consumption elasticity and in income.

The persistence of hazardous facilities and land use close to residential neighbourhoods might be a reflection of the affected residents' small political power. If political power is associated with economic power it can be expected that the more affluent the neighbourhood, the more effective the lobbying against hazardous facilities and land use in its vicinity. In other words, a strong association between residential location and income might contribute to the persistence of undesired sources of environmental hazards in and near low-income neighbourhoods.

To facilitate the examination of the relationship between the population's residential dispersion and income distribution, the following analysis considers the case described in section 4, in which personal income is not affected by the environmental quality of the place of residence, and assumes that there exists an interior solution to the location choice problem of each person. The analysis considers firstly the case where all people have the same health sensitivity to environmental quality and the same utility's consumption elasticity.

Let $\mathbf{a}_j = \mathbf{a}$ and $\mathbf{b}_j = \mathbf{b}$ for every person $j = 1, 2, 3, \dots, N$ and recall Eq. (13).

Then, the residential location of each person j is given by

$$\mathbf{q}_j^* = \left(\frac{\mathbf{a}}{\mathbf{a} + \mathbf{b}} \right) \left[\frac{y_j - R_0}{R_1 - R_0} \right]. \quad (18)$$

Corollary 9: When all people have identical health sensitivity to environmental quality and identical utility's consumption elasticity, high (low) income earners reside in less (more) exposed neighbourhoods to hazardous facilities and land use.

Furthermore, the residential-location mean is

$$E(\mathbf{q}) = \left(\frac{\mathbf{a}}{\mathbf{a} + \mathbf{b}} \right) \left[\frac{E(y) - R_0}{R_1 - R_0} \right] \quad (19)$$

and the residential-location variance is

$$VAR(\mathbf{q}) = [\mathbf{a} / (\mathbf{a} + \mathbf{b})(R_1 - R_0)]^2 VAR(y). \quad (20)$$

Compatibly with the statistical notion of the concentration coefficient, the residential-dispersion coefficient (RDC) is defined as the ratio of residential-location variance to the residential-location mean

$$RDC = \left(\frac{\mathbf{a}}{(\mathbf{a} + \mathbf{b})(R_1 - R_0)} \right) \frac{VAR(y)}{E(y) - R_0} \quad (21)$$

where $VAR(y) / [E(y) - R_0]$ can be interpreted as the base-rent-adjusted income-dispersion coefficient.

Corollary 10: When all people have identical health sensitivity to environmental quality and identical utility's consumption elasticity, the residential-dispersion coefficient is proportional to the base-rent adjusted income-dispersion coefficient. (See Appendix D for proof.)

Corollary 11: When all people have identical health sensitivity to environmental quality and identical utility's consumption elasticity, the effect of the base-rent adjusted income-dispersion coefficient on the residential-dispersion coefficient is intensified by the population's health sensitivity to environmental quality and moderated by the population's consumption-elasticity of utility and the market rent-gradient. (See Appendix D for proof.)

Let us now consider the case where people's health sensitivities to environmental quality and people's consumption elasticities are not identical and let us denote the population means of health sensitivity to environmental quality, utility's consumption elasticity and income by \mathbf{m}_a , \mathbf{m}_b and \mathbf{m}_y , respectively. Considering the first-order approximation of the interior solution displayed by Eq. (13) in the vicinity of these means, the variance of the chosen environmental quality of the place of residence within the population is given by:

$$\begin{aligned} \text{VAR}(\mathbf{q}) \cong & \left(\frac{\mathbf{m}_y - R_0}{(\mathbf{m}_a + \mathbf{m}_b)^2 (R_1 - R_0)} \right)^2 \{ \mathbf{m}_b^2 \text{VAR}(\mathbf{a}) + \mathbf{m}_a^2 \text{VAR}(\mathbf{b}) + [\mathbf{m}_a (\mathbf{m}_a + \mathbf{m}_b) / (\mathbf{m}_y - R_0)]^2 \text{VAR}(y) \\ & - \mathbf{m}_b \mathbf{m}_a \text{COV}(\mathbf{a}, \mathbf{b}) + [\mathbf{m}_a (\mathbf{m}_a + \mathbf{m}_b) / (\mathbf{m}_y - R_0)] [\mathbf{m}_b \text{COV}(\mathbf{a}, y) - \mathbf{m}_a \text{COV}(\mathbf{b}, y)] \}. \quad (22) \end{aligned}$$

As the computation of the RDC and the assessment of its properties, in this case, are tedious, the following corollaries are focused on the residential variance.

Corollary 12: If \mathbf{m}_a is positive (zero), $\frac{\partial \text{VAR}(\mathbf{q})}{\partial \text{VAR}(y)}$ is positive (zero) and increasing in

\mathbf{m}_a and decreasing with \mathbf{m}_b and $R_1 - R_0$. (See proof in Appendix D.)

This corollary implies that as long as some people's health is sensitive to environmental quality, the residential-location variance increases with the income-variance. The larger the group of such people and the higher their health sensitivity to environmental quality, the more profound the effect of the income variance on the residential-location variance. This effect is moderated by the average utility's consumption elasticity within the population and by the market rent-gradient.

Corollary 13: The residential-location variance increases with the variance of the health sensitivity to environmental quality and with the variance of the consumption elasticity of utility within the population. (See proof in Appendix D.)

Corollary 14: The residential-location variance increases with the covariance between income and environmental sensitivity within the population, but decreases with the covariance between environmental sensitivity and consumption elasticity of utility and with the covariance between income and consumption elasticity of utility within the population. (See proof in Appendix D.)

Corollary 15: If the mean of the environmental sensitivity is larger (smaller) than the mean of the consumption elasticity of utility within the population, the moderating effect of the covariance between income and consumption elasticity of utility dominates (is dominated by) the intensifying effect of the covariance between income and environmental sensitivity on the residential-location variance. (See proof in Appendix D.)

7. Conclusion

A lifetime utility function was constructed under the assumption that life is uncertain and the probability of survival depends upon the environmental quality of the place of residence and the length of the period of exposure to hazardous facilities and land use. Lifetime utility from consumption is increased by the probability of survival. Using this environmental-quality adjusted utility function, the effect of environmental-quality improvement on the highest bid on a residential property was found to be positive and intensified by income and environmental sensitivity, but moderated by the consumption elasticity of utility, rate of time preference and rate of decline of survival probability stemming from continued exposure to hazardous facilities and land use. It was shown that when income is not affected by environmental quality, the optimal environmental quality of the place of residence is proportional to the ratio of the difference between the individual income and the rent in the most hazardous location to the difference between the rents in the least and most hazardous locations. The proportion-coefficient rises with the individual's sensitivity to environmental quality and declines with her utility's consumption-elasticity.

The possibility that productivity is positively affected by environmental quality was considered. It was found that when an interior solution to the location choice problem exists, the optimal environmental quality of the place of residence increases with the personal sensitivity to the environmental quality, with the marginal effect of the environmental quality on income and with the income-rent differential in the most hazardous location, but decreases with the utility's consumption elasticity and with the marginal effect of the environmental quality on rent. Under certain

circumstances the solution to the location-choice problem is corner. When income in the most hazardous location is at least as large as the rent and when the environmental-quality effect on income is at least as large as its effect on rent, people maximise their consumption and minimise their risk of dying by residing in the least hazardous location. However, when income in the most hazardous location is equal to the rent and the environmental-quality effect on the market rent exceeds its effect on income people reside in the most hazardous location as they cannot afford renting, or bearing the user cost of, a property in a less hazardous location.

When people are endowed with identical health sensitivities to environmental hazards and consumption elasticities of utility, their residential dispersion is closely related to the distribution of income. In the more likely case of heterogeneous population, the level of residential dispersion increases with the variances of income, environmental sensitivity and utility's consumption elasticity and with the covariance between income and environmental sensitivity within the population, but decreases with the covariances between environmental sensitivity and utility's consumption elasticity and between income and utility's consumption elasticity within the population. The moderating effect of the covariance between income and utility's consumption elasticity on the level of residential dispersion dominates the intensifying effect of the covariance between income and environmental sensitivity on the level of residential dispersion when the mean of environmental sensitivity is larger than the mean of utility's consumption elasticity within the population.

Frequently, political power is related to economic power, in which case a strong association between residential location and income might contribute to the persistence of hazardous facilities and land use in the vicinity of poor

neighbourhoods, whose lobbying effort is the least effective. The interior solution to the location-choice problem implies that a high level of income inequality within a population of rational people is a necessary, but not sufficient, condition for a strong association between residential location and income. A low correlation between income and sensitivity to environmental quality and a high correlation between income and utility's consumption elasticity within the population may lead to residential dispersion where household income is not correlated with neighbourhood's environmental quality. However, the existence of hazardous facilities and land use in, or near, populated areas is perpetuated by a strong association between residential location and income, and, in turn, intensifies the association between residential location and income, when there is a high correlation between income and health sensitivity to environmental quality and a low correlation between income and utility's consumption elasticity within the population. Furthermore, the higher the income-disparity level the greater the likelihood of persistent hazardous facilities and land use in the vicinity of poor neighbourhoods. When the level of income disparity is very high, corner solutions to individuals' location choice problems are likely to be a common phenomenon, leading to residential polarisation of the population: low-income earners living in low-environmental-quality locations and high-income earners residing in high-environmental-quality neighbourhoods.

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Appendix A: An explanation of the transition from Eq. (1) to Eq. (2)

Recall that

$$f(t) = F'(t) \tag{A1}$$

where the subscript j is omitted for convenience. Then, Eq. (1) can be rendered as

$$V = \int_0^{\infty} F'(t) \left\{ \int_0^t e^{-rt} u_t dt \right\} dt = \int_0^{\infty} v dU \tag{A2}$$

where,

$$v = \int_0^t e^{-rt} u_t dt \tag{A3}$$

and

$$U = -(1 - F(t)). \tag{A4}$$

The integration by parts rule suggests that

$$V = \int_0^{\infty} v dU = UV - \int_0^{\infty} U dv. \tag{A5}$$

Note, however, that

$$UV = - \left[(1 - F(t)) \int_0^t e^{-rt} u_t dt \right]_0^{\infty} = 0 \tag{A6}$$

because when evaluated at the lower limit

$$UV = - \left[(1 - F(0)) \int_0^0 e^{-rt} u_t dt \right] = 0 \tag{A7}$$

and when evaluated at the upper limit

$$UV = - \left[(1 - F(\infty)) \int_0^{\infty} e^{-rt} u_t dt \right] = 0 \tag{A8}$$

as

$$F(\infty) = 1. \quad (\text{A9})$$

Hence,

$$V = -\int_0^{\infty} U dv. \quad (\text{A10})$$

By virtue of equation (A3)

$$dv = e^{-rt} dt \quad (\text{A11})$$

and the substitution of equations (A4) and (A11) into (A10) implies

$$V = \int_0^{\infty} e^{-rt} u_t \Omega(t) dt \quad (\text{A12})$$

where

$$\Omega(t) \equiv -U_t = 1 - F(t) \quad (\text{A13})$$

and indicating the probability of living at least until t .

Appendix B: Proofs of corollaries 1 and 2

Proof of Corollary 1: For $0 < \mathbf{a}_j \leq 1$ the fact that $\mathbf{q}_B / \mathbf{q}_A < 1$ implies that

$u_j(y_j - R_B^j) > u_j(y_j - R_A^j)$ and, recalling that $u'_j > 0$, $R_A^j > R_B^j$. For $\mathbf{a}_j = 0$,

$u_j(y_j - R_B^j) = u_j(y_j - R_A^j)$ and, recalling $u'_j > 0$, $R_A^j = R_B^j$.

Proof of Corollary 2: By differentiating Eq. (10),

$\frac{\partial(P_A^j - P_B^j)}{\partial(\mathbf{q}_A / \mathbf{q}_B)} = \frac{\mathbf{a}_j (y_j - R_A^j)}{\mathbf{b}_j (\mathbf{r}_j + \mathbf{m}_j)} (\mathbf{q}_A / \mathbf{q}_B)^{-(\mathbf{a}_j / \mathbf{b}_j + 1)} > 0$ and increases with y_j and \mathbf{a}_j but

decreases with \mathbf{b}_j , \mathbf{r}_j and \mathbf{m}_j .

**Appendix C: First and second order conditions for maximum and proofs of
corollaries 3-7**

The location-choice problem is

$$\max_{\mathbf{q}} \{V = \mathbf{q}^{\mathbf{a}} u(c(\mathbf{q})) / (\mathbf{r} + \mathbf{m})\}$$

where the index j is omitted for tractability and $c \equiv y - R$.

The first-order condition (f.o.c.) for maximum

$$\frac{dV_j}{d\mathbf{q}} = \mathbf{q}^{\mathbf{a}-1} [\mathbf{a}u(c(\mathbf{q})) + \mathbf{q}u'(c(\mathbf{q}))c'(\mathbf{q})] = 0$$

implies

$$\mathbf{q} = \frac{\mathbf{a}u(c(\mathbf{q}))}{c'(\mathbf{q})u'(c(\mathbf{q}))}.$$

The second-order condition (s.o.c.) for maximum is

$$\frac{d^2V_j}{d\mathbf{q}^2} = (\mathbf{a} - 1)\mathbf{q}^{\mathbf{a}-2} \underbrace{[\mathbf{a}u + \mathbf{q}u'c']}_0 + \mathbf{q}^{\mathbf{a}-1} \{(1 + \mathbf{a})u'c' + \mathbf{q}[u''c'^2 + u'c'']\} < 0.$$

From the f.o.c., the first term on the r.h.s. of the s.o.c. is equal to zero, and as $c(\mathbf{q})$ is taken to be linear, the s.o.c. can be rendered as

$$(1 + \mathbf{a})u'(c(\mathbf{q})) < \mathbf{q}u''(c(\mathbf{q}))c'(\mathbf{q}).$$

Recalling also that $u = c^{\mathbf{b}}$,

$$(1 + \mathbf{a})\mathbf{b}c(\mathbf{q})^{b-1} < \mathbf{q}\mathbf{b}(\mathbf{b} - 1)c(\mathbf{q})^{b-2}c'(\mathbf{q})$$

or, equivalently,

$$(1 + \mathbf{a})c(\mathbf{q}) < \mathbf{q}(\mathbf{b} - 1)c'(\mathbf{q}).$$

By rearranging terms,

$$\frac{c(\mathbf{q})}{-c'(\mathbf{q})} > \frac{1 - \mathbf{b}}{1 + \mathbf{a}}\mathbf{q}.$$

When only rent is affected by location,

$$c(\mathbf{q}) = y - [R_0 + (R_1 - R_0)\mathbf{q}]$$

and the s.o.c. for maximum requires

$$\frac{y - R_0 - (R_1 - R_0)\mathbf{q}}{R_1 - R_0} > \frac{1 - \mathbf{b}}{1 + \mathbf{a}}\mathbf{q}$$

or, equivalently,

$$\frac{y - R_0}{R_1 - R_0} > \left[1 + \frac{1 - \mathbf{b}}{1 + \mathbf{a}}\right]\mathbf{q}$$

where, by virtue of our assumption ($0 \leq \mathbf{a} \leq 1$ and $0 < \mathbf{b} < 1$), the term on the r.h.s. is positive.

When income is also affected by location

$$c(\mathbf{q}) = [y_0 + (y_1 - y_0)\mathbf{q}] - [R_0 + (R_1 - R_0)\mathbf{q}]$$

the s.o.c. for maximum requires

$$\frac{(y_0 - R_0) + [(y_1 - y_0) - (R_1 - R_0)]\mathbf{q}}{-[(y_1 - y_0) - (R_1 - R_0)]} > \frac{1 - \mathbf{b}}{(1 + \mathbf{a})}\mathbf{q}$$

or, equivalently,

$$\frac{(y_0 - R_0)}{(R_1 - R_0) - (y_1 - y_0)} > [1 + \frac{1 - \mathbf{b}}{1 + \mathbf{a}}]\mathbf{q}$$

where the term on the r.h.s. is positive.

Proof of Corollary 3: If $\frac{y_j - R_0}{R_1 - R_0} > [1 + \frac{1 - \mathbf{b}_j}{1 + \mathbf{a}_j}]\mathbf{q}_j^*$ there exists an interior solution to the

location-choice problem. By substituting Eq. (12), $R' = R_1 - R_0$ and $u = c^b$ into Eq.

(11), $\mathbf{q}_j^* = \left(\frac{\mathbf{a}_j}{\mathbf{a}_j + \mathbf{b}_j} \right) \left[\frac{y_j - R_0}{R_1 - R_0} \right]$. The rest is straightforward from this expression.

Proof of Corollary 4: From Eq. (13), $\mathbf{q}_j^* = 1$ if $\left(\frac{\mathbf{a}_j}{\mathbf{a}_j + \mathbf{b}_j} \right) \left[\frac{y_j - R_0}{R_1 - R_0} \right] \geq 1$. By

rearranging the terms in this inequality, $\mathbf{q}_j^* = 1$ if

$$y_j \geq R_0 + (1 + \mathbf{b}_j / \mathbf{a}_j)(R_1 - R_0) = R_1 + (\mathbf{b}_j / \mathbf{a}_j)(R_1 - R_0).$$

Proof of Corollary 5: The s.o.c. requires that $\frac{(y_0 - R_0)}{(R_1 - R_0) - (y_1 - y_0)} > [1 + \frac{1 - \mathbf{b}}{1 + \mathbf{a}}]\mathbf{q}$, where

the term on the r.h.s. is positive. If $y_0 - R_0 > 0$ the satisfaction of the s.o.c. requires

that $R_1 - R_0 > y_1 - y_0$. If $y_0 - R_0 < 0$ the satisfaction of the s.o.c. requires

that $R_1 - R_0 < y_1 - y_0$.

Proof of Corollary 6: Straightforward from Eq. (16).

Proof of Corollary 7: $\frac{(y_{0j} - R_0)}{(R_1 - R_0) - (y_{1j} - y_{0j})} < 0$ and hence the s.o.c. for maximum is

not satisfied and the solution to the location-choice problem is corner: $\mathbf{q}_j^{**} = 0$ or 1.

When $y_{0j} \geq R_0$ and $y_{1j} - y_{0j} \geq R_1 - R_0$ environmentally sensitive individuals maximise their consumption and minimise their risk of dying by residing in the least hazardous environment. When $y_{0j} = R_0$ and $y_{1j} - y_{0j} < R_1 - R_0$ people reside in the most hazardous location since safer ones are not affordable.

Proof of Corollary 8: From Eq. (16), $\mathbf{q}_j^* = 1$ if

$$\left(\frac{\mathbf{a}_j}{\mathbf{a}_j + \mathbf{b}_j} \right) \left[\frac{y_{0j} - R_0}{(R_1 - R_0) - (y_{1j} - y_{0j})} \right] \geq 1.$$

By rearranging the terms in this inequality, $\mathbf{q}_j^* = 1$ if

$$y_{1j} \geq R_0 + (1 + \mathbf{b}_j / \mathbf{a}_j)[(R_1 - R_0) - (y_{1j} - y_{0j})] = R_1 + (\mathbf{b}_j / \mathbf{a}_j)[(R_1 - R_0) - (y_{1j} - y_{0j})]$$

Appendix D: Proofs of corollaries 9 and 15

Proof of Corollary 9: Straightforward from Eq. (18).

Proof of Corollary 10: Straightforward from Eq. (21).

Proof of Corollary 11: Straightforward from Eq. (21).

Proof of Corollary 12: From Eq. (22), $\frac{\partial VAR(\mathbf{q})}{\partial VAR(y)} = [\mathbf{m}_a / (\mathbf{m}_a + \mathbf{m}_b)(R_1 - R_0)]^2 \geq 0$ as

$\mathbf{m}_a \geq 0$, which further implies that $\frac{\partial^2 VAR(\mathbf{q})}{\partial VAR(y)\partial \mathbf{m}_a} > 0$, $\frac{\partial^2 VAR(\mathbf{q})}{\partial VAR(y)\partial \mathbf{m}_b} < 0$ and

$$\frac{\partial^2 VAR(\mathbf{q})}{\partial VAR(y)\partial (R_1 - R_0)} < 0.$$

Proof of Corollary 13 and Corollary 14: Straightforward from Eq. (22) as long as $\mathbf{m}_y - R_0 > 0$. Supply and demand consideration and the assumption that rent increases with environmental quality imply that $\mathbf{m}_y - R_0 > 0$. Otherwise, people with income lower than the mean, as well as people earning the mean income and consume, could not afford renting a residential property.

Proof of Corollary 15: From Eq. (22),

$$\frac{\partial VAR(\mathbf{q})}{\partial COV(\mathbf{b}, y)} - \frac{\partial VAR(\mathbf{q})}{\partial COV(\mathbf{a}, y)} \cong \left(\frac{\mathbf{m}_y - R_0}{(\mathbf{m}_a + \mathbf{m}_b)^2 (R_1 - R_0)} \right)^2 (\mathbf{m}_a - \mathbf{m}_b) \begin{matrix} > \\ < \end{matrix} 0 \text{ as } (\mathbf{m}_a \begin{matrix} > \\ < \end{matrix} \mathbf{m}_b).$$