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Some comments on the ranking procedure based on deviation variables

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Abstract

Ghasemi et al. (2019) (Improving discriminating power in data envelopment models based on deviation variables framework. European Journal of Operational Research 278, 442– 447) propose a procedure for ranking efficient units in data envelopment analysis (DEA) based on the deviation variables framework. They claim that their procedure improves the discriminating power of DEA and can be an alternative to the super-efficiency model which is well known to have the infeasibility problem. We demonstrate, in this short note, that their procedure does not provide added value over the existing cross-efficiency-based ranking method by showing that the optimal values of deviation variables are just cross-efficiencies when they are properly normalized. We also show that their development is based upon inappropriate use of deviation variables and incorrect characterization of optimal weights.

Keywords: Data envelopment analysis; Deviation variables; Cross-inefficiency.

1. Introduction

The multiple criteria approach of Li and Reeves (1999) to data envelopment analysis (DEA) models has received extensive attention in the literature. Li and Reeves discuss the necessity, or possibility, of considering alternative criteria in the objective function of DEA models such as the Charnes, Cooper, and Rhodes (CCR) model (Charnes et al., 1978). The main idea behind the multiple criteria approach, distinguishing it from the classical CCR model, is to find optimal weights that maximize the total efficiency of all decision making units (DMUs) and/or the efficiency of the worst-performing DMU rather than only the DMU under evaluation. They capture this by introducing variables known as deviation variables, each of which denotes the gap between the weighted sum of inputs and the weighted sum of outputs at each of the constraints. They then introduce two new additional objective functions known as the *minsum* and *minimax* objectives to establish their multiple criteria model.

In its basic form, Li and Reeves' model, hereafter referred to as the deviation model, is an alternative to the input-oriented CCR model. The deviation model minimizes the deviation variable (measuring the inefficiency) for the DMU under evaluation rather than maximizing the weighted sum of outputs (measuring the efficiency) as in the classical input-oriented CCR model. Li and Reeves show that these two variations (maximizing efficiency and minimizing $(1 -$ deviation variable$)$ or inefficiency are equivalent but they do not necessarily lead to the same optimal multiplier values. Li and Reeves indeed advocate the optimal weights obtained from minimizing inefficiency since they believe that these weights are distributed more evenly.

There have been some recent developments, extensions and applications of the multiple criteria approach developed by Li and Reeves (see for example Bal et al., 2010; Ghasemi et al., 2014; De Carvalho Chaves et al., 2016; Dos Santos Rubem et al., 2017; Ghasemi et al., 2019). The motivation for this paper is to critically review the work of Ghasemi et al. (2019), which is one of the recent extensions of the deviation variable model. This extension aims to provide a complete ranking of DMUs and is claimed to be an alternative to the super-efficiency (Andersen and Petersen, 1993) and the cross-efficiency approaches (Sexton et al., 1986; Doyle and Green, 1994).

We will, however, argue that Ghasemi et al.'s ranking approach, hereafter referred to as the deviation-based ranking approach, is not consistent with the definitions they provide in their paper. Their ranking approach and the results provided by their approach do not serve their initial purpose, assumptions, and definitions. In particular, we argue that Ghasemi et al. have: (1) misinterpreted the meaning of deviation variables in the Li and Reeves model and (2) misinterpreted the meaning of "even distributions of weights" by relating it to the unique/non-unique input and output optimal weights. More specifically, they claim that since the deviation model can provide optimal weights which are distributed more evenly than the optimal weights provided by the classical CCR model (maximizing efficiency), their model can avoid non-unique solutions (see Ghasemi et al., 2019, p. 446). However, it is obvious that the issue of even distribution of optimal weights and the issue of non-uniqueness of optimal weights are in no way related to each other. We will provide a counter-example to demonstrate the possible presence of non-unique optimal solutions to their deviation model. This contradicts Ghasemi et al.'s argument that the deviation model avoids multiple optimal solutions. The purpose of our paper is, therefore, to clarify the fallacy of Ghasemi et al.'s work focusing on the two problems briefed above.

In the following Section 2, we will introduce Li and Reeves's deviation model and then Ghasemi et al.'s deviation-based ranking approach. In order to explain how Ghasemi et al. misinterpreted deviation variables, we bring Ghasemi et al.'s approach into a cross-inefficiency matrix format. We notice that Ghasemi et al. have argued that their approach should not be confused with the cross-efficiency approach. However, we show that their model leads to a cross-inefficiency approach when the deviation variables are correctly interpreted and applied. We then provide a counter-example in this section to show the presence of multiple optimal solutions using one of the data set used by Ghasemi et al. Section 3 concludes the paper.

2. Deviation form model

Model (1) is the linear CCR model that maximizes the efficiency score of the DMU under evaluation (DMU$_o$).

$$
\begin{array}{lll}
\max & \sum_{r=1}^{s} u_{ro}\, y_{ro} & \\
\text{s.t.} & \sum_{i=1}^{m} v_{io}\, x_{io} = 1, & \\
 & \sum_{r=1}^{s} u_{ro}\, y_{rj} - \sum_{i=1}^{m} v_{io}\, x_{ij} \leq 0 & j = 1, \dots, n, \\
 & u_{ro} \geq 0 & r = 1, \dots, s, \\
 & v_{io} \geq 0 & i = 1, \dots, m,
\end{array}
\tag{1}
$$

where j is the DMU index, $j = 1, \dots, n$, r is the output index, $r = 1, \dots, s$, i is the input index, $i = 1, \dots, m$, x_{io} is the value of the ith input for DMU$_o$, x_{ij} is the value of the ith input for DMU$_j$, y_{ro} is the value of the rth output for DMU$_o$, y_{rj} is the value of the rth output for DMU$_j$, v_{io} is the weight of the ith input for DMU$_o$ and u_{ro} is the weight of the rth output. Model (1) can also be equivalently expressed in deviation variable form, as it was named by Li and Reeves. Model (2) is equivalent to Model (1) in determining the efficiency of DMUs where the efficiency of DMU_o is given by $1 - d_o^*$. Model (2) is to minimize the deviation variable associated with DMU_o while satisfying all the constraints involving other deviation variables.

$$\begin{array}{lll} \min & d_o & \\ \text{s.t.} & \sum_{i=1}^{m} v_{io}\, x_{io} = 1, & \\ & \sum_{r=1}^{s} u_{ro}\, y_{rj} - \sum_{i=1}^{m} v_{io}\, x_{ij} + d_j = 0 & j = 1, \dots, n, \\ & u_{ro} \geq 0 & r = 1, \dots, s, \\ & v_{io} \geq 0 & i = 1, \dots, m, \\ & d_j \geq 0 & j = 1, \dots, n. \end{array} \quad (2)$$

Li and Reeves expressed the CCR model in deviation variable form for two reasons. First, they argue that the model obtains optimal weights of inputs and outputs that are distributed more evenly than those from the classical CCR model which maximizes efficiency, i.e. $\sum_{r=1}^{s} u_{ro}\, y_{ro}$. Second, they use deviation variables to incorporate two new objective functions for their multiple criteria approach. These two new objective functions are developed to minimize the inefficiency of the worst-performing DMU (by minimizing the maximum of d_j, $j = 1, \dots, n$) and to minimize the total inefficiency of all DMUs (by minimizing $\sum_{j=1}^{n} d_j$), respectively.

There exists extensive DEA literature that deals with the problem of improving the discriminatory power of DEA models. After solving a DEA model such as Models (1) and (2), we may arrive at a large number of efficient DMUs with $d_o^* = 0$ or $1 - d_o^* = 1$, which may become problematic due to the lack of discrimination among DMUs. This situation is sometimes undesirable as the decision-maker might be interested in a complete ranking of all DMUs. Recently, Ghasemi et al. (2019) develop a new procedure to achieve a complete ranking of DMUs based on the optimal deviation variables (d_j^*) in Model (2). Their ranking approach consists of the following steps. Firstly, run Model (2) for DMU_o to obtain the inefficiency (d_o^*) or efficiency score ($1 - d_o^*$) of the DMU. Secondly, if DMU_o is efficient, calculate $\frac{1}{n}\sum_{j=1}^{n} d_j^*$. The value of d_j^* is obtained based on the optimal weights of the DMU_o. Thirdly, repeat the first and second steps for all DMUs and use the value of $\frac{1}{n}\sum_{j=1}^{n} d_j^*$ as the final score of each DMU. Sort DMUs based on this score from the smallest to largest. DMUs with smaller $\frac{1}{n}\sum_{j=1}^{n} d_j^*$ obtain a better ranking.

We now show that this ranking approach misinterprets the meaning of deviation variables. Ghasemi et al. have developed a ranking approach for both constant returns to scale (CRS) and variable returns to scale (VRS) technologies based on the misinterpreted deviation variables. Ghasemi et al. have misinterpreted the meaning of $\sum_{j=1}^{n} d_j^*$ as "the total value of inefficiency associated with the efficient DMU$_o$" or $\frac{1}{n}\sum_{j=1}^{n} d_j^*$ as "the average inefficiency of DMU$_o$". However, **<u>Model (2) does not intend to minimize the total inefficiency of DMU$_o$ while calculating its efficiency score.</u>** The key to identifying the main issue with this approach is that the optimal values of d_j^* are obtained by solving Model (2) for DMU$_o$. The correct interpretation of this measure ($\frac{1}{n}\sum_{j=1}^{n} d_j^*$) is "the average inefficiency of all DMUs (DMU$_j$, $j = 1, \dots, n$) rated with the optimal weights chosen by DMU$_o$". One cannot use the sum or average of other DMUs' inefficiency **<u>to represent an efficiency measure of DMU$_o$ relative to others</u>**. Therefore, the ranking approach based on $\frac{1}{n}\sum_{j=1}^{n} d_j^*$ is not consistent with, and does not serve the initial purpose of Ghasemi et al.

The obvious reason for our claim is that when Model (2) is solved for DMUo, d_j^* represents indeed the inefficiency of DMUj rated with the optimal weights of DMUo and does not represent the inefficiency of DMUo itself rated with the optimal weights of DMU$_j$. Therefore, it is not reasonable to rank DMUo based on the average inefficiencies of other DMUs ($\frac{1}{n}\sum_{j=1}^{n} d_j^*$). This is the main problem associated with the deviation-based ranking approach. The fact that other DMUs on average achieve smaller inefficiency scores with the optimal weights of DMUo does not indicate a better performance of DMUo. This fallacy alone is sufficient to suggest that the deviation-based ranking approach needs some corrections.

In order to clarify the correct interpretation and use of the deviation variables, we bring the deviation variables in Model (2) into a cross-inefficiency matrix format. This contradicts Ghasemi et al.'s argument that their ranking approach should not be confused with the cross-efficiency approach (see p. 446). We show that their approach and the measure they suggested to rank DMUs ($\frac{1}{n}\sum_{j=1}^{n} d_j^*$) is indeed row averages in the cross-inefficiency matrix shown in Table 1. The matrix is shown for an example involving six DMUs. This example is similar to the cross-efficiency matrix of Doyle and Green (1994) except that the efficiency score elements of the matrix are replaced by inefficiency scores (deviation variables). Each element of the matrix represents the inefficiency score of a DMU using either its own optimal weights or the optimal weights chosen by other DMUs.

Table 1
Cross-inefficiency matrix.

Rating DMU	Rated DMU 1	2	3	4	5	6	Averaged inefficiency appraisal of peers
1	$\boldsymbol{d_{11}}$	d_{12}	d_{13}	d_{14}	d_{15}	d_{16}	$\boldsymbol{\bar{d}_{1j}}$
2	d_{21}	$\boldsymbol{d_{22}}$	d_{23}	d_{24}	d_{25}	d_{26}	$\boldsymbol{\bar{d}_{2j}}$
3	d_{31}	d_{32}	$\boldsymbol{d_{33}}$	d_{34}	d_{35}	d_{36}	$\boldsymbol{\bar{d}_{3j}}$
4	d_{41}	d_{42}	d_{43}	$\boldsymbol{d_{44}}$	d_{45}	d_{46}	$\boldsymbol{\bar{d}_{4j}}$
5	d_{51}	d_{52}	d_{53}	d_{54}	$\boldsymbol{d_{55}}$	d_{56}	$\boldsymbol{\bar{d}_{5j}}$
6	d_{61}	d_{62}	d_{63}	d_{64}	d_{65}	$\boldsymbol{d_{66}}$	$\boldsymbol{\bar{d}_{6j}}$
Averaged inefficiency appraisal by peers	$\boldsymbol{\bar{d}_{j1}}$	$\boldsymbol{\bar{d}_{j2}}$	$\boldsymbol{\bar{d}_{j3}}$	$\boldsymbol{\bar{d}_{j4}}$	$\boldsymbol{\bar{d}_{j5}}$	$\boldsymbol{\bar{d}_{j6}}$	

For instance, d_{12} is the inefficiency score of DMU 2 rated with DMU 1's optimal weights, d_{21} is the inefficiency score of DMU 1 rated with DMU 2's optimal weights and d_{34} is the inefficiency score of DMU 4 rated with DMU 3's optimal weights. The leading diagonal shows the self-appraisal inefficiency score of DMUs when DMUs are evaluated based on their own best possible set of weights. In developing the cross-inefficiency matrix above, we borrow terminologies such as peer-appraisal, self-appraisal, rating DMU, rated DMU, appraisal by peers and appraisal of peers from Doyle and Green (1994).

A key to avoiding misinterpretation of the deviation variables (as in Ghasemi et al.) is to clarify the difference between taking the average of the columns and rows in the cross-inefficiency matrix[1]. Similar to the definitions provided by Sexton et al. (1986) and Doyle and Green (1994) but in the context of a cross-inefficiency matrix, the average values of columns represent average inefficiency appraisal by other DMUs while the average values of rows represent the average inefficiency appraisal of other DMUs. Based on Sexton et al. (1986, p. 90), rows averages measure "the average efficiency of all DMUs according to DMU*k*" where *k* represents the DMU under evaluation. The cross-inefficiency matrix clearly explains an incorrect interpretation of the deviation variables in the deviation-based ranking approach. In the deviation-based ranking approach, average values of rows are used to rank the rating DMU itself, which is evidently not serving Ghasemi et al.'s definitions and intension, i.e. to use the total or average value of inefficiency associated with the efficient DMU$_o$. For instance, $\bar{d}_{1j}$ is used as the final score of DMU 1. This simply means that the average of other DMUs' inefficiencies rated with DMU 1's optimal weights is used to rank the performance of DMU 1.

[1] This matrix was not shown by Ghasemi et al. and we have developed the matrix to clarify the role of deviation variables and to provide an easier interpretation of the deviation variables and the deviation-based ranking approach.

This incorrect interpretation of the deviation variables can be corrected by using the average values of columns. For instance, $\bar{d}_{j1}$ is the average inefficiency scores of DMU 1 rated with the optimal weights of other DMUs. If one wants to be consistent with Ghasemi et al.'s intention and definitions for using the total or average values of inefficiencies associated with the efficient DMUs, he/she should use the averages of columns to discriminate (and rank) the DMUs. In other word, $\frac{1}{n}\sum_{j=1}^{n} d_{jo}^{*}$ which is obtained after applying n linear programs (LPs) should be used rather than $\frac{1}{n}\sum_{j=1}^{n} d_{oj}^{*}$ which is obtained after applying only one LP for DMU$_o$. Therefore, the results derived from Ghasemi et al.'s deviation variable based ranking approach are just cross-inefficiencies when row averages are used rather than columns[2] and when they are properly normalized. Ghasemi et al.'s ranking approach (if the deviation variables are correctly interpreted and normalized) can also be seen as an equivalent to Liang et al. (2008). Deviation variables can be normalized by dividing them by the weighted sum of inputs as below:

$$\widehat{d_j} = \frac{d_j}{\sum_{i=1}^{m} v_i^* x_{ij}}$$

Then, we have $0 \le \widehat{d_j} < 1$ as required. However, note that

$$\widehat{d_j} = \frac{d_j}{\sum_{i=1}^{m} v_i^* x_{ij}} = 1 - \frac{\sum_{r=1}^{s} u_r^* y_{rj}}{\sum_{i=1}^{m} v_i^* x_{ij}}$$

Also, note that $\widehat{d_j}$ is just a conventional cross-inefficiency of DMU$_j$ evaluated by DMU$_o$'s optimal weights. Therefore, with this normalization scheme, there is no difference between Ghasemi et al.'s approach, i.e. deviation based ranking approach (if the deviation variables are correctly interpreted and applied as suggested in this paper) and the existing conventional cross-efficiency (inefficiency) approach.

In a very important inspection, Sexton et al. (1986) have empirically found that rows and columns averages in a cross-efficiency matrix tend to be negatively correlated. This basically means that DMUs with high cross-efficiency scores will achieve a low ranking if Ghasemi et al.'s ranking approach is applied.

In the following, we provide three numerical examples. Consider Example 1 in Table 2 with 2 DMUs, 2 inputs, and 2 outputs to demonstrate Ghasemi et al.'s ranking approach (using

[2]Note that seminal works in the cross-efficiency literature such as Sexton et al. (1986) and Doyle and Green (1994) use columns averages to rank DMUs and not rows.

row averages) and rankings based on the correct interpretation of the deviations variables (column averages).

Table 2
Example 1.

DMU	x_1	x_2	y_1	y_2	d_o^*	Rankings based on Ghasemi et al.	Rankings based on columns averages
1	10	8	10	15	0.51	1	2
2	6	5	14	19	0	2	1

DMU 2 dominates DMU 1 in all inputs and outputs ($\boldsymbol{x_{12} < x_{11}, x_{22} < x_{21}, y_{12} > y_{11}, y_{22} > y_{21}}$). DMU 1's CCR efficiency score is equal to 0.4934 ($d_1^* = 0.5066$). DMU 2's efficiency score is equal to 1 ($d_2^* = 0$). When Model (2) is solved for DMU 1, we have the optimal values of deviation variables as: $d_1^* = 0.5066$, $d_2^* = 0$. As suggested by Ghasemi et al., we take the average of the deviation variables as $\frac{d_1^*+d_2^*}{2} = 0.2533$. Then the optimal values of deviation variables for DMU 2 are: $d_1^* = 0.9524$, $d_2^* = 0$ and $\frac{d_1^*+d_2^*}{2} = 0.4762$. Based on Ghasemi et al.'s ranking approach, DMU 1 is better than DMU 2 (smaller average is preferred). The last column of the table shows the DMUs' rankings when our normalization approach explained above, the optimal solutions to Model (2) and the column averages are applied. As expected, DMU 2 is ranked higher than DMU 1.

Example 2 in Table 3, includes 3 DMUs all with an efficiency score equal to 1 ($d_o^* = 0$). These DMUs use 3 different inputs and provide 3 different outputs. Inspecting the data shows that DMU 3 is not dominated by DMUs 1 and 2 in any of the inputs and outputs. In fact, this DMU performs better than DMUs 1 and 2 in some inputs and outputs. As shown in column 9 of Table 3, DMU 3 is ranked as the worst DMU using Ghasemi et al.'s ranking approach while the column averages, as expected, rank this DMU as the best DMU.

Table 3
Example 2.

DMU	x_1	x_2	x_3	y_1	y_2	y_3	d_o^*	Rankings based on Ghasemi et al.	Rankings based on columns averages
1	10	8	6	4	10	15	0	1	3
2	10	8	6	4	11	16	0	2	2
3	10	7	6	4	12	16	0	3	1

For Example 3 in Table 4, we apply a data set from Li and Reeves which involves 6 DMUs, 2 inputs, and 2 outputs. The efficiency scores based on different approaches are reported in Table 4. Columns 6 – 8 of the table (e_1, e_2 and e_3) show the results based on Li and Reeves's model when different criteria (minimizing d_o, minimax and minsum) are applied. Other columns show the results based on benevolent (e_4) and aggressive (e_5) formulations of cross-efficiency (Sexton et al., 1986; Doyle and Green, 1994; Liang et al. 2008), super-efficiency (e_6) (Andersen and Petersen, 1993), cross-inefficiency approach when columns averages and the normalization approach introduced in this paper is applied (e_7) and Ghasemi et al.'s ranking approach (e_8). Based on Ghasemi et al.'s ranking approach, DMU 1 is ranked as the worst DMU while this DMU is ranked as a top DMU by other approaches.

Table 4
Example 3.

DMU	x_1	x_2	y_1	y_2	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8
A	1.50	0.2	1.40	0.35	1.00	1.00	1.00	1.00	0.76	2.00	1.00	0.82
B	4.00	0.7	1.40	2.10	1.00	0.95	0.86	0.95	0.70	1.40	0.98	0.94
C	3.20	1.2	4.20	1.05	1.00	0.88	0.83	0.89	0.64	1.41	0.86	0.95
D	5.20	2.0	2.80	4.20	1.00	1.00	1.00	1.00	0.72	1.13	1.00	0.97
E	3.50	1.2	1.90	2.50	0.98	0.97	0.98	0.97	0.70	0.98	0.98	0.95
F	3.20	0.7	1.40	1.50	0.87	0.85	0.87	0.85	0.61	0.87	0.86	0.93

e_1: Minimizing d_o; e_2: Minimax; e_3: Minsum; e_4: Benevolent cross-efficiency; e_5: Aggressive cross-efficiency; e_6: Super-efficiency; e_7: Columns averages with new normalization; e_8: Ghasemi et al.

Examples 1 – 3 also confirm Sexton et al.'s inspection that rows averages in a cross-efficiency matrix tend to be negatively correlated with the averages of the columns. Although with a sample size it might not be possible to make a statistical inference, however, the correlation matrix in Table 5 gives an indication of Sexton et al.'s finding of a negative correlation between rows (Ghasemi et al.) and columns averages in a cross-efficiency matrix. Interestingly, Ghasemi et al.'s results are negatively correlated with all other approaches explained above and applied to the data set in Table 4. All other correlation coefficients (except the last row - Ghasemi et al.) are positive values. This explains the misinterpretation and misuse of the deviation variables by Ghasemi et al.

Table 5
Correlation matrix.

	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8
e_1	1							
e_2	0.7115	1						
e_3	0.2988	0.8393	1					
e_4	0.7241	0.9998	0.8303	1				
e_5	0.7119	0.9505	0.7693	0.9515	1			
e_6	0.5933	0.4340	0.1993	0.4418	0.6786	1		
e_7	0.6024	0.9717	0.8016	0.9699	0.9220	0.3684	1	
e_8	-0.0842	-0.2683	-0.3237	-0.2655	-0.5305	-0.8042	-0.2614	1

Ghasemi et al. also falsely link the problem of even distribution of weights to the problem of the non-uniqueness of optimal weights. Specifically, they argue that the optimal weights obtained from Model (2) are unique. However, we note that Model (2) can have non-unique optimal weights. We provide a counter-example to support this. Table 6 re-produces one of the numerical examples used by Ghasemi et al. to demonstrate their ranking approach. The data set includes seven university departments with 3 inputs and 3 outputs. The inefficiency and efficiency scores of the DMUs based on Model (2) are presented in the last two columns of the table.

Table 6
Seven university departments' inputs and outputs.

DMU	Inputs			Outputs			Inefficiency	Efficiency
	x_1	x_2	x_3	y_1	y_2	y_3	d_o^*	$1 - d_o^*$
1	12	400	20	60	35	17	0	1
2	19	750	70	139	41	40	0	1
3	42	1500	70	225	68	75	0	1
4	15	600	100	90	12	17	0.18	0.82
5	45	2000	250	253	145	130	0	1
6	19	730	50	132	45	45	0	1
7	41	2350	600	305	159	97	0	1

Source: Ghasemi et al. (2019)

According to Ghasemi et al. (p. 446), the optimal weights provided by Model (2) should be unique. They state that: "in other words, we provide the deviation variables for each efficient DMU using DEA model (3), in which the input-output weights are **distributed more evenly** and this is the ability of the proposed model vs. DEA cross-efficiency evaluation, thus **avoiding non-uniqueness** of the DEA optimal input-output weights or multiple optimal weights. To the best of our knowledge, we have not come across a method that provides a full ranking procedure which can still retain the DEA technology for both CRS and VRS, while **avoiding non-uniqueness of weights** and infeasibility problems". Here we show their claim is not correct. Table 7 presents some alternative optimal weights for DMU 7, as an example, where they all assign an inefficiency score of zero for this DMU. Indeed, there exist multiple optimal weights to Model (2) with this data set. With each set of weights this DMU achieves a different score ($\frac{1}{n}\sum_{j=1}^{n} d_j^*$) based on the deviation-based ranking approach, as indicated by the results shown in the last column of the table. The score reported for this DMU by Ghasemi et al. is 0.045 which is one of the multiple optimal solutions reported in the last column of Table 7. This counter-example justifies our argument as there are a number of non-unique solutions.

Note that, our discussions in this paper about non-unique optimal solutions, incorrect interpretation of deviation variables, equivalence between Ghasemi et al.'s ranking approach and rows-averages in a cross-inefficiency matrix are not limited to the CRS model and the same implications also apply to the VRS model developed and used by Ghasemi et al. Moreover, in addition to the more fundamental issues explained above, there are also some calculation errors in Ghasemi et al. such as ranking DMUs from smallest to largest in Table 2 and then ranking DMUs from largest to smallest in Table 5 in their paper.

Table 7
Multiple optimal solutions for DMU 7 and different scores based on the deviation-based approach.

v_{1o}^*	v_{2o}^*	v_{3o}^*	u_{1o}^*	u_{2o}^*	u_{3o}^*	d_o^*	$\frac{1}{n}\sum_{j=1}^{n} d_j^*$
0.00994030	0.00025211	0	0	0.00628931	0	0	0.12034507
0.01774148	0	0.00045433	0	0.00628931	0	0	0.11071864
0.00731881	0.00029784	0	0.00120670	0.00333687	0.00104531	0	0.04507096
0.02439024	0	0	0	0.00628931	0	0	0.21874521
0.01667318	0	0.00052733	0.00129192	0.00377201	0.00006407	0	0.04890276
0.02309235	0.00002264	0	0.00327869	0	0	0	0.09970111
0.02439024	0	0	0.00155779	0	0.00541109	0	0.07909654
0.02439024	0	0	0.00327869	0	0	0	0.10853944
0.02385025	0	0.00003690	0.00327869	0	0	0	0.09976581
0.02439024	0	0	0	0.00356345	0.00446816	0	0.14666853
0.00759931	0.00029295	0	0.00164670	0.00313054	0	0	0.04905326

3. Concluding remarks

We have revisited the ranking procedure based on deviation variables recently developed by Ghasemi et al. (2019) and critically discussed a few issues related to their approach. We have shown that Ghasemi et al. (i) misinterpret the meaning of deviation variables derived from the Li and Reeves model – Model (2), (ii) misinterpret the meaning of even distribution of weights by linking it to the problem of the uniqueness of optimal weights, (iii) incorrectly claim that unlike the cross-efficiency approach their deviation based ranking approach provides a unique solution, and (iv) incorrectly claim that their ranking approach should not be confused with the cross-efficiency approach. We have also demonstrated how one should correctly interpret deviation variables and apply them for ranking DMUs.

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