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# Rational Non-addictive Eating: Cycles, Overweightness, and Underweightness

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# RATIONAL NON-ADDICTIVE EATING:

## CYCLES, OVERWEIGHTNESS AND UNDERWEIGHTNESS

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**Abstract**

Although a deviation from the physiologically optimal weight increases the probability of dying, the steady state for a lifetime expected-utility maximiser is a state of overweightness. However, even a small initial deviation from this rationally stationary weight is followed by explosive oscillations. These oscillations might lead to severe and chronic underweightness in a late stage of life. In the presence of socio-cultural norms of appearance, the rationally stationary weight of fat people is lower than otherwise and the rationally stationary weight of lean people is greater than otherwise. (*JEL* I12)

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## 1. Introduction

Cycles in food consumption (binges and strict diets in particular) and overweightness and underweightness (obesity and anorexia in particular) can result from psychological, physiological and environmental problems. They may also reflect attempts to conform to social and cultural norms of appearance. This paper shows that in a stylised world where there are no psychological, physiological and environmental problems and where no social and cultural pressures exist the empirically observed overweightness, underweightness and cyclical food consumption can be caused by a rational non-addictive eating.

In an attempt to explain cycles in food consumption Dockner and Feichtinger (1993) have modified Becker and Murphy's (1988) rational addiction model by considering two negatively correlated stocks of consumption capital: an addictive stock and weight. The present analysis presents a different approach: eating is considered to be non addictive and the possibility of rational cyclical food consumption, as well as overweightness and underweightness, is explained by assuming that utility is derived by consuming food (taken as a homogeneous aggregate), that for any individual there is a *physiologically optimal weight*, that the larger the deviation from the physiologically optimal weight the higher the probability of dying, and that a rational person considers the risk stemming from becoming overweight, or underweight, and plans his, or her, food-consumption trajectory so as to maximise his, or her, expected lifetime utility.

A *rationally optimal weight* trajectory is defined as the weight trajectory associated with the food-consumption path which maximises the individual's expected lifetime utility. The *rationally optimal weight* may not necessarily be equal, or converging, to the *physiologically optimal weight*. A positive difference between the rationally optimal weight and the physiologically optimal weight indicates the individual's rationally optimal level of overweightness, whereas a negative weight differential reflects his, or her, rationally optimal level of underweightness. Overweightness and underweightness can be considered to be *chronic* if the rationally optimal weight-trajectory does not converge to the physiologically optimal weight and can be diagnosed as *acute* if the rationally optimal weight trajectory divergs from the physiologically optimal weight.

A manageable, stylised, basic, dynamic model of rational, non-addictive, eating model is developed. It is shown that when physiological, psychological, environmental and socio-cultural reasons for divergence from a physiologically optimal weight do not exist, the steady state is a state of overweighness. However, it is asymptotically unstable: even a small initial deviation from this stationary weight is followed by explosive oscillations and binges which might lead in a late stage of life to severe and chronic underweightness. The model is extended to the case where socio-cultural norms of appearance exist. It is demonstrated that in the presence of such norms the stationary weight of fat people is lower than otherwise and the stationary weight of lean people is greater than otherwise.

The paper is structured as follows. A basic dynamic model of rational non-addictive eating is introduced in section 2. The stationary weight advocated by the model is described in section 3. The possibility of convergence to, or divergence from, an initial weight to the stationary rationally optimal weight and cyclical consumption is analysed in section 4. The effect of socio-culturally preferred weight is analysed in section 5. Comments about possible extensions of the basic model are made in section 6.

## 2. A basic model of rational non-addictive eating

It is assumed that the probability of dying ( $p$ ) at time $t$ is quadratic in the deviation of weight ($W$) from the physiologically optimal weight ($W*$). Let $r$ be the rate of time preference, $T$ the upper-bound on life expectancy, and $u$ a concave instantaneous utility function of food consumption ($c$). Then the decision problem of a rational food-consumer can be formally presented as

$$\max_{\{c\}} \int_0^T p((W(t)-W*)^2)\left\{\int_0^t e^{-rt}u(c(t)dt\right\}dt$$

subject to a weight-motion equation where weight is gained by consuming food and is lost in proportion to the individual's weight

$$\dot{W}(t) = c(t) - d(W(t)). \qquad (1)$$

In this setting $d$ is a positive scalar indicating the marginal, and average, effect of weight on burning calories, and consequently reducing weight, in performing various physiological functions.

Integrating by parts, the rational food-consumer's objective can be rendered as

$$\max_{\{c\}} \int_0^T e^{-rt} u(c(t))\Phi((W(t) - W^*)^2)]dt$$

where $\Phi$ is equal to 1 minus the cumulative density function associated with $p$ and hence indicates the probability of living beyond $t$. It is assumed that $\Phi$ is diminishing and concave in $(W - W^*)^2$. This assumption and the concavity of $u$ ensure that an interior solution exists.

The corresponding Hamiltonian is

$$H = e^{-rt} u(c(t))\Phi((W(t) - W^*)^2) + l(t)[c(t) - dW(t)] \qquad (2)$$

where $l$ is a co-state variable. The necessary conditions for maximum are:

$$\dot{l} = -e^{-rt} u(c)\Phi_W((W - W^*)^2) + ld \qquad \text{(the adjoint equation)} \qquad (3)$$

$$e^{-rt}u_c\Phi((W-W^*)^2)+ l = 0 \qquad \text{(the optimality condition)} \qquad (4)$$

the weight-motion equation (1) and the transversality condition $l(T)W(T)=0$.

The necessary conditions and the associated singular control imply that the rationally optimal food-consumption and weight trajectories should satisfy the following no-arbitrage rule:

$$\frac{u_{cc}}{u_c}\dot{c}+\frac{\Phi_W}{\Phi}\dot{W}-\frac{u}{u_c}\frac{\Phi_W}{\Phi}=r+d\,. \qquad (5)$$

The implications of this rule for the rationally optimal stationary levels and trajectories of food-consumption and weight and the possibility of cycles are discussed in the following sections with analytically convenient explicit specifications of $\Phi$ and $u$.

## 3. The steady state: rational overweightness

By setting of $\dot{c}$ and $\dot{W}$ to be equal to zero equation (5) implies that the stationary levels of the rationally optimal food-consumption and weight should satisfy

$$-\frac{u}{u_c}\frac{\Phi_W}{\Phi}=r+d\,. \qquad (6)$$

Let the probability of living beyond $t$ be given by

$$\Phi=\Phi_0 e^{-m(W-W^*)^2} \qquad (7)$$

where $m$ is a positive scalar and $0 < \Phi_0 < 1$ is the intrinsic probability of living beyond $t$ (i.e., $\Phi$ for a person having the physiologically optimal weight); and let the instantaneous satisfaction from eating be given by

$$u = c^b \tag{8}$$

where $0 < b < 1$ is the utility elasticity. Then the stationary levels of the rationally optimal food-consumption and weight should satisfy

$$c_{ss}(W_{ss} - W^*) = \frac{(r+d)b}{2m}. \tag{9}$$

Recalling also that the weight-motion equation (1) implies that in steady state

$$c_{ss} = dW_{ss} \tag{10}$$

equation (9) can be rendered as a second-order polynomial of $W_{ss}$ whose only relevant and feasible solution is

$$W_{ss} = 0.5W^* + 0.5\sqrt{W^{*2} + 2b(r+d)/dm}. \tag{11}$$

This result suggests that the rationally optimal stationary weight is greater than the physiologically optimal weight. It also suggests that the rationally optimal stationary level of overweightness rises with the rate of time preference and the elasticity of utility but declines with the rate of burning calories ($d$) and the rate of decline ($m$) of the probability of living beyond $t$ stemming from an infinitesimal rise in the quadratic deviation from the physiologically optimal weight.

**4. Can rational non-addictive eating be cyclical? Would it lead to the rationally optimal stationary level of overweightness?**

To answer these questions let us construct the possible trajectories of weight and food-consumption satisfying equations (1) and (5) in the phase-plane diagram. In recalling equations (7) and (8), equation (5) implies that the optimal rate of change in food consumption is

$$\frac{\dot{c}}{c} = \frac{(r+d) + \left(\frac{2m(b-1)}{b}\right)(W - W^*)c - 2mdW^2 + 2mdW^*W}{b-1}. \quad (12)$$

Hence, the isocline $\dot{c} = 0$ is given by

$$(r+d) + \left(\frac{2m(b-1)}{b}\right)(W - W^*)c - 2mdW^2 + 2mdW^*W = 0 \quad (13)$$

and its slope by

$$\left.\frac{dc}{dW}\right|_{\dot{c}=0} = -\left[\frac{db}{1-b} + \frac{c + \frac{db}{1-b}}{W - W^*}\right] \quad (14)$$

which is negative for $\tilde{W} > W > W^*$ and positive for $\tilde{W} < W < W^*$ as displayed in Figure 1, where

$$\tilde{W} = W^* - \left[1 + \left(\frac{1-b}{db}\right)c\right]. \quad (15)$$

The direction of the change in food consumption above and below the isocline $\dot{c} = 0$ is obtained by differentiating equation (12) with respect to $W$ and displayed by the vertical arrows.

By virtue of equation (1), the isocline $\dot{W} = 0$ is the locus of all the combinations of $c$ and $W$ for which

$$c = dW. \tag{16}$$

It is displayed in Figure 1 by a line whose slope is equal to $d$. Above this line $\dot{W} > 0$ and below it $\dot{W} < 0$ as displayed by the horizontal arrows.

***Figure 1 to be inserted here.***

The rationally optimal stationary level of overweightness is displayed by the intersection point of the two isoclines. The directions of the horizontal and vertical arrows in the phase-plane diagram suggest that the stationary point can be either a spiral or a centre. The steeper the isocline $\dot{c} = 0$ and the flatter the isocline $\dot{W} = 0$ (i.e., the smaller $d$) the larger the fluctuations around the stationary point. That is, the model suggests that binges and strict diets, and more generally cyclical food consumption and weight, may be rational. As demonstrated in the following, the food consumption and weight cycles are explosive and the stationary level of overweighness is unstable.

The linearization of the dynamic system consisting of equations (1) and (12) at the vicinity of steady state implies that the 2x2 state-transition (or Jacobian) matrix $a$ have the following elements:

$$a_{11} = 1 \tag{17a}$$

$$a_{12} = -d < 0 \tag{17b}$$

$$a_{21} = -\left(\frac{r+d}{1-b}\right) + \left(\frac{2c_{ss}}{b} + \frac{dW_{ss}}{1-b}\right) 2m(W_{ss} - W^*) \tag{17c}$$

$$a_{22} = \frac{2mc_{ss}^2}{b} + \frac{4mdc_{ss}}{1-b}(W_{ss} - 0.5W^*) > 0 \tag{17d}$$

where the sign of $a_{21}$ is likely to be positive for a large value of $m$ (i.e. large effect of diverging from the physiologically optimal weight on the probability of dying), for a large value of $W_{ss} - W^*$ (i.e., obesity) and for a low rate of time preference $r$ (i.e., far-sighted individual), but negative otherwise. The eigenvalues of $a$ are

$$E_{1,2} = 0.5\left[\underbrace{(a_{11} + a_{22})}_{Positive} \pm \sqrt{\underbrace{(a_{11} + a_{22})^2 - 4(a_{11}a_{22} - a_{21}a_{12})}_{Negative}}\right]. \tag{18}$$

Recalling that the directions of the horizontal and vertical arrows in the phase-plane diagram imply that the stationary point is either a spiral or a centre, the above discriminant is negative and the eigenvalues are conjugate-complex pair. Since their real part $a_{11} + a_{22}$ is positive the stationary point is asymptotically unstable as illustrated in Figure 1 by a diverging spiral. That is, there is no convergence to the stationary level of overweightness but rather explosive oscillations around it. It is illustrated further by the trajectory starting at point $A$ near the steady state that there is a possibility of a chronic loss of weight even below the physiologically optimal level in a late stage of life. The greater $d$, the steeper the isocline $\dot{W} = 0$ and the greater the likelihood of becoming chronically and fatally underweight.

**5. The effect of social and cultural preferences on weight**

Overweightness and underweightness might also stem from conforming to social and cultural norms and perceptions. For instance, in some oriental countries overweightness is associated with beauty, contentment, grace and wisdom, whereas in some occidental countries slenderness is much more appreciated. If this associations between physical attractiveness and weight is valid it is expected that, *ceteris paribus*, oriental slims and occidental fats are the less fortunate people in their domestic markets of partners. It is shown in the following, however, that despite their failure to conform to the socio-cultural norms of physical appearance, their weight is closer to the socio-culturally preferred weight than otherwise.

The socio-culturally preferred weight can be incorporated into the basic model by assuming that the individual suffers from a loss of utility by not conforming to the socio-culturally preferred weight ($W^{sp}$). In this case, the rational food-consumer's objective can be rendered as

$$\max_{\{c\}} \int_0^T e^{-rt} u(c(t),(W(t)-W^{sp})^2)\Phi((W(t)-W^*)^2)]dt$$

subject to a weight-motion equation (1). The necessary conditions for maximum and the associated singular control equation imply that the rationally optimal food-consumption and weight trajectories should satisfy the no-arbitrage rule

$$\frac{u_{cc}}{u_c}\dot{c}+\left[\frac{u_{cW}}{u_c}+\frac{\Phi_W}{\Phi}\right]\dot{W}-\left[\frac{u}{u_c}\frac{\Phi_W}{\Phi}+\frac{u_W}{u_c}\right]= r+d \qquad (19)$$

and that in steady state

$$-\left[\frac{u}{u_c}\frac{\Phi_W}{\Phi}+\frac{u_W}{u_c}\right]=r+d\,. \tag{20}$$

In order to compare the stationary weight in this case ($W_{ss}'$) to the stationary weight case where no socio-cultural pressure exists ($W_{ss}$), the probability of living beyond $t$ is specified as depicted earlier by equation (7) and the instantaneous utility from eating described by equation (8) is deflated by not conforming to the socio-culturally preferred weight as follows:

$$u(t)=\frac{c^b}{(W(t)-W^{sp})^2}\,. \tag{21}$$

These specifications imply that the stationary weight when there is a socio-culturally preferred weight should satisfy

$$W_{ss}'[m(W_{ss}'-W^*)+(W_{ss}'-W^{sp})^{-1}]=0.5b(r+d)/d \tag{22}$$

whereas the stationary weight when there is no socio-cultutrally preferred weight should satisfy

$$W_{ss}[m(W_{ss}-W^*)]=0.5b(r+d)/d\,. \tag{23}$$

If $W_{ss}'>W^{sp}$, which is the likely characteristic of occidental people who are less successful in the matching game, then $W_{ss}'<W_{ss}$. That is, when there exists a socio-

cultural norm of appearance their stationary weight is lower than otherwise. If, however, $W_{ss}' < W^{sp}$, which is the likely case of oriental people who are less successful in the matching game, then $W_{ss}' > W_{ss}$. That is, when there exists a socio-cultural norm of appearance their stationary weight is greater than otherwise.

**6. Conclusion**

The paper presented a manageable stylised basic model of non-addictive eating that can explain overweightness, underweightnes and cyclical food consumption by assuming that the satisfaction from eating is counterbalanced by increasing probability of dying as weight deviates from the physiologically optimal level. It was found that when physiological, psychological, environmental and socio-cultural reasons for divergence from a physiologically optimal weight do not exist, the steady state for a lifetime expected-utility maximiser is a state of overweighness. The rationally optimal stationary level of overweightness was shown to be rising with the individual’s rate of time preference and elasticity of utility but declining with his, or her, rate of calories burning and rate of decline of the probability of continuing living caused by an infinitesimal rise in the quadratic deviation from the physiologically optimal weight. It was shown, however, that even a small initial deviation from this rationally optimal stationary weight is followed by explosive oscillations, i.e., binges and strict diets, and possibly a severe and chronic underweightness in a late stage of life. The incorporation of socio-cultural norms into the basic model revealed that when there exists a socio-cultural norm of appearance the stationary weight of fat people is lower than otherwise and the stationary weight of lean people is greater than otherwise.

Some other possible effects of deviation from the physiologically optimal weight and budget considerations were not incorporated into the model for sake of simplicity. In a broader framework, instantaneous utility may be presented as derived from eating and devoting a fraction ($\ell$) of an instance to leisure activities (the rest, $1-\ell$, is devoted to income-generating activities), and a deviation from the physiologically optimal weight might also lead to a loss of satisfaction from leisure activities and a loss of efficiency in generating income. Intuitively, it is expected that the incorporation of these elements would reduce the difference between the rationally optimal stationary weight and the physiologically optimal weight, dampen the oscillations of the food-weight joint trajectory, and increase the likelihood of convergence to the stationary weight. An extension of the basic model along these lines is described in the Appendix. However, the complexity of the extended model rendered the assessment of this intuitive assertion and the properties of the optimal trajectories of food consumption, leisure and weight impossible.

**References**


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**APPENDIX: An Extension of the Basic Model**

An intensive effort has been made in extending the basic model. By incorporating the elements indicated in the concluding section into the basic model, the modified decision problem of a rational food-consumer can be portrayed as follows:

$$\max_{\{c,\ell\}} \int_0^T e^{-rt} u(c(t),[1-g\,(W(t)-W^*)^2\,]\ell(t))\Phi((W(t)-W^*)^2\,)dt$$

where weight is gained by consuming food and is lost by spending energy on work and leisure activities and proportionally to its current level

$$\dot{W}(t) = c(t) - [d_1\ell + d_2(1-\ell)]W(t) \qquad \text{(A1)}$$

and where

$$c(t) = \underbrace{[1-q(W(t)-W^*)^2\,]\,f\,(1-\ell(t))}_{Income}. \qquad \text{(A2)}$$

In this framework, $d_1$ and $d_2$ are positive scalars indicating the marginal effect of weight on burning calories, and subsequently weight, in leisure activities and work, respectively; $g$ and $q$ are positive scalars indicating the marginal effects of being overweight or underweight on satisfaction from leisure activities and on income generation, respectively; and $f$ is a concave earning function ( $f\,'>0$ and $f\,''<0$ ) for a person having physiologically optimal weight.

Finally, the degree of complexity was largely increased when equation (7) was modified as follows

$$\Phi = \Phi_0 e^{-m\frac{(W(t)-W^*)^2}{T-t}} \tag{A3}$$

to incorporate the possible effect of age and the moderating influence of the distance from the upper-bound on life expectancy on the adverse effect of a deviation from the physiological weight on the probability of living beyond $t$.

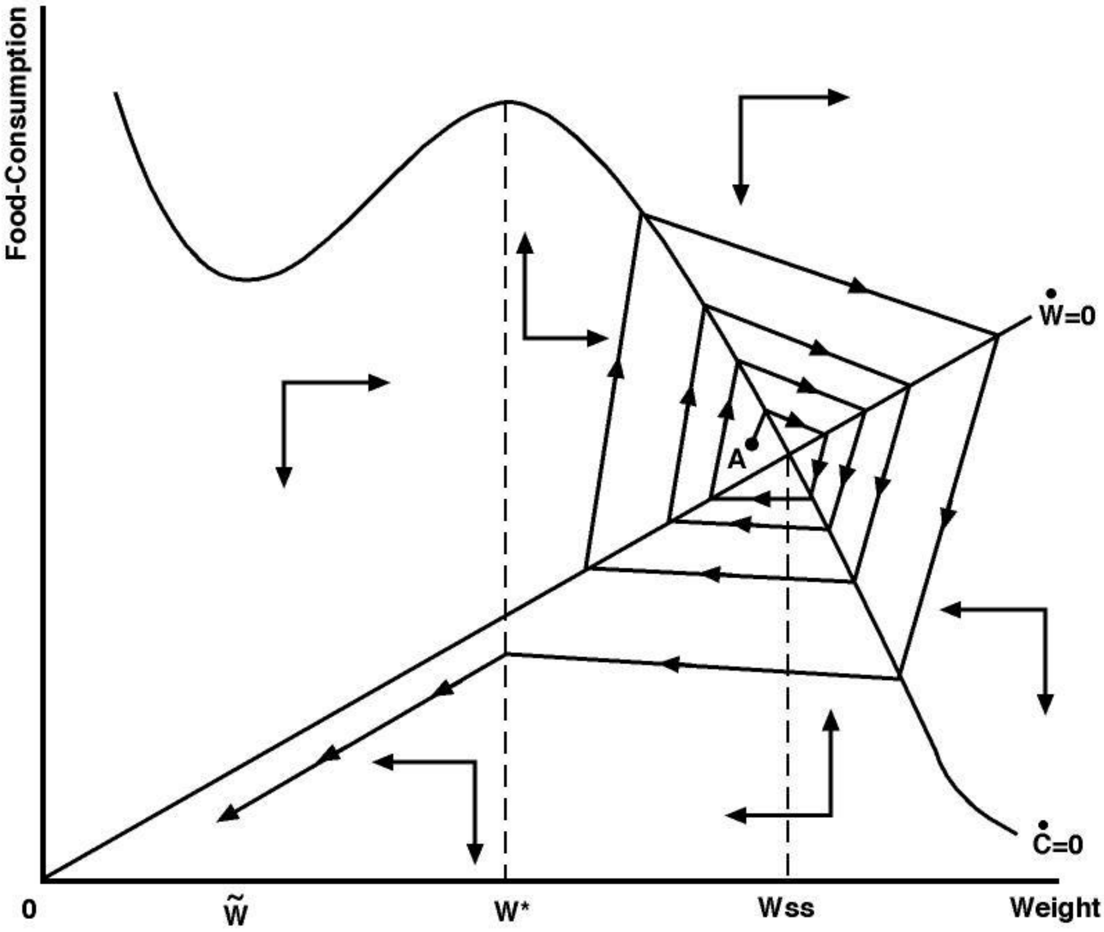


**Figure 1. Phase-plane diagram of food consumption and weight**