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## School of Accounting Economics and Finance
## Working Paper Series 2020

# Economically Sustainable Control of Epidemics with Insights from COVID-19

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**WP 20-02**
*June 2020*

**Economically Sustainable Control of Epidemics with Insights from COVID-19**

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**Abstract**

The novel coronavirus pandemic's morbidity and mortality data imply that the isolation of people with underlying (pre-existing) health conditions is necessary. An epidemic control framework that allows the people without underlying health conditions to function and sustain the economy is constructed. Herd-immunity, medical capacity, costs and avoidance of economic collapse are taken into account. The full cost includes grief for lost lives, foregone output and spending on control measures and medical capacity. The full cost is subtracted from the epidemic-free income to obtain a grief-adjusted net-income and, subsequently, community-wellbeing. An intertemporal community-wellbeing-maximizing decision on control measures is outlined and simulated.

*JEL* classification: I18, I31, J17, J18, C61

Keywords: Epidemics; Herd-Immunity; Medical Capacity; Full Cost; Sustainability

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## 1. Introduction

It takes at least four months to develop a vaccine against a new viral disease and several more months until it becomes widely available. The release of information about the emergence in China of the acute respiratory disease caused by the novel coronavirus was delayed until February 2020. The subsequent uncontrolled spread of the coronavirus disease (COVID-19) to the rest of the world and the reactions of individuals and policy makers to the expected long period without vaccines, to the numbers of infections and deaths and to the realised capacity of health-care industries to test and treat have led to the adoption of measures that strongly restricted work, attendance of schools, meetings, visits to personal care and entertainment venues, travels and international trade. The implemented distancing measures and quarantines have devastated the world economy: shortage of consumer goods, sharp drop in the value of stocks, soaring unemployment and dependency on unsustainable welfare payments. In view of the already experienced aftermath of the implemented measures, and as future outbreaks of new viral epidemics are unavoidable, it is necessary to consider a sustainable approach for managing epidemics—an approach that distinguishes between highly vulnerable people and less vulnerable people and takes into account the trade-off between medical-capacity based considerations and the economy.

In the absence of absolutely reliable testing of the entire population, the real number of infected people is unknown. Estimates of the possible person-to-person spread of COVID-19 have been made by Atkeson (2020) with Wang et al.'s (2020) model. In that model, the current transition from the group of exposed people to the group of infected people is linear in the current number of infected people and also not (directly) dependent on the number of recovered people. These estimates ignore the possibility of herd-immunity and its moderating effect on the rate of spread of the disease. The slow mutation of the novel coronavirus suggests that immunity gained by recovered people, and also through infusion of plasma from recovered people to susceptible people, may last over a significantly long period and, consequently, highlights the possible important role of herd-immunity in curving the spread of COVID-19 and other future viral epidemics.

Insight on non-linearly spreading epidemics and their control may be gained from Levy et al's (2006) analysis and control of a logistically diffused use of light drugs. The said paper proposes that, due to high full costs of control and the convex-concave nature of the diffusion of drug-use within the population, beyond a critical number of users it is no longer optimal to eliminate drug-use but to reduce the control effort gradually and allow convergence to a steady state that accommodates a larger number of users. In the case of viral epidemics, this proposition is reinforced by herd immunity, by the binding capacity constraint of the health-care system to test and treat, by irreversible downturns in economic activities that might stem from the introduction of stay-at-home and lock-up orders, and by avoidance of economic meltdown.

In large and densely populated places, the test-and-trace method is efficient in controlling the spread of a disease during the early stage. It is evident that in later stages the spread of COVID-19 outpaces the tracing. As the chains of infection from different sources converge to one another, testing-and-tracing becomes more and more difficult and, consequently, expensive and inefficient. Randomised testing is cheaper. It also provides a more accurate assessment of the prevalence of the epidemic.

Our analysis of the economically sustainable epidemic control is structured as follows. The current numbers of fatalities and recoveries and, consequently, the cumulative number of people with acquired immunity depend on the number of diagnostic tests. A notion and a formulation of the economically sustainable number of diagnostic tests are introduced in section 2.

Our formulation of the epidemic progression expands the logistic growth function, which was developed by Verhulst (1838) and firstly applied to demography and epidemiology by Pearl and Reed (1920), to include herd-immunity. The logistic spread of the epidemic with herd-immunity is formulated in section 3 for any number of diagnostic tests and then presented for the economically sustainable largest one. The epidemic spread is indirectly affected by the medical industry's capacity to treat infected people. This capacity and, subsequently, the medical care constraint are also formulated in section 3, under the assumption that the medical capacity is depreciated in a rate that increases with the exposure of staff to hospitalised people.

The full (pecuniary and non-pecuniary) cost of the epidemic includes the grief of the surviving members of the community for the lost lives, foregone output and spending on testing and preventative control measures, on hospitalisation of the infected people and people with underlying health conditions and on investment in the medical industry's capacity. These components of the epidemic's full cost are formulated in section 4. Community-wellbeing is derived from the grief-adjusted net-income obtained by subtracting the full cost from the epidemic-free income. Section 4 presents a general community-wellbeing-maximising intertemporal decision problem on the number of people to be tested, on the fraction of the people tested positive to be hospitalized and on the investment in the medical industry's capacity. The corresponding optimal control problem is presented and subsequently solved with some simplifying assumptions.

Due to the possibility of open borders, mutation, imperfect protection provided by vaccines and incomplete immunity acquired by recovered people, epidemics might prevail indefinitely. The steady-state number of infected people in an indefinitely prevailing epidemic is derived in section 5. The steady state of the epidemic and its comparative statics are simulated in section 6 for three mutually exclusive hypothetical degrees of immunity acquired by recovered people: low, medium and high.

## 2. Economically sustainable number of diagnostic tests

Viral epidemics, such as COVID-19, breakout unnoticedly. It takes a while for public planners to realise the existence of an epidemic and devise a plan for its expected remaining duration, *T*—the expected time without a universally available vaccine. We take each unit of time, *t*, in the interval $(0, T)$ as comprising the incubation and recovery, or dying, period (about five and fourteen days, respectively, in the case of COVID-19). For tractability, we construct our dynamic modelling of the epidemic and its control with continuous time and, technically, condense the period from incubation to recovery, or dying, to an instance.

A major lesson from COVID-19 is that there is a high-risk of morbidity and mortality for people (elderly ones, most notably) with one, or more, underlying health conditions such as diabetes, lung disease, cancer, immunodeficiency, heart disease, hypertension, asthma, kidney disease and GI/liver disease (Worldometer, 2020). Hence, it is paramount to test such people as soon as an acute respiratory disease becomes noticeable, hospitalise the ones testing positive and quarantine the rest. We therefore assume that the $Z_0$ members of the group of people with underlying health conditions who survived to $t = 0$ are safely kept in isolation until *T*. Taking these members of the community to be adequately sheltered from the epidemic and assuming that they are provided with sufficient medical care for managing their chronic conditions, we focus on the spread and control of the disease within the group of the less vulnerable members of the community only. Yet, the cost of isolation for the $Z_0$ people with underlying health conditions is taken into account in the form of lost production of goods and services—mainly tangible ones for those in the labour force and less tangible ones (derived from hobbies and participation in family, social, spiritual and recreational activities) for retirees and elderly.

Epidemics cannot be eliminated quickly. There is also a possibility of a second and third wave. Expansion of a long blanket-quarantine to the group of people that do not have underlying health conditions is not economically and socially sustainable. A short while after imposing a blanket-quarantine the hardship for non-affluent people from a loss of income, employment and freedom is likely to outweigh the health gains. Sporadic violations might escalate to demonstrations and riots. Hence, an economically and socially viable long-term method of control should be based on screening by testing and quarantining only those testing positive.

Consistent with some evidence from COVID-19, we conservatively assume that people without underlying health conditions prefer work to quarantine as long as they do not feel very sick. Worried about the adverse effects of quarantine on their income and employment, they refrain from voluntary testing. They regard themselves sufficiently fit and entitled to work and perceive their personal contribution to the spread of the disease to be negligible. We assume that the public planner is aware of the said private disincentive to be tested and hence authorizes proactive testing of the people without underlying health conditions. Yet, due to limited resources, the number of people tested at any *t* is smaller than the number of such people.

Let,
$N(t)$ denote the number of residents in a place under consideration at *t*;
$S(t)$, the number of residents carrying the epidemic virus (some might be asymptomatic) at *t*;
$q(t)$, the number of residents tested at *t*;
$i(t) \in (0,1)$, the proportion of the residents testing positive in $q$ (the confirmed infection rate) at *t*, ($i(t) > S(t)/N(t)$ when residents showing symptoms of the disease receive priority in testing), $\lim_{q(t)\to N(t)} i(t) = S(t)/N(t)$;
$y$, the output (including the production of less tangible goods and services) of neither a hospitalised nor isolated (quarantined is used interchangeably) resident at *t*, equal to all (in nominal terms) and time-invariant (for simplicity);
$\varepsilon$, the portion of personal output lost due to being quarantined, or hospitalised, at *t*, $0 \le \varepsilon \le 1$, time-invariant (for simplicity); and
$y_{\min}$, the minimum per capita income required for avoiding widespread protests, maintaining social order and preventing economic collapse at *t*, time-invariant (for simplicity).

With the said notations and definitions, the number of residents testing positive at *t* is $i(t)q(t)$. Some of these residents are hospitalised and the rest are quarantined. Recalling that the $Z_0$ residents with underlying health conditions are also quarantined, the loss of output in the place under consideration at $t$ is $[Z_0 + i(t)q(t)]\varepsilon y$. In order to avoid economic collapse, the per capita income of the place under consideration must not fall below $y_{\min}$ at any *t*:

$$\frac{\{N(t) - [Z_0 + i(t)q(t)]\varepsilon\} y}{N(t)} \ge y_{\min} . \tag{1}$$

By rearranging terms, the economically sustainable current number of diagnostic tests should satisfy the following inequality:

$$q(t) \leq \frac{[1-(y_{\min}/y)]N(t)-\varepsilon Z_0}{\varepsilon i(t)}. \tag{2}$$

The economically sustainable largest number of diagnostic tests is:

$$q_{\max}(t) = \frac{[1-(y_{\min}/y)]N(t)-\varepsilon Z_0}{\varepsilon i(t)}. \tag{3}$$

This number of diagnostic tests increases with the population size proportionally to the distance of the place from economic collapse, $1-(y_{\min}/y)$, and decreases with the fraction of output lost from quarantining a resident, $\varepsilon$, proportionally to the confirmed infection rate, *i*, and the number of residents with pre-existing health problems, $Z_0$.

**3. Epidemic progression, population size and medical industry's capacity**

Due to limited hospital capacity, it is possible that only a fraction of the people that test positive are admitted. The rest are quarantined. To formulate the spread of the epidemic we let:

$h(t) \in (0,1)$ denote the proportion of hospitalised residents and $[1-h(t)]$ the proportion of quarantined residents in the group of people that tested positive at *t*;
$m \in (0,1)$, a standard, time-invariant, level of medical care given to any hospitalised person infected by the epidemic virus at *t*;
$r$ the recovery rate of non-tested and, consequently, non-quarantined or non-hospitalised infected residents at *t*, time-invariant (for simplicity);
$1-r$, the mortality rate of non-hospitalised infected residents at *t*;
$r+\varphi m$, the recovery rate of hospitalised people at *t*, with $0<\varphi<1$ indicating the effectiveness of the medical treatment and $r+\varphi m \leq 1$;
$1-r-\varphi m$, the mortality rate of hospitalised residents at *t*;
$o(t)$, the current number of recoveries from the disease at *t*;
$p(t)$, the number of deaths caused by the disease at *t*;
$R(t)$, the cumulative number of recoveries from the disease at *t*; and
$\beta \in (0,1)$, the degree of immunity against the virus acquired by a recovered person, where $\beta = 0$ represents no acquired immunity (recovered people have no antibody response to the virus and hence remain fully susceptible) and $\beta = 1$ represents complete immunity.

We further let the number of residents contracting the disease at *t* be generated by a logistic process of diffusion with an intrinsic rate of person-to-person infection $\alpha > 0$ that reflects local conditions (population density, sanitary standards, climate and openness). In our specification of the logistic diffusion, the endogenously determined number of susceptible residents is affected by the immunity acquired by recovered people, $0 \leq \beta \leq 1$. Consequently, the progression of the epidemic is depicted as:

$$\frac{dS(t)}{dt} = \alpha S(t)\left[1 - \frac{S(t)}{N(t) - Z_0 - \beta R(t)}\right] - o(t) - p(t) \,. \tag{4}$$

The third term on the right-hand side of this equation is the number of hospitalised residents, quarantined residents and non-tested infected residents, who die at *t*:

$$\begin{aligned} p(t) &= (1 - r - \varphi m)h(t)i(t)q(t) + (1 - r)[1 - h(t)]i(t)q(t) + (1 - r)[S(t) - i(t)q(t)] \\ &= (1 - r)S(t) - \varphi m h(t)i(t)q(t) \end{aligned} \quad .(5)$$

The second term on the right-hand side of equation (4) is the number of residents who were infected by the disease and recovered (in hospitals, in quarantine and outside of quarantine) at *t*:

$$\begin{aligned} o(t) &= (r + \varphi m)h(t)i(t)q(t) + r[1 - h(t)]i(t)q(t) + r[S(t) - i(t)q(t)] \\ &= rS(t) + \varphi m h(t)i(t)q(t) \end{aligned} \quad .(6)$$

The first term on the right-hand-side of equation (4) is the number of residents that contract the disease at *t*, with $R(t)$ indicating the *cumulative* number of recovered residents by *t*:

$$R(t) \equiv \int_0^t o(\tau)\,d\tau = \int_0^t [rS(\tau) + \varphi m h(\tau)i(\tau)q(\tau)]\,d\tau \,. \tag{7}$$

The product of $\beta$ and $R(t)$ is the herd-immunity at *t*. Substracting $\beta R(t)$ and $Z_0$ from $N(t)$ yields the number of susciptible residents at *t*.

By substituting the right-hand sides of equations (5), (6) and (7) into equation (4) for $p(t)$, $o(t)$ and $R(t)$, respectively, and collecting terms, the epidemic progression equation is:

$$\frac{dS(t)}{dt} = \alpha S(t)\left[1 - \frac{S(t)}{N(t) - Z_0 - \beta \int_0^t [rS(\tau) + \varphi m h(\tau)i(\tau)q(\tau)]\,d\tau}\right] - S(t) \,. \tag{8}$$

At the same time, the population size changes as follows:

$$\frac{dN(t)}{dt} = \eta N(t) - p(t) = \eta N(t) - (1 - r)S(t) + \varphi m h(t)i(t)q(t) \tag{9}$$

with $\eta$ denoting the population's natural growth-rate and taken to be time-invariant, for simplicity.

Recalling equation (3), the herd-immunity-moderated diffusion of the epidemic under maximum economically sustainable number of diagnostic tests is:

$$\frac{dS(t)}{dt} = \alpha S(t)\left[1 - \frac{S(t)}{N(t) - Z_0 - \beta\int_0^t \{rS(\tau) + \varphi mh(\tau)\{[1-(y_{\min}/y)][N(\tau)/\varepsilon] - Z_0\}\}d\tau}\right] - S(t) \tag{10}$$

while the population-size changes according to:

$$\frac{dN(t)}{dt} = \eta N(t) - (1-r)S(t) + \varphi mh(t)\{[1-(y_{\min}/y)][N(t)/\varepsilon] - Z_0\}. \tag{11}$$

The evolution of $S(t)$ and $N(t)$ are indirectly affected by the medical industry's capacity. We take the medical industry's capacity of the place under consideration, $M(t)$, to be represented by a number of equal-size hospitalisation units endowed with fixed proportions of equal-quality medical inputs (structure, equipment, materials and staff) at *t*. We also let the medical care given at any *t* to each quarantined resident with underlying health conditions be $m_z(t)$ and at least as large as the minimum care ensuring their survival ($m_z^{\min}$). Consequently, the medical-support constraint is portrayed as:

$$mh(t)i(t)q(t) + m_z(t)Z_0 = M(t), \quad m_z(t) \geq m_z^{\min}. \tag{12}$$

With the medical capacity fully utilised, the intensity of medical care given to each resident with underlying health conditions is:

$$m_z(t) = [M(t) - mh(t)i(t)q(t)]/Z_0. \tag{13}$$

We take any investment in the medical capacity of the place under consideration to be reversible and completed immediately. The underlying rationale is that the public planner can quickly and reversibly transform public assets and resources (army and emergency structures, equipment, material and staff) and private assets and resources (hotels, sport arenas, cruise-ships and retired and unemployed nurses, paramedics and doctors) into hospitals. With $I(t)$ being a control variable and denoting the investment in medical capacity at *t* and with $0 < \delta_1 + \delta_2 h(t)i(t)q(t) < 1$ and $C_5 = c_5 I(t)^2$ displaying an increasing rate of depreciation (linear, for simplicity) in the number of people hospitalised at *t*; due to infection of staff by patients; the medical-capacity's motion equation is:

$$\frac{dM(t)}{dt} = I(t) - [\delta_1 + \delta_2 h(t)i(t)q(t)]M(t). \tag{14}$$

## 4. Full cost, community-wellbeing and control

### *4.1 Non-pecuniary and pecuniary costs*

The full cost of epidemics comprises non-pecuniary and pecuniary components. Evidently, of a major concern is the loss of life and the associated grief of the surviving community members. We assume that the grief of the surviving community members is proportional to the lost years of life. With $v \geq 0$ denoting the value of a life-year lost for the surviving members of the community, $L_0$ the average life expectancy of the population on the eve of the epidemic and $A_0$ the average age of the population on the eve of the epidemic, and with the number of deaths caused by the disease at *t* being given by equation (5), the grief of the surviving community members generated by the loss of lives at *t* is:

$$g(t) = v(L_0 - A_0)p(t) = v(L_0 - A_0)[(1-r)S(t) - \varphi mh(t)i(t)q(t)]. \tag{15}$$

There is also a loss of output at *t* from quarantining residents with pre-existing health conditions and from quarantining, or hospitalising, residents with no underlying health conditions that tested positive: $[Z_0 + i(t)q(t)]\varepsilon y$.

There are other costs associated with testing, quarantining and hospitalising residents and from investing in medical capacity. We let,

$C_1(q(t))$ denote the cost of testing $q(t)$ residents at *t*, $C_1' > 0, C_1'' \geq 0$;
$C_2((1-h(t))i(t)q(t))$, the cost of facilitating and enforcing a quarantine on the non-hospitalised residents testing positive at *t*, $C_2' > 0, C_2'' \geq 0$;
$C_3(m)$, the cost of hospitalising a person testing positive at *t*, $C_3' > 0, C_3'' \geq 0$;
$C_4(m_z(t))$, the cost of medical care given to each person with underlying conditions at *t*, $C_4' > 0, C_4'' \geq 0$; and
$C_5(I(t))$, the cost of investment in medical capacity at *t*, $C_5' > 0, C_5'' \geq 0$.

By substituting the right-hand side of the medical capacity constraint (13) into $C_4$ for $m_z(t)$ and summing up, the full cost of the epidemic for the place under consideration at any $t \in (0,T)$ is:

$$\begin{aligned} c(t) = {} & v(L_0 - A_0)[(1-r)S(t) - \varphi mh(t)i(t)q(t)] \\ & + [Z_0 + i(t)q(t)]\varepsilon y + C_1(q(t)) + C_2((1-h(t))i(t)q(t)) \\ & + h(t)i(t)q(t)C_3(m) + Z_0 C_4([M(t) - mh(t)i(t)q(t)]/Z_0) + C_5(I(t)) \end{aligned} \quad .(16)$$

*4.2 Community-wellbeing and control*

Personal (instantaneous) utility is assumed to be derived from the grief-adjusted net personal income. For simplicity, the utility functions of the $N(t)$ residents of the place under consideration are assumed to be identical. The grief-adjusted net-income of each of the $N(t)$ residents at *t* is taken to be the equally shared (for simplicity) difference between the epidemic-free instantaneous aggregate income, $N(t)y$, and the instantaneous full cost of the epidemic, $c(t)$, which includes the loss of income caused by the epidemic. Hence, the utility of each resident at *t* is $u((N(t)y - c(t))/N(t))$ and, by summation, the community-wellbeing at *t* is: $N(t)u((N(t)y - c(t))/N(t))$. Clearly, a blanket quarantine is optimal only when the value of a lost year of life for the surviving members of the community ($v$) is sufficiently high for the grief to dominate the said expression of the community-wellbeing.

In the absence of vaccines until the expected date *T*, the public planner may choose the joint trajectories of the number of diagnostic tests $(q)$, rate of hospitalisation of people testing positive $(h)$ and investment in medical capacity $(I)$ so as to maximise the sum of the discounted instantaneous levels of the community-wellbeing during the period $(0,T)$. With $\rho$ representing the rate of time-preference of the residents, the public planner's optimal control problem is postulated to be:

$$\max_{\{q(t),h(t),I(t)\}} \int_0^T e^{-\rho t}[N(t)u((N(t)y-c(t))/N(t))]dt \tag{17}$$

subject to the epidemic progression equation (8), the population's net-growth equation (9), the motion-equation of the medical capacity (14), the medical support constraint (12) and the collapse-avoidance condition (1), where $c(t)$ is given by equation (16).

Due to computational complexity, the aforesaid optimal control problem cannot be solved, and the properties of the optimal trajectories cannot be derived. A special, more tractable, case is considered.

*4.3 Control with fully protected medical staff and economically viable maximum tests*

In the present case, the medical staff is fully protected from infection by patients and hence the depreciation of the medical industry's capacity does not depend on the number of people hospitalised:

i. $\delta_2 = 0$ (fully protected staff) and $\delta \equiv \delta_1$ (the medical capacity's depreciation rate).

In addition, the number of control variables is reduced to two, *h* and *I*, and so also the number of state variables to *S* and *M*, by assuming that the natural growth of the population is offset by the epidemic fatalities and that, in order to gain the best possible assessment of the prevalence of the disease, the public planner makes the maximum economically viable number of diagnostic tests at random, which implies that the average income at every *t* is equal to the minimum level required for economic sustainability. We further assume that by doing so, the assessment of the spread of the epidemic is correct. Formally,

ii. $dN(t)/dt = 0$ (zero net-growth of the population), $N(t) = N_0$;
iii. $q(t) = q_{\max}(t)$ (the economically viable largest number of diagnostic tests) and, consequently, $y(t) = y_{\min}$; and
iv. $i(t) = S(t)/N(t)$ (representative testing).

For tractability, the following specifications of the cost functions are considered:

v. $C_1 = c_1 q(t)$, $C_2 = c_2(1-h(t))i(t)q(t)$, $C_3 = c_3 m$, $C_4 = c_4[M(t) - mh(t)i(t)q(t)]/Z_0$ and $C_5 = c_5 I(t)^2$, with $c_1$ indicating the cost of a diagnostic test, $c_2$ the quarantine's enforcement cost per person, $c_3$ the cost of hospitalising a person with the best possible treatment ($m = 1$), $c_4$ the cost of medical care given to a person with pre-existing conditions, and $c_5 = C_5(I = 1)/2$.

In the present setting,

$$i(t) = S(t) / N_0 \tag{18}$$

and, recalling equation (3),

$$q(t) = \frac{[1-(y_{\min} / y)]N_0 - \varepsilon Z_0}{\varepsilon[S(t) / N_0]} > 0 \text{ as long as } [1-(y_{\min} / y)]N_0 - \varepsilon Z_0 > 0 . \tag{19}$$

By substituting equations (18) and (19) and condition (v) into equation (16) and collecting terms, the full cost of the epidemic is:

$$\begin{aligned}
c(t) = {} & Z_0\varepsilon y + (c_2 + \varepsilon y)\big[[1-(y_{\min} / y)](N_0 / \varepsilon) - Z_0\big] \\
& + \nu(L_0 - A_0)(1-r)S(t) + c_4 M(t) + c_5 I(t)^2 \\
& + c_1\{[1-(y_{\min} / y)](N_0^{\,2} / \varepsilon) - Z_0 N_0\} / S(t) \\
& + \{[c_3 - c_4 - \nu(L_0 - A_0)\varphi]m - c_2\}\big[[1-(y_{\min} / y)](N_0 / \varepsilon) - Z_0\big]h(t)
\end{aligned} \tag{.20}$$

Assuming, for simplicity, that the residents' instantaneous utilities are proportional to their grief-adjusted net-income, the planner's optimal control problem is reduced to choosing the trajectories of the hospitalisation rate ($h$) of infected residents and investment in medical capacity ($I$) so as to maximise the sum of the discounted instantaneous differences between the epidemic-free aggregate income ($N_0 y$) and the full cost ($c(t)$) of the epidemic during the planning horizon (0,$T$), subject to the state equations (8) and (14), where $q(t)$ is given by (19) and $i(t)$ by (18) and $\delta_2 = 0$ in the state equations. As detailed in Appendix A, the solution to this optimal control problem leads to the following equality:

$$\begin{aligned}
& \rho\{[c_3 - c_4 - \nu(L_0 - A_0)\varphi]m - c_2\}\big[[1-(y_{\min} / y)](N_0 / \varepsilon) - Z_0\big] \\
& + (\alpha - 1)\{[1-(y_{\min} / y)](N_0 / \varepsilon) - Z_0\}\{[c_4 + \nu(L_0 - A_0)\varphi - c_3]m + c_2\} \\
& - 2\{[c_4 + \nu(L_0 - A_0)\varphi - c_3]m + c_2\}\{[1-(y_{\min} / y)](N_0 / \varepsilon) - Z_0\}\{[dS(t) / dt] / S(t)\} \\
& - 2\{[c_4 + \nu(L_0 - A_0)\varphi - c_3]m + c_2\}\frac{\{[1-(y_{\min} / y)](N_0 / \varepsilon) - Z_0\}\beta[dR(t) / dt]}{[N_0 - Z_0 - \beta R(t)]} \\
& + \frac{\alpha\beta\varphi m\{[1-(y_{\min} / y)](N_0 / \varepsilon) - Z_0\}}{[N_0 - Z_0 - \beta R(t)]^2} c_1\{[1-(y_{\min} / y)](N_0^{\,2} / \varepsilon) - Z_0 N_0\} \\
& + \frac{\alpha\beta\{[1-(y_{\min} / y)](N_0 / \varepsilon) - Z_0\}\{\nu(L_0 - A_0)\varphi m(1-2r) - [(c_4 - c_3)m + c_2]r\}}{[N_0 - Z_0 - \beta R(t)]^2} S(t)^2 \\
& - \frac{2\alpha\{[1-(y_{\min} / y)](N_0 / \varepsilon) - Z_0\}\{[c_4 + \nu(L_0 - A_0)\varphi - c_3]m + c_2\}}{[N_0 - Z_0 - \beta R(t)]} S(t) = 0
\end{aligned} \tag{.21}$$

Equation (21) is used in the following sections for assessing the steady-state prevalence of an indefinite epidemic.

## 5. Indefinite epidemic and its steady state

*5.1 Indefinite epidemic*

In the case of viral diseases, vaccines are not fully effective. For instance, it is estimated that the influenza vaccine was effective against the 2019-20 seasonal influenza A and B for forty-five percent of the vaccinated people in the United States (Dawood et al., 2020). In view of the possibility of mutation and the hazardous nature of an acute respiratory viral disease, such as COVID-19, we consider, henceforth, a scenario where the public planner does not expect vaccines to terminate the epidemic and also believes that, due to the non-absolute nature of the immunity acquired by recovered residents, eradiction of the epidemic virus is impossible. This impossibility is reenforced by unsealed borders. In such a scenario, the upper bound of the public planner's time-horizon is infinite and, consequently, a steady state may be reached. Clearly, as $t \to \infty$ all the $N_0 - Z_0$ people, who were not quarantined at $t = 0$, have become infected at one point of time or another. Recalling, further, that the cases of death are assumed to be offset by the population's natural growth at every *t*, the number of recovered people in steady state is $R_{ss} = N_0 - Z_0$. Hence, the susceptible population in steady state under non-absolute immunity acquired by recovered people $(0 < \beta < 1)$ is equal to: $(N_0 - Z_0) - \beta R_{ss} = (1-\beta)(N_0 - Z_0)$.

*5.2 Steady state of the indefinite epidemic*

By evaluating equation (21) in steady state (that is, $dS/dt = 0$ and $dR/dt = 0$) with the assumption that the immunity acquired by recovered people is not absolute, the following second order polynomial is obtained

$$\pi \equiv AS_{ss}^{\ 2} - BS_{ss} + C = 0 \tag{22}$$

and the possible steady-state levels of the prevalence of the epidemic are:

$$S_{ss1,2} = \frac{B \pm \sqrt{B^2 - 4AC}}{2A} \tag{23}$$

where,

$$A \equiv \frac{\alpha\beta\{[1-(y_{\min}/y)](N_0/\varepsilon) - Z_0\}\{v(L_0 - A_0)\varphi m(1-2r) - [(c_4 - c_3)m + c_2]r\}}{[(1-\beta)(N_0 - Z_0)]^2} \tag{24}$$

$$B \equiv \frac{2\alpha\{[1-(y_{\min}/y)](N_0/\varepsilon) - Z_0\}\{[c_4 + v(L_0 - A_0)\varphi - c_3]m + c_2\}}{[(1-\beta)(N_0 - Z_0)]} \tag{25}$$

and

$$\begin{aligned} C \equiv\ & \{[c_3 - c_4 - v(\rho L_0 - A_0)\varphi]m - c_2\}\{[1-(y_{\min}/y)](N_0/\varepsilon) - Z_0\} \\ & + \{[1-(y_{\min}/y)](N_0/\varepsilon) - Z_0\}\{[c_4 + v(L_0 - A_0)\varphi - c_3]m + c_2\}(\alpha - 1) \\ & + \frac{\alpha\beta\varphi m c_1\{[1-(y_{\min}/y)](N_0/\varepsilon) - Z_0\}\{[1-(y_{\min}/y)](N_0^{\ 2}/\varepsilon) - Z_0 N_0\}}{[(1-\beta)(N_0 - Z_0)]^2} \end{aligned} \quad .(26)$$

## 6. Comparative statics: some numerical simulations

For assessing the effects of the model parameters on the steady-state levels of the prevalence of the epidemic we used mathematical software to totally differentiate equation (22) with respect to each of the model parameters (e.g., $dS_{ss} / dr = -(\partial\pi / \partial r) / (\partial\pi / \partial S_{ss})$). Due to the complexity of the resultant expressions, claims on the directions of the effects of the model parameters on $S_{ss}$ cannot be made. To obtain a glimpse into the possible directions, the said expressions are calculated by mathematical software with a heuristicly assumed set of parameter values: a city with an initial popoulation of 1,500,000 people ($N_0 = 1{,}500{,}000$) enjoying an average income of \$40,000 per annum on the eve of the epidemic, and where hospitalised and quarantined people cannot work ($\varepsilon = 1$) and social upheaval and, subsequently, economic collapse are inevitable when the average income drops below half the initial level ($y_{\min} / y = 0.5$). With the cost parameters adjusted to fourteen days (the recovery period from COVID-19, as a time unit), the rest of the parameter values for that place are: $r = 0.75$, $\varphi = 0.30$, $m = 0.75$ (implying a death-rate of hospitalised infected people, $1-(r+\varphi m)$, of 0.025), $\alpha = 6$ (the first sick person infects six residents), $\rho = 0.05$, $c_1 = \$20$, $c_2 = \$10$, $c_3 = \$14{,}000$ (the cost of the highest quality ($m = 1$) hospitalisation of an infected person for fourteen days), $c_4 = \$1{,}000$ (the cost of the highest quality ($m_z = 1$) medical support given to a quarantined person with underlying health conditions for fourteen days), $c_5 = 70{,}000$ (assuming that a unit of investment in medical capacity accommodates ten patients for two weeks in a demountable structure with ten beds, a full-time nurse and a part-time doctor, $c_5 = C_5(I = 1) / 2 = 10 \times 14{,}000 / 2 = 70{,}000$), $Z_0 = 300{,}000$ people (twenty percent of the population of the city), $L_0 = 85$ years, $A_0 = 45$ years, and $v = \$80{,}000$ (the value of a lost year of life is twice the initial average annual income). Three degrees of acquired immunity ($\beta$) are alternatively considered: 0.25 (low), 0.5 (medium) and 0.75 (high).

With the aforesaid set of parameter values, the only valid steady-state level of the number of infected residents is $S_{ss} = [B + (B^2 - 4AC)^{0.5}] / 2A$. This steady-state number and the effects of the model parameters on it are summarised in Table 1.

Insert Table 1 here

As can be seen from the first two rows of Table 1, the higher the degree of the acquired immunity the more effective the herd-immunity in moderating the spread of the epidemic and, consequently, the lower the steady-state number of infected people: about 24, 15 or 8 per cent of the population when the degree of the acquired immunity is 0.25, 0.5 or 0.75, respectively. The rest of the rows of Table 1 suggest that, for any degree of acquired immunity, the steady-state number of infected people:

i. increases with the proximity of the economically sustainable minimum-income to the epidemic-free per capita income ($y_{\min} / y$), due to the adverse effect of that proximity on the maximum number of economically sustainable diagnostic tests, and the increment is largest when the degree of the acquired immunity is medium;
ii. decreases with the effectiveness of the medical treatment ($\varphi$), and the decline is largest when the degree of the acquired immunity is medium;

iii. decreases with the rate of recovery without hospitalisation $(r)$, and the decline is largest when the degree of the acquired immunity is medium;

iv. increases with the intrinsic diffusion rate of the epidemic $(\alpha)$, and the increment is largest when the degree of the acquired immunity is low;

v. decreases with the degree of acquired immunity $(\beta)$, and the decline is largest when the degree of the acquired immunity is low;

vi. decreases with the fraction of output lost by hospitalisation or quarantine $(\varepsilon)$, and the decline is largest when the degree of the acquired immunity is high;

vii. decreases with the rate of time preference $(\rho)$, and the decline is largest when the degree of the acquired immunity is low;

viii. increases with the cost of a diagnostic test $(c_1)$, and the increment is largest when the degree of the acquired immunity is high;

ix. decreases with the cost of ensuring that a person is quaranined, and the decline is largest when the degree of the acquired immunity is medium;

x. increases with the marginal cost of the medical treatment given to infected residents $(c_3)$, equally dereases with the marginal cost of the medical treatment given to residents with underlying health conditions, and the said changes are largest when the degree of the acquired immunity is medium;

xi. is insignificantly affected by the coefficient of the medical capacity investment cost function (due, technically, to its relatively very large size);

xii. increases with the initial number of residents $(N_0)$, equally decreases with the number of residents with underlying health conditions $(Z_0)$, as they are taken to be quarantined, and the said changes are largest when the degree of the acquired immunity is low;

xiii. decreases with the value assigned by surviving members of the community to a lost year of life $(v)$, and the decline is largest when the degree of the acquired immunity is medium; and

xiv. decreases with the average numbr of remaining years of life $(L_0 - A_0)$ on the eve of the epidemic, and the decline is largest when the degree of the acquired immunity is medium.

## 7. Conclusion

In an ever more congested and integrated world, the spread of hazardous viruses is inevitable and becomes easier and faster and hence more difficult and costly to control. A component of the full cost of epidemics is the grief generated by the loss of lives. For communities that assign a dominant weight to lost years of life in the formation of their sense of wellbeing, a blanket quarantine during the entire presence of an epidemic is optimal. In the absence of perfect vaccines and/or remedies and in view of the economic costs and the possibility of collapse, communities may have to learn to live with epidemics. As evident from COVID-19, the vast majority of the deceased people in acute respiratory epidemics have underlying health conditions. An early isolation of such members of the community can drastically reduce the loss of lives. It is also evident from COVID-19 that a complete lockout of the rest of the population; for minimising infections and, in turn, the probability of clinics and hospitals becoming overwhelmed; brings about economic hardship.

The purpose of this paper was to formulate an economically sustainable control of epidemics within the less vulnerable part of the population. Herd-immunity, capacity of the health-care industry, value of life and grieving for lost lives, costs of precautionary measures and avoidance of economic collapse were taken into account in modelling the spread and control of epidemics such as COVID-19. Our formulation of the full cost of epidemics included grief, foregone output and spending on testing and preventative control measures, hospitals and hospitalisation. The full cost was subtracted from the epidemic-free income to obtain the grief-adjusted net income and, subsequently, wellbeing of the community. An intertemporal wellbeing-maximising decision on tests, hospitalisation, quarantine, intensity of medical care to people testing positive and investment in the medical industry's capacity was outlined and then solved with some simplifying assumptions. A steady state and its comparative statics were derived and simulated for the case of indefinite epidemics. The simulations highlighted the role of the degree of immunity acquired by recovered people in determining the steady-state prevalence of the epidemic and the effects of the model parameters on that prevalence. As can be intuitively expected, it was found that the steady-state prevalence of the epidemic is highest when the degree of immunity acquired by recovered people is low. Yet, it was found that the effects of many of the model parameters on the said steady state are most pronounced when the degree of the immunity acquired by the recovered people is medium.

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Table 1. Steady-state numbers and rates of infected people and parameter effects

| Degree of acquired immunity | Low ($\beta = 0.25$) | Medium ($\beta = 0.5$) | High ($\beta = 0.75$) |
|---|---|---|---|
| $S_{ss}$ | 362262 | 236052 | 115528 |
| $S_{ss} / N$ | 0.241508 | 0.157368 | 0.077019 |
| $dS_{ss} / d(y_{\min} / y)$ | 14458.9 | 17292.9 | 11739.8 |
| $dS_{ss} / d\varphi$ | -401.991 | -488.802 | -338.445 |
| $dS_{ss} / dr$ | -35206.6 | -42908.2 | -29607.4 |
| $dS_{ss} / d\alpha$ | 12504.5 | 7976.33 | 3829.56 |
| $dS_{ss} / d\beta$ | -517266 | -492968 | -471670 |
| $dS_{ss} / d\varepsilon$ | -1.12431 | -3.50437 | -7.85518 |
| $dS_{ss} / d\rho$ | -71454.1 | -45578.6 | -21883.2 |
| $dS_{ss} / dc_1$ | 0.0282945 | 0.0812173 | 0.233964 |
| $dS_{ss} / dc_2$ | -0.0123928 | -0.0151046 | -0.0104279 |
| $dS_{ss} / dc_3$ | 0.0092946 | 0.0113284 | 0.0078209 |
| $dS_{ss} / dc_4$ | -0.0092946 | -0.0113284 | -0.0078209 |
| $dS_{ss} / dc_5$ | 0 | 0 | 0 |
| $dS_{ss} / dN_0$ | 0.301885 | 0.196711 | 0.0962739 |
| $dS_{ss} / dZ_0$ | -0.301886 | -0.196712 | -0.096276 |
| $dS_{ss} / d\nu$ | -0.00151454 | -0.00185331 | -0.00132766 |
| $dS_{ss} / d(L_0 - A_0)$ | -110.333 | -131.728 | -89.3749 |

**Appendix A**

With $\lambda_1(t)$ and $\lambda_2(t)$ denoting the shadow prices of the cumulative number of infected people (*S*) and the medical capacity (*M*) at *t*, the Hamiltonian associated with the optimal control problem presented in section 4.3 is:

$$H(t) = e^{-rt}[N_0 y - c(t)]$$
$$+\lambda_1(t)\left\{\alpha S(t)\left[1-\frac{S(t)}{N_0 - Z_0 - \beta\int_0^t [rS(\tau)+\varphi m\{[1-(y_{\min}/y)](N_0/\varepsilon)-Z_0\}h(\tau)]d\tau}\right]-S(t)\right\}. \quad (A1)$$
$$+\lambda_2(t)[I(t)-\delta M(t)]$$

where $c(t)$ is given by (20). Note that $c(t)$ is linear in $h(t)$ and, recalling that $[1-(y_{\min}/y)](N_0/\varepsilon)-Z_0>0$, the epidemic-progression function is concave in $h(t)$. Consequently, the Hamiltonian is concave in the control variable $h(t)$. As $c(t)$ is convex in $I(t)$ and the change in the medical industry's capacity is linear in $I(t)$, the Hamiltonian is also concave in this second control variable. The set of the necessary conditions includes the state-variable equations and the following adjoint equations and optimality conditions:

$$d(\lambda_1(t))/dt = -\frac{\partial H(t)}{\partial S(t)} = -e^{-\rho t}c_1\{[1-(y_{\min}/y)](N_0{}^2/\varepsilon)-Z_0N_0\}/S(t)^2$$
$$+e^{-\rho t}v(L_0-A_0)(1-r)-\lambda_1(t)\left[(\alpha-1)-\frac{2\alpha S(t)[N_0-Z_0-\beta R(t)]+\alpha\beta r S(t)^2}{[N_0-Z_0-\beta R(t)]^2}\right] \quad (A2)$$

$$d(\lambda_2(t))/dt = -\frac{\partial H(t)}{\partial M(t)} = e^{-\rho t}c_4 + \lambda_2(t)\delta \quad (A3)$$

$$\frac{\partial H(t)}{\partial h(t)} = -e^{-\rho t}\{[c_3-c_4-v(L_0-A_0)\varphi]m-c_2\}\left[[1-(y_{\min}/y)](N_0/\varepsilon)-Z_0\right]$$
$$-\lambda_1(t)\frac{\alpha\beta\varphi m\{[1-(y_{\min}/y)](N_0/\varepsilon)-Z_0\}S(t)^2}{[N_0-Z_0-\beta R(t)]^2}=0 \quad (A4)$$

$$\frac{\partial H(t)}{\partial I(t)} = -e^{-\rho t}2c_5 I(t)+\lambda_2(t)=0\,. \quad (A5)$$

From (A4), the shadow price of the stock of infected people is:

$$\lambda_1(t)=\frac{e^{-\rho t}\{[c_4+v(L_0-A_0)\varphi-c_3]m+c_2\}[N_0-Z_0-\beta R(t)]^2}{\alpha\beta\varphi m S(t)^2}\,. \quad (A6)$$

From (A5), the shadow price of the medical industry's capacity is:

$$\lambda_2(t) = e^{-\rho t} 2c_5 I(t)\,. \tag{A7}$$

By differentiating the optimality conditions (A4) and (A5) for $h$ and $I$ with respect to $t$ and substituting (A2) and (A6) for $d\lambda_1 / dt$ and $\lambda_1$ and (A3) and (A7) for $d\lambda_2 / dt$ and $\lambda_2$ into the resultant singular control equations, multiplying by $e^{\rho t}$ and collecting terms, the following equalities are obtained:

$$\begin{aligned}
&\rho\{[c_3 - c_4 - \nu(L_0 - A_0)\varphi]m - c_2\}\big[[1-(y_{\min}/y)](N_0/\varepsilon) - Z_0\big] \\
&+(\alpha-1)\{[1-(y_{\min}/y)](N_0/\varepsilon) - Z_0\}\{[c_4 + \nu(L_0 - A_0)\varphi - c_3]m + c_2\} \\
&-2\{[c_4 + \nu(L_0 - A_0)\varphi - c_3]m + c_2\}\{[1-(y_{\min}/y)](N_0/\varepsilon) - Z_0\}\{[dS(t)/dt]/S(t)\} \\
&-2\{[c_4 + \nu(L_0 - A_0)\varphi - c_3]m + c_2\}\frac{\{[1-(y_{\min}/y)](N_0/\varepsilon) - Z_0\}\beta[dR(t)/dt]}{[N_0 - Z_0 - \beta R(t)]} \\
&+\frac{\alpha\beta\varphi m\{[1-(y_{\min}/y)](N_0/\varepsilon) - Z_0\}}{[N_0 - Z_0 - \beta R(t)]^2} c_1\{[1-(y_{\min}/y)](N_0^{\,2}/\varepsilon) - Z_0 N_0\} \\
&+\frac{\alpha\beta\{[1-(y_{\min}/y)](N_0/\varepsilon) - Z_0\}\{\nu(L_0 - A_0)\varphi m(1-2r) - [(c_4 - c_3)m + c_2]r\}}{[N_0 - Z_0 - \beta R(t)]^2} S(t)^2 \\
&-\frac{2\alpha\{[1-(y_{\min}/y)](N_0/\varepsilon) - Z_0\}\{[c_4 + \nu(L_0 - A_0)\varphi - c_3]m + c_2\}}{[N_0 - Z_0 - \beta R(t)]} S(t) = 0
\end{aligned} \tag{A8}$$

and

$$dI(t)/dt - (\rho + \delta)I(t) - (c_4 / 2c_5) = 0\,. \tag{A9}$$

where,

$$R(t) = \int_0^t \{rS(\tau) + \varphi m\{[1-(y_{\min}/y)]N_0/\varepsilon - Z_0\}h(\tau)\}\,d\tau \tag{A10}$$

and

$$dR(t)/dt = rS(t) + \varphi m\{[1-(y_{\min}/y)]N_0/\varepsilon - Z_0\}h(t)\,. \tag{A11}$$