A Theory of Rational Junk-Food Consumption

by

Amnon Levy
University of Wollongong

Abstract: The expected lifetime-utility maximizing diet requires a balance between the marginal satisfaction from junk-food consumption, discounted by both the individual’s time preference and prospects of survival, and the value of the marginal damage to the individual health caused by consuming junk-food … To be completed!

JEL I12

Key words: junk food, health food, relative price, relative taste, risk, natural recovery, full capacity income, expected lifetime utility, rational consumption, health, health value

Corresponding Author: Amnon Levy, Economics, School of Economics and Information Systems, Faculty of Commerce, University of Wollongong, Wollongong, NSW 2522, Australia. Tel: 61-2-42213658. E-mail: amnon_levy@uow.edu.au
A Theory of Rational Junk-Food Consumption

I. Introduction

Food can be classified in accordance with its fat, sugar and salt contents as junk or healthy. Because of its high concentration of sugar, fat and salt, junk food is often tastier than its low calories, leaner and less salty substitute. Due to its relatively expensive ingredients, health food is often more expensive than junk food.¹ The short-term taste and price advantages of junk food are, at least, partially offset by the long-term adverse effects of junk food on health and life expectancy. Rational food consumers are not myopic - they are aware of the short-term advantages and long-term disadvantages of junk-food consumption. In addition to the taste and price differentials, rational consumers incorporate the risk differential into an expected lifetime utility maximization analysis of the composition of junk-food and health-food products in their diet.

Taste, price and risk differences are not exclusive to junk-food products and their healthier substitutes. They may also provide an explanation to decisions on the consumption of commodities such as coffee, tea, beer and self-rolled cigarettes. The comparison of the taste, price and health impeding effects of coffee, tea, beer and self-rolled cigarettes to those of their healthier substitutes (decaffeinated coffee, herbal tea, light beer and filter cigarettes, respectively) within a lifetime utility maximization framework with uncertain life expectancy constitutes a complementary approach to the rational addiction model proposed by Gary Becker and Kevin Murphy (1988) and applied by Frank Chaloupka (1991), Gary Becker, Michael Grossman and Kevin Murphy (1994), Nilss Olekalns and Peter Bardsley (1996), Michael Grossman, Frank

¹ For instance, diet ice cream is relatively expensive because of its synthetic sweetening and creaming inputs. Yet, for many consumers, diet ice cream is not as tasty as its buttery and sugary rival.
Chaloupka and Ismail Sirtalan (1998) and many others to the consumption of cigarettes, alcohol and coffee.

It is possible that junk food and/or health food are addictive for some people. John Cawley’s (1999) empirical findings on the consumption of calories lend support to the hypothesis that some types of junk food are addictive. However, addiction and, in particular, the controversial concept of rational addiction are not the scope of the present paper. Consistently with Karen Dynan’s (2000) empirical findings with panel household data, the present analysis assumes that food consumption is neither addictive nor a formed habit. That is, the stocks of junk-food consumption and health-food consumption are not considered as moderating the individual’s level of satisfaction from the flows of these commodities and hence are not introduced into the individual’s utility function. Instead, the analysis focuses on the roles of price, taste and risk differences in explaining the individual’s choice of junk-food and health-food consumption flows.

An intertemporal consumer model incorporating the taste, price and risk differences between junk food and its healthier substitute is proposed. The building blocks of the model generating a rational choice of a diet of junk food and health food are presented in section II. Similar to Amnon Levy (2000, 2002a and 2002b), life expectancy is taken to be random, the probability of dying is related to health and age, and rational behavior is defined as expected lifetime-utility maximization. The expected lifetime-utility maximization problem is presented in section III and the properties of the rational diet of junk-food and value of health are discussed in section IV. The rational long-run (stationary) consumption of junk food and the consumer’s stationary health are presented in section V.
II. Building Blocks

The analysis employs the following notations:

\( t = \) a continuous time index, \( t \in (0, T) \) where \( T \) is a positive scalar indicating the upper bound on a consumer’s life expectancy;

\( c_j(t) = \) the individual’s consumption of junk food at instance \( t \);

\( c_h(t) = \) the individual’s consumption of health food at instance \( t \);

\( x(t) = \) the individual’s age-adjusted health condition at instance \( t \), a unit interval index \( 0 \leq x(t) \leq 1 \) with \( x = 0 \) representing a terminally ill person and \( x = 1 \) a perfectly healthy person;

\( p(t) = \) the junk food-health food price ratio;

\( \alpha = \) the junk food-health food taste ratio;

\( y(t) = \) the individual’s income at instance \( t \);

\( \hat{y} = \) a positive scalar indicating the full capacity income;

\( \phi(t) = \) the probability of dying at instance \( t \);

\( u(t) = \) the individual’s satisfaction from food at instance \( t \); and

\( \rho(t) = \) the individual’s rate of time preference at instance \( t \).

As indicated in the notations, the individual’s health condition, \( x \), is adjusted to the adverse effects of normal aging. That is, \( x \) indicates the individual’s health relative to her age. This definition of \( x \) is used for distinguishing between the effect of age (i.e., youth vis-à-vis old age) and the effect of health on the individual’s probability of survival. (See assumption 7.) This definition also explains why age (and thereby aging) is not included in the motion equation of the individual’s health. (See assumption 6.)
The building blocks of the rational junk-food consumption model are summarized by the following assumptions.

**Assumption 1 (relative price):** Junk food is cheaper than health food. That is, \( p(t) < 1 \).

**Assumption 2 (instantaneous satisfaction):** The individual’s instantaneous satisfaction from eating is represented by a utility function \( u(c_j(t), c_h(t)) \) having the following properties. Food is essential -- \( u(0,0) = 0 \). However, neither junk food nor health food is essential -- \( u(0, c_h) > 0 < u(c_j, 0) \). The marginal satisfaction with respect to each type of food is positive and diminishing -- \( u_j, u_h > 0 \) \( u_{jj}, u_{hh} < 0 \) -- and health food and junk food are substitutes -- \( u_{jh} < 0 \).

**Assumption 3 (relative taste):** Junk food is tastier than health food. For equal intakes, the marginal satisfaction from junk food is higher than that from health food. That is, \( u_j > u_h \) for every \( c_j \leq c_h \).

Consistent with assumptions 2 and 3, the following explicit utility function is considered

\[
    u(t) = [\alpha c_j(t) + c_h(t)]^\beta
\]

where \( \alpha > 1 \) is the relative taste coefficient and \( 0 < \beta < 1 \) is the elasticity of the individual’s satisfaction from food.
Assumption 4 (instantaneous income): The ratio of the individual’s instantaneous income to the full capacity income is equal to the individual’s age-adjusted health condition. That is,

\[ y(t) = x(t)\tilde{y} \]

revealing that the full capacity income could only be attained by a perfectly healthy individual, and that the income of a terminally ill person is nil. To simplify matters, the full capacity income is assumed to be independent of age.

Assumption 5 (instantaneous budget constraint): There is no borrowing or lending (for simplicity sake) and the individual’s instantaneous income is fully spent on buying junk food and health food. Taking the price of health food as a numeraire, the budget constraint is given by

\[ p(t)c_j(t) + c_h(t) = x(t)\tilde{y}. \]

Assumption 6 (health change): The individual’s age-adjusted health is deteriorated by eating junk food and improved by a natural recovery process. Health-food only helps maintaining the individual’s health relative to her age at the same level.\(^2\) Correspondingly, the instantaneous change in the individual’s age-adjusted health is given by a logistic function displaying a diminishing intrinsic health-improvement rate \((r_0)\) in junk-food consumption, a diminishing health-improvement rate \((r)\) in the

\[^2\text{Health-food fans may argue that, ceteris paribus, health food not only helps maintain personal health but also improves personal health. The incorporation of the latter assertion complicates the analysis and renders the model unsolvable.}\]
level of health, and a unit upper bound and a zero lower bound on the individual health. That is, it is assumed that

\[
\dot{x}(t) = \frac{r_0}{r} \left[1 - \delta c_j(t)(1 - x(t))x(t)\right]
\]

where, \( \delta \) is a positive scalar indicating the marginal adverse effect of junk-food consumption on the intrinsic rate of improvement of the individual’s age-adjusted health. Loosely interpreted, \( \delta \) is the health sensitivity to junk food.

\textit{Assumption 7 (survival probability):} There is an upper bound \((T)\) on life expectancy and the probability of survival at any given point in time declines with the individual’s age and rises with the individual’s age-adjusted health. It converges to zero as the individual’s age approaches the upper-bound on life expectancy and as her health is completely deteriorated \((x = 0)\).

This assumption is formally presented as follows. Let \( F(t) \) be the cumulative distribution function associated with the probability of dying \( \phi(t) \). Then \( \Phi(t) = 1 - F(t) \) indicates the probability of living beyond \( t \) (i.e., survival at \( t \)). It is assumed that \( \Phi(t) = \Phi(x(t), T - t) \) with \( \Phi_{T-t} > 0 \) (the youth effect), \( \Phi_x > 0 \) (the age-adjusted health effect) and \( \Phi(0, T-t) = 0 = \Phi(x(t), 0) \).

\textit{Assumption 8 (time-consistent preferences):} The individual’s rate of time preference is positive and time invariant. That is, \( \rho(t) = \rho \) for every \( t \in (0, T) \).

\(^3\) The intrinsic health-improvement \((r_0)\) is the rate of health improvement \((r)\) at the vicinity of the lower bound on health \((x = 0)\).
III. Rational Choice

It is postulated that rational individuals chose their junk and health food diet path so as to maximize their expected lifetime satisfaction from food subject to their health motion equation. Since life expectancy is random, expected-lifetime-satisfaction-maximizing food consumers multiply their accumulated satisfaction from food between the starting point of their planning horizon, 0, to their possible time of death \( t \) (i.e., multiply \( \int_0^t e^{-\rho \tau} u(\tau) d\tau \) by the probability of dying at time \( t \) (i.e., \( \phi(t) \)).

The products of \( \phi(t) \) and \( \int_0^t e^{-\rho \tau} u(\tau) d\tau \) associated with any possible life expectancy \( 0 \leq t \leq T \) are considered by such rational consumers. The sum of all these products is these consumers’ expected lifetime-satisfaction from food. It is given by the following double-integral expression

\[
V = \int_0^T \int_0^t e^{-\rho \tau} u(\tau) d\tau dt .
\]  

(5)

Integrating by parts, this expected lifetime-satisfaction is equivalently rendered by a mathematically more manageable single-integral expression:

\[
V = \int_0^T \Phi(x(t), T-t) e^{-\rho t} u(t) dt .
\]  

(6)

A detailed mathematical explanation is given in Appendix A.

The analysis of the rational diet trajectory is further simplified by expressing \( c_h \) as a function of \( c_j \). Recalling the instantaneous budget constraint,

\[
c_h(t) = x(t) \hat{y} - p(t) c_j(t) .
\]  

(7)
The substitution of Eq. (7) into Eq. (1) renders the instantaneous satisfaction function as
\[ u(t) = \left[ (\alpha - p(t))c_j(t) + x(t)\tilde{y} \right]^{\beta}. \] (8)

By virtue of assumptions 1 and 3, the difference between the relative taste and the relative price is positive. That is, \( \alpha - p(t) > 0 \). Hence, the marginal instantaneous satisfaction from junk food, in this concentrated form, is positive and diminishing. In turn, \( V \) is concave in the control variable \( c_j \).

By substituting Eq. (8) into Eq. (7) for \( u(t) \) the individual’s expected lifetime-satisfaction maximizing junk-food consumption path can now be found by
\[
\max_{\{c_j\}} \int_0^T \Phi(x(t), T - t) e^{-\rho t} \left[ (\alpha - p(t))c_j(t) + x(t)\tilde{y} \right]^{\beta} \, dt
\]
subject to the health motion equation 4.

IV. Optimal Junk-Food Consumption and the Shadow Value of Health

The present-value Hamiltonian corresponding to the aforementioned constrained maximization problem is
\[
H(t) = \Phi(x(t), T - t) e^{-\rho t} \left[ (\alpha - p(t))c_j(t) + x(t)\tilde{y} \right]^{\beta} + \lambda(t) [1 - \partial c_j(t)] [1 - x(t)] x(t)
\]
where the co-state variable \( \lambda(t) \) indicates the shadow present value of the individual’s age-adjusted health at \( t \). Since \( 0 < \beta < 1 \) and \( \alpha - p(t) > 0 \), \( H \) is concave in both the control variable \( c_j \) and the state variable \( x \) and hence, in addition to the state equation (Eq. (4)), the following conditions are necessary and sufficient for maximum expected lifetime satisfaction from junk-food consumption:\n
\footnote{The time-index \( t \) is omitted for tractability.}
\[
\dot{\lambda} = -\frac{\partial H}{\partial x} = -[\Phi x Z^\beta u(t) - \Phi Z^\beta u_x(t)] e^{-\rho t} - \lambda (1 - 2x)(1 - c_j) \tag{10.1}
\]

and

\[
\frac{\partial H}{\partial c_j} = \Phi e^{-\rho t} Z^\beta u_x(t) - \lambda \dot{\delta}(1 - x) = 0. \tag{10.2}
\]

The optimality condition, Eq. (10.1), indicates that along the optimal junk-food consumption path there should be a balance between the marginal satisfaction from junk-food consumption, discounted by both the individual’s time preference and prospects of survival, and the value of the marginal damage to the individual health caused by consuming junk-food.

The adjoint equation, Eq. (10.1), implies, in conjunction with the optimality condition, that along the individual’s optimal junk-food consumption path the rate of change of the shadow value of health is given by

\[
\frac{\dot{\lambda}(t)}{\lambda(t)} = \left[ \frac{(\eta / \beta) c_j + [1 + (\eta / \beta)] x^\gamma}{\alpha - p} \right] \delta(1 - x) - (1 - 2x)(1 - c_j) \tag{11}
\]

where \( \eta \) denotes the survival elasticity \( (\Phi_x / \Phi) \) which, for simplicity, is henceforth assumed to be constant. As \( \frac{\partial (\dot{\lambda} / \lambda)}{\partial c_j} > 0 \) for \( x = (1 + \eta / \beta)/(2 + \eta / \beta) \), the individual’s health is appreciated (devaluated) by junk-food consumption when the individual’s health is better (worse) than a critical level, which rises with the ratio of the elasticity of survival to the elasticity of satisfaction from eating \( (\eta / \beta) \).
The optimal change in junk-food consumption over time is given by:

\[
\dot{c}_j = \frac{\dot{A}}{(1-2x)(1-c_j - \rho)\beta Z(\alpha - p) + \delta(1-x)(\eta Z^2 + \beta Z\dot{y})}{\beta(1-\beta)(\alpha - p)}
\]

\[
+ \left[ \frac{B}{\eta/(1-2x)/(1-x)x[\alpha - p]Z - (1-\beta)^p} \right] \dot{y} = \left[ \frac{Z}{(1-\beta)(\alpha - p)} \right] \dot{p} + \left[ \frac{Z}{(1-\beta)} \right] \Phi
\]

(12)

This equation and Eq. (4) portray the optimal joint evolution of the individual’s junk-food consumption and health. They lead to the following conclusions.

Recalling assumptions 1 and 3, \((1-\beta)(\alpha - p) > 0\). Hence, the direction of the effect of an improvement in the individual’s health on junk food consumption depends on the sign of \(B\). That is, \(\frac{d\dot{c}_j}{d\dot{x}} = \frac{B}{(1-\beta)(\alpha - p)} > 0\) as

\[
\frac{\eta}{x} + \frac{1-2x}{(1-x)x} > (1-\beta)^p.
\]

The direct effects of changes in the prospects of survival and the relative price of junk food on junk-food consumption are given by differentiating Eq. (12) with respect to these factors. Recalling Eq. (4), these direct effects on junk food consumption affect the individual’s age-adjusted health at a rate of \(-\delta\), which, by virtue of Eq. (12), also affects junk food consumption. The full effects of changes in the prospects of survival and the relative price of junk food on junk-food consumption are equal to the sum of these direct and indirect effects.

---

5 Eq. (12) is obtained by differentiating Eq. (10.2) with respect to time, substituting the right-hand sides of Eq. (10.1) and Eq. (10.2) for \(\dot{\lambda}\) and \(\lambda\), multiplying both sides of the resultant equation by \(e^{\dot{\lambda}} / \Phi Z^\beta/2 \beta(1-\beta)(\alpha - p)\) and collecting terms.
As can be seen from Eq. (12) and assumptions 1 and 3, the adverse effect of age on survival \((\Phi < 0)\) has a direct moderating effect \((Z/(1-\beta)\Phi)\) on the individual’s junk-food consumption over time. However, this decline in consumption of junk food improves the individual’s age-adjusted health by \(\delta Z/(1-\beta)\Phi\) and hence indirectly changes junk-food consumption by \(\delta ZB/(1-\beta)^2\Phi(\alpha - p)\). Recalling that \(\frac{d\hat{c}_j}{dx} = \frac{B}{(1-\beta)(\alpha - p)}\), the indirect effect of aging on junk-food consumption is positive (negative) if \(\frac{\eta + \frac{1-2x}{(1-x)x}}{(1-x)x}\) is greater (smaller) than \(\frac{(1-\beta)\hat{y}}{(\alpha - p)Z}\) and hence dimming (amplifying) the direct moderating effect of age on junk-food consumption.

As can be expected, a rise in the relative price of junk food over time has a moderating direct effect \((- Z/(1-\beta)(\alpha - p))\) on junk-food consumption. This decline in junk-food consumption leads to an improvement in the individual’s age-adjusted health by \(\delta Z/(1-\beta)(\alpha - p)\) and hence indirectly changes junk-food consumption by \(\delta ZB/[(1-\beta)(\alpha - p)]^2\). Recalling that \(\frac{d\hat{c}_j}{dx} = \frac{B}{(1-\beta)(\alpha - p)}\), this indirect effect of a rise in the relative price of junk food on junk-food consumption is positive (negative) if \(\frac{\eta + \frac{1-2x}{(1-x)x}}{(1-x)x}\) is greater (smaller) than \(\frac{(1-\beta)\hat{y}}{(\alpha - p)Z}\) and hence dimming (amplifying) the direct moderating effect of a relative price rise on junk-food consumption.

V. Rational Stationary Junk-Food Consumption and Health Index of Forever-Young Feeling People

The notion of steady state (SS) is used in this section to indicate possible long-run levels. Of course, the derivation of stationary junk-food consumption and
stationary health index is inconsistent with the assumption that \( \Phi_{T-t} > 0 \). Hence, this assumption is now relaxed. The following analysis is conducted under the assumption that some people ignore aging (\( \Phi_{T-t} = 0 \)) and believe that their survival depends only on their health. For these forever-young feeling people the evolution of junk-food consumption is given by

\[
\dot{c}_j = \frac{\mathcal{A}}{\mathcal{B}} \frac{[\eta x + (1-2x)(1-x)](\alpha - p) + \delta (1-x)[\eta Z^2 + \beta Zx\dot{y}]}{\beta (1-\beta)(\alpha - p)} 
\]

\[
+ \left[ \frac{[\eta x + (1-2x)(1-x)](\alpha - p)Z - (1-\beta)\dot{y}}{(1-\beta)(\alpha - p)} \right] \dot{x} - \left[ \frac{Z}{(1-\beta)(\alpha - p)} \right] \dot{p} 
\]

(13)

The substitution of \( \dot{p} = \dot{c}_j = \dot{x} = 0 \) and the definition of \( Z \) into Eq. (13) implies that in steady state

\[
[(1-2x_{ss})(1-c_{jss}) - \rho]\beta (\alpha - p) + \delta (1-x_{ss})[\eta (\alpha - p)c_{jss} + (\beta + \eta)x_{ss}\dot{y}] = 0 \quad (14)
\]

and, as the substitution of \( \dot{x} = 0 \) into Eq. (4) implies that \( c_{jss} = 1/\delta \), it is obtained that

\[
x_{ss}^2 - \left[ 1 - \frac{(\alpha - p)[2(1-1/\delta) + \eta/\beta]}{\delta (1+\eta/\beta)\dot{y}} \right] x_{ss} + \frac{(\alpha - p)[1/\rho + \rho - 1 - \eta/\beta]}{\delta (1+\eta/\beta)\dot{y}} = 0. \quad (15)
\]

The solution of this quadratic equation yields double stationary age-adjusted health conditions

\[
x_{ss}^f = 0.5 \left[ 1 - \frac{(\alpha - p)[2(1-1/\delta) + \eta/\beta]}{\delta (1+\eta/\beta)\dot{y}} \right] + 0.5 \left[ 1 - \frac{(\alpha - p)[2(1-1/\delta) + \eta/\beta]}{\delta (1+\eta/\beta)\dot{y}} \right]^2 - 0.5 \left[ 1 - \frac{(\alpha - p)[2(1-1/\delta) + \eta/\beta]}{\delta (1+\eta/\beta)\dot{y}} \right] \cdot \frac{0.5}{\delta (1+\eta/\beta)\dot{y}}
\]

(16)

and
Numerical simulations are used for assessing the effects of the model’s parameters on the stationary levels of health. The simulations reveal that for various choices of parameter-values only $x_{ss}^H$ is, as required by construction, within the unit interval $(0,1)$. Hence, the reported simulation results are generated by using Eq. (17). The reported simulations refer to a forever-young feeling person for whom:

- junk-food is fifty per cent tastier than health food, $\alpha = 1.5$;
- junk-food is fifty per cent cheaper than health food, $p = 0.5$;
- the elasticity of satisfaction from eating is $\beta = 0.5$;
- the elasticity of survival is $\eta = 1$ (i.e., $\Phi = x$);
- the marginal (adverse) effect of junk-food consumption on the intrinsic rate of improvement of the individual health is $\delta = 0.0003$;
- the daily rate of time preference is $\rho = 0.00026$ (which is equivalent to about 10 per cent per annum); and
- the daily full-capacity income is $\hat{y} = $100.

For this person, the stationary health index is 0.578; namely, 57.8 per cent of a perfectly healthy individual in her cohort.

The numerical simulations reveal that this stationary health index is not sensitive to changes in the relative taste of junk food, in the relative price of junk food, in the elasticity of satisfaction from eating, in the elasticity of survival, and in the full-capacity income.

In contrast, and as can be expected, the numerical simulations indicate that the stationary health index is considerably lowered by the rate of time preference. For
instance, a one-percent rise in $\rho$ from the aforementioned benchmark level, all other things remain the same, reduces $x_{ss}$ by 0.998 percent.

It is also found the stationary health index rises considerably with the marginal effect of junk-food consumption on the intrinsic rate of improvement of the individual health. The rise of the stationary health index is due to the moderating effect of an increase in $\delta$ on the stationary consumption of junk food ($c_{jss} = 1/\delta$). For instance, a one-percent rise in $\delta$ from the aforementioned benchmark level, all other things remain the same, increases $x_{ss}$ by 1.006 percent.

However, the trajectories of health index and junk-food consumption of the “forever-young feeling” (otherwise rational) people neither converge to, nor orbit, the stationary combination. (See Appendix B.)

VI. Conclusion

To be completed!

References


Cawley, John C. “Rational Addiction, the Consumption of Calories, and Body Weight”, *Ph.D. Dissertation*, Department of Economics, University of Chicago, August 1999.


APPENDIX

Appendix A: An explanation of the transition from Eq. (6) to Eq. (7)

$F(t)$ is the cumulative density function associated with the probability of dying at $t$ (i.e., the probability of living up to $t$). Hence,

$$
\phi(t) = F'(t) \quad (A1)
$$

and Eq. (6) can be rendered as

$$
J = \int_{0}^{T} F'(t) \left[ \int_{0}^{t} e^{-\rho \tau} u(\tau) d\tau \right] dt = \int_{0}^{T} v(t) dU \quad (A2)
$$

where,

$$
v = \int_{0}^{t} e^{-\rho \tau} u(\tau) d\tau \quad (A3)
$$

and

$$
U = -(1 - F(t)) \quad (A4)
$$

The integration by parts rule suggests that

$$
J = \int_{0}^{T} v dU = Uv - \int_{0}^{T} Ud\nu \quad (A5)
$$

Note, however, that
\[ U_v = \left[ (1 - F(t)) \int_0^t e^{-\rho \tau} u(\tau) d\tau \right]_0^T = 0 \]  
\text{(A6)}

Because when evaluated at the lower limit

\[ U_v = \left[ (1 - F(0)) \int_0^0 e^{-\rho \tau} u(\tau) d\tau \right] = 0 \]  
\text{(A7)}

And when evaluated at the upper limit

\[ U_v = \left[ (1 - F(T)) \int_0^T e^{-\rho \tau} u(\tau) d\tau \right] = 0 \]  
\text{(A8)}

As

\[ F(T) = 1. \]  
\text{(A9)}

Hence,

\[ J = -\int_0^T U dv . \]  
\text{(A10)}

By virtue of equation (A3)

\[ dv = e^{-\rho \tau} d\tau \]  
\text{(A11)}

And the substitution of equations (A4) and (A11) into (A10) implies

\[ J = \int_0^T e^{-\rho \tau} u(t) \Omega(t) dt \]  
\text{(A12)}

Where

\[ \Omega(t) \equiv -u(t) = 1 - F(t) \]  
\text{(A.13)}

And indicating the probability of living at least until \( t \).
Appendix B: The nature of the steady-state combination

In order to find whether the individual’s health and consumption of junk food convergence to the aforementioned stationary levels of 0.578 and 3333.333, respectively, the system of equations (13) and (4) is linearized at the vicinity of this stationary point. The eigenvalues of the state-transition matrix are given by

\[
\lambda_{1,2} = 0.5\left\{ [M_{c_j}(ss) + N_x(ss)] \pm \sqrt{[M_{c_j}(ss) + N_x(ss)]^2 - 4[M_{c_j}(ss)N_x(ss) - N_{c_j}(ss)M_x(ss)]} \right\}
\]

(B.1)

with \( M_{c_j}(ss) = 2068.761 \) and \( M_x(ss) = 45,288,789 \) indicating the stationary values of the derivatives the right-hand-side of Eq. (13) with respect to \( c_j \) and \( x \), and \( N_{c_j}(ss) = -7.32298E-05 \) and \( N_x(ss) = 0 \) (as it is proportional to \( 1 - \delta c_j^{ss} = 0 \)) the stationary values of the derivatives the right-hand-side of Eq. (4) with respect to \( c_j \) and \( x \). As \( \lambda_1 \) and \( \lambda_2 \) are both positive (2067.156 and 1.604, respectively) the individual’s health and junk-food consumption trajectories neither converge to, nor orbit, the stationary combination.